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MATHEMATICAL MODELING AND STABILITY ANALYSIS OF VISUAL LOCALIZATION ALGORITHMS UNDER BRIGHTNESS AND NOISE VARIATIONS

Abstract. Visual localization algorithms are an integral part of modern robotics and navigation systems, providing object position determination based on visual features or images. However, their effectiveness is largely dependent on external factors, such as image brightness and noise level, which directly affect landmark recognition and coordinate accuracy. **Subject of research:** analysis of the impact of image brightness and noise on the accuracy and stability of adaptive localization algorithms. **The purpose of the work** is to quantify the impact of image parameters on the robustness of various localization methods and to identify algorithms most suitable for real-time operation under unstable visual conditions. **Research methods:** A two-factor experimental design with brightness and noise level variables was applied, within which a series of localization experiments were conducted. Mathematical modeling was performed to obtain analytical dependences of the minimum, average, and maximum localization errors for four algorithms – Proximity, Centroid, Weighted Centroid, and Lateration. Based on the obtained models, a stability coefficient was introduced as an indicator of the algorithm's robustness. **Results:** the constructed regression models demonstrated high adequacy and allowed us to visualize the influence of brightness and noise on localization accuracy. It was found that the Weighted Centroid and Lateration methods provide the highest stability of operation, maintaining low error variation when changing image parameters, while the Proximity and Centroid algorithms showed greater sensitivity to noise and lighting fluctuations.

Keywords: visual localization; adaptive algorithms; *regression model;* stability coefficient; robustness; image noise; brightness.

Introduction

Visual navigation systems are actively used in autonomous robotic platforms, unmanned aerial vehicles, object tracking systems, augmented reality, and industrial vision. They allow determining the position and orientation of an object in space based on the analysis of visual landmarks, which makes them especially important for tasks where satellite navigation signals are absent or unstable. However, the key problem of such systems remains the stability of localization when external environmental factors change. The accuracy and reliability of determining coordinates are significantly affected by lighting conditions, scene contrast, sensor noise, image artifacts, as well as changes in the geometry or structure of landmarks. In real operating conditions, these factors change dynamically – for example, during the flight of a drone when moving from a lit area to a shadow, or when a mobile robot moves indoors with different lighting and glare.

Most existing algorithms provide high positioning accuracy in laboratory or controlled conditions, but lose their effectiveness when brightness changes or noise appears. This is due to the fact that traditional approaches often do not take into account nonlinear relationships between image parameters and localization error characteristics. As a result, even small variations in illumination can lead to noticeable shifts in the coordinates of certain landmarks, and the appearance of noise can lead to a decrease in the accuracy of detecting key points and correspondences between frames. Therefore, a current task is to quantitatively assess the robustness of localization algorithms and develop analytical models that allow describing the behavior of the system when external factors change. This approach provides the possibility of analytically predicting localization error, optimizing algorithm parameters, and increasing the stability of visual systems in real conditions. Special attention should be paid to creating a universal stability indicator that would allow comparing different algorithms with each other regardless of the experimental conditions. The introduction of the stability coefficient as an integral criterion makes it possible to assess not only the average accuracy, but also the variation of the error in response to changes in environmental parameters, which is important for the design of adaptive and autonomous navigation systems.

Literature analysis. Modern visual navigation systems are actively developing in the areas of autonomous transport, unmanned aerial vehicles and robotics. They are based on algorithms for image processing, landmark recognition and spatial position reproduction of an object. A comprehensive review of approaches to building intelligent transport systems is given in [1], which emphasizes the role of visual sensors in autonomous control systems.

The problem of the influence of landmark coordinate errors on the accuracy of visual localization is considered in detail in [2], where both systematic and random errors are analyzed. Modeling of visual guidance systems of unmanned aerial vehicles is described in [3], which allows us to reproduce various flight scenarios and investigate the robustness of algorithms.

A review of modern visual localization methods for autonomous navigation systems is given in [4], which classifies approaches by sensor types and level of integration. In [5], navigation algorithms for unmanned aerial vehicles are considered and the main factors of loss of accuracy are identified, including changing lighting and noise. Methods of simultaneous localization and mapping (SLAM) in variable lighting conditions are described in [6], and improvements to localization algorithms based on data filtering are given in [7]. Works [8], [9] are devoted to the construction of lightweight semantic maps and hybrid positioning based on the combination of visual and radio frequency sensors. Research [10] demonstrates the effectiveness of such approaches for underwater autonomous systems.

In [11–14], the use of deep neural networks (YOLOv5, DeepSORT, SSD) to improve the accuracy of visual landmark recognition and tracking stability is considered. In particular, it is shown that the positioning accuracy depends on the lighting parameters and the contrast level of the scene.

Algorithms for automatic landing of UAVs using computer vision are presented in [15], and methods for assessing image quality and determining target motion parameters are presented in [16, 17]. An adaptive approach to visual positioning of UAVs, which takes into account changes in lighting conditions, is described in [18].

In [19], stochastic optimization methods that can be used to improve the stability of navigation systems are considered. Image preprocessing to improve recognition quality is presented in [20], where the effect of filtering and contrasting on the efficiency of algorithms is shown.

In [21], the use of a full-factor experiment for optimizing signal processing parameters is presented, which is consistent with the methodology of this work. The study [22] confirms the importance of the stability of visual sensors in variable lighting conditions. The methods of sensor integration and full-factor analysis are discussed in detail in [23], where the influence of the choice of factors on the accuracy of the models is shown. Similar experimental approaches have been applied in engineering problems of optimizing technological processes [24, 25], which demonstrates the versatility of the chosen planning method.

The review of sources shows that, despite the active development of visual localization methods, the issues of assessing the stability of algorithms when changing brightness and image noise remain insufficiently addressed. This justifies the relevance of the research aimed at building regression models of stability and determining the zones of effective operation of visual localization algorithms.

Problem statement and research objective. The main problem considered in the study is the instability of visual localization algorithms under conditions of changes in illumination and image noise. In real operating conditions, such algorithms demonstrate a significant deterioration in the accuracy of determining coordinates when external factors fluctuate - brightness, scene contrast, or sensor noise. This instability limits the reliability of visual navigation systems in a dynamic environment and complicates their integration into autonomous robotic platforms and unmanned aerial vehicles.

Most existing approaches are focused primarily on improving the average localization accuracy, without taking into account the variation of results and the robustness of algorithms when changing image parameters. Therefore, there is a need for quantitative and analytical assessment of the impact of brightness and noise on the stability of localization algorithms.

The purpose of the study is to build and test mathematical models that describe the change in localization error depending on the brightness and noise level of the image, as well as introduce a stability coefficient that allows for a comparative assessment of the effectiveness of different algorithms. The resulting models are designed to determine the areas of stable operation and identify those localization methods that are most robust and suitable for adaptive visual navigation systems capable of operating in a changing environment.

Research methodology

The research methodology is based on a combination of experimental modeling, statistical data analysis, and regression modeling to evaluate the effectiveness and robustness of visual localization algorithms under conditions of changing image brightness and noise levels.

At the initial stage, input data is generated, which includes image parameters — brightness (b) and noise level (n). These factors determine the conditions under which localization occurs and affect the stability of the results.

Next, a preliminary numerical analysis is performed to determine the allowable factor space within which the visual localization algorithms remain operational. At this stage, extreme values of parameters that lead to loss of contrast or noise overload are analyzed, and realistic limits for changing the factors are determined.

After that, the experiment is planned - a second-order two-factor design is constructed, which allows us to study not only the separate effects of brightness and noise, but also the effect of their interaction. This approach provides complete information for further mathematical modeling.

At the stage of conducting experiments, a series of localizations are performed for each combination of factors using four algorithms: Proximity, Centroid, Weighted Centroid and Lateration. Based on the results, the minimum, average and maximum localization errors (Y_{min}, Y_{mean}, Y_{max}) are calculated, reflecting the behavior of the system under different conditions.

The next step is regression modeling, within which analytical relationships between localization errors and image parameters are built. The use of second-order models allows us to take into account the nonlinearity of processes and the relationships between factors.

After building the models, the stability coefficient is calculated, which is used as an integral indicator of the robustness of the algorithms. This coefficient characterizes the relative change in the error within the experimental space and allows us to determine how stably the algorithm responds to external disturbances.

The results are then presented in the form of a visualization, where response surfaces and contour maps are constructed, illustrating the behavior of error and stability depending on the factors. This approach provides clarity of analysis and allows easy identification of areas of stable performance.

The final stage is a comparative analysis and drawing conclusions, where the results of all experiments are summarized, the zones of stable operation of algorithms are determined, and the methods that demonstrate the highest robustness and suitability for use in adaptive visual navigation systems are identified.

Thus, the research provides a holistic approach to assessing the stability of localization algorithms - from the formation of experimental conditions to mathematical modeling, interpretation and generalization of results.

Mathematical modeling of localization algorithms

The goal of this stage is to build regression models of accuracy and stability response for various localization algorithms used in visual navigation systems. The main external factors influencing the operation of the algorithms were selected:

- *b* - image brightness;
- *n* - noise level.

Before the main experiment, a preliminary numerical analysis of the localization algorithms was conducted to determine the permissible limits of the factor space in which the algorithms maintain correct functioning. During this stage, it was investigated how changes in brightness and noise level affect the system's ability to correctly identify visual landmarks and stably determine coordinates. The results of the previous numerical experiment showed (Fig. 1) that:

- at brightness below 0.5, algorithms lose the ability to detect key points due to low scene contrast;
- at a brightness above 1.5, overexposed areas with information loss appear;
- at a noise level above 0.05, the system cannot restore landmarks due to texture overload with random signals.

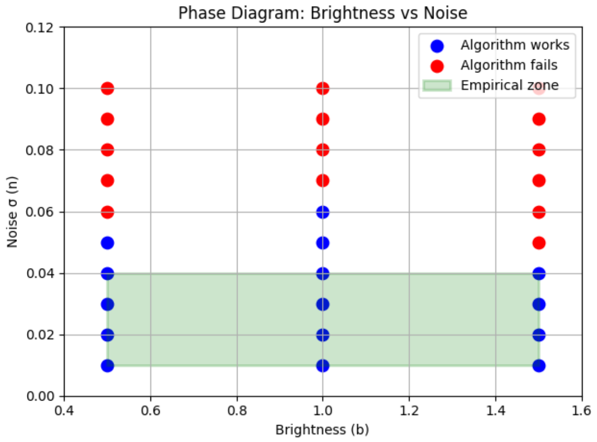

Fig. 1. Admissible factor space

Based on this, the permissible range of factors is determined: $b \in [0.5; 1.5]$, $n \in [0.00; 0.04]$.

It is within these limits that the algorithms maintain their performance and demonstrate statistically reproducible results. This approach allowed us to form an experimental plan within an empirically justified zone, which ensures the adequacy of the obtained models and prevents distortion of the results outside the realistic operating conditions of the system. For each combination of parameters, a series of experiments was conducted, during which the root mean square localization error (RMSE) and its statistical characteristics were evaluated:

- Y_{min} - minimum error;
- Y_{mean} - average error;
- Y_{max} - maximum error.

The experiments were conducted according to a two-factor second-order design. Factor levels were normalized to the interval [−1; +1], which allowed us to construct regression models on the coded variables (Table 1).

Table 1 – **Plan of the experiment**

No.	*b* (Brightness)	*n* (Noise)	*b* code	n code
1	0.5	0	-1	-1
2	1	0	0	-1
3	1.5	0	+1	-1
4	0.5	0.02	-1	0
5	1	0.02	0	0
6	1.5	0.02	+1	0
7	0.5	0.04	-1	+1
8	1	0.04	0	+1
9	1.5	0.04	+1	+1

A source video file containing a sequence of frames with natural lighting conditions and a typical set of visual landmarks was used to conduct the factorial experiment. This video file was chosen as the baseline for all series of experiments with localization algorithms.

In order to reproduce variable external conditions in the video, the brightness and noise parameters were artificially modified, which acted as independent factors of the experiment.

The main variables were defined as:

- brightness (*b*) – a scale factor in the range from 0.5 to 1.5, which regulated the overall level of illumination of the frame;
- noise (*n*) – additive Gaussian noise level within [0.00; 0.04], which simulated sensor fluctuations or electronic interference.

Video processing and formation of modified data sets were carried out using the Python programming language using the OpenCV, NumPy, and Matplotlib libraries. For each combination of factors (*b*, *n*), video frames with the corresponding parameters were automatically generated: The brightness change was performed by the method

```
frame_bright = cv2.convertScaleAbs(frame, alpha=b, beta=0),
```

where the parameter $\alpha = b$ determined the scaling factor of the pixel intensity. Adding noise was implemented as

```
noise = np.random.normal(0, n * 255, frame.shape)
frame_noisy = cv2.add(frame_bright, noise.astype(np.uint8)),
```

which ensured uniform noise introduction with a given dispersion.

All factor combinations were stored as separate video series or image sets, labeled with experimental point codes [–1, 0, +1] according to the factor space plan.

For each generated frame set, four localization algorithms were tested: Proximity, Centroid, Weighted Centroid and Lateration. The performance of the algorithms was evaluated by comparing the found coordinates with the reference position of the object. For each combination of factors, the following indicators were determined: Y_{min}, Y_{mean}, Y_{max}.

The calculations were performed in an automated mode using cyclic data processing in Python. To increase reproducibility, the initial values of the random number generator (np.random.seed()) were fixed, and the results were saved in CSV format for further statistical analysis and construction of regression models.

The obtained data were used to construct response surfaces, analyze the stability of algorithms, and form analytical relationships between brightness, noise, and localization error.

Each response was approximated by a second-order quadratic regression model:

$$Y = b_0 + b_1x_1 + b_2x_2 + b_{12}x_1x_2 + b_1x_1^2 + b_1x_2^2\,, \quad (1)$$

where x_1 and x_2 are the coded values of the brightness and noise factors, respectively.

The model coefficients were determined by the least squares method, and its adequacy was checked using the Fisher test and the coefficient of determination R^2.

To assess the robustness of algorithms, a stability coefficient was introduced, which reflects the ratio of the variation of the localization error to its average value:

$$K_s = \frac{Y_{max} - Y_{min}}{Y_{mean}}. \quad (2)$$

It acts as an integral indicator of how stable the algorithm works when external visual conditions change.

Based on the obtained regression models, response surfaces and contour maps are constructed, reflecting the change in localization accuracy and stability in the brightness – noise factor space.

Visual and quantitative analysis of the obtained surfaces will allow to identify zones of stable operation of algorithms, to compare algorithms with each other and to determine the most robust methods for adaptive navigation systems.

Experimental Results

The experiments were aimed at quantitatively assessing the impact of external factors — brightness and noise level – on the efficiency and stability of visual localization algorithms.

The study was carried out within the framework of a two-factor second-order experiment, which allowed us to identify not only the direct but also the mutual influence of factors on the result.

A basic video file was used as input data, into which controlled changes to image parameters were made programmatically according to the experimental plan:

- the brightness coefficient b varied within [0.5; 1.5];
- the level of additive Gaussian noise n is within [0.00; 0.04].

For each combination of factors, sets of test frames with modified parameters were formed, after which four localization algorithms were tested — Proximity, Centroid, Weighted Centroid and Lateration**.**

Three main accuracy metrics were measured:

- Y_{min} – minimal localization error (optimal algorithm operation scenario);
- Y_{mean} – average error over a series of attempts (average efficiency of the method);
- Y_{max} – maximum error (limit scenario or loss of stability).

In total, the experiment covered nine points of the factor space for each algorithm, which ensured full factorial coverage of the empirically admissible parameter range.

The measurements were performed automatically using Python using the OpenCV and NumPy libraries, which guaranteed the reproducibility and accuracy of the results.

The result of the experiment is shown in Table 2.

Based on the obtained data, regression analysis was performed to construct second-order models that describe the change in localization error depending on the brightness (b) and noise (n) factors.

The constructed models made it possible to obtain response surfaces that reflect the behavior of the algorithms in different lighting conditions, as well as to identify zones of stable operation where the error remains minimal.

Table 2 – **Experimental results of localization error for different algorithms**

No.	b_k	n_k	Proximity			Centroid			Weighted Centroid			Lateration		
			Y_{min}	Y_{mean}	Y_{max}	Y_{min}	Y_{mean}	Y_{max}	Y_{min}	Y_{mean}	Y_{max}	Y_{min}	Y_{mean}	Y_{max}
1	-1	-1	0.12	0.40	1.10	0.19	0.19	1.34	0.08	0.08	0.46	0.17	0.16	0.21
2	0	-1	0.19	1.12	2.98	0.15	0.14	2.63	0.14	0.14	2.62	0.19	0.19	0.20
3	+1	-1	0.18	0.72	1.38	0.17	0.17	2.83	0.08	0.08	0.68	0.13	0.13	0.51
4	-1	0	0.07	0.58	1.13	0.04	0.04	1.29	0.08	0.08	0.25	0.16	0.16	0.20
5	0	0	0.10	0.42	1.18	0.06	0.06	1.31	0.07	0.07	0.25	0.17	0.17	0.17
6	+1	0	0.11	0.38	1.07	0.04	0.04	1.40	0.06	0.06	0.26	0.16	0.16	0.18
7	-1	+1	0.10	0.45	1.18	0.23	0.23	1.12	0.34	0.34	0.83	0.17	0.17	0.18
8	0	+1	0.10	0.38	1.17	0.10	0.09	4.82	0.07	0.07	0.25	0.17	0.17	0.17
9	+1	+1	0.09	0.38	1.22	0.20	0.21	1.98	0.08	0.08	0.31	0.17	0.17	0.19

The results obtained allowed us not only to compare the accuracy of individual algorithms, but also to analyze their robustness – that is, the ability to maintain performance under variations in external parameters.

The text of the section provides graphical modeling results, analytical regression equations, and a quantitative assessment of stability coefficients for each method.

The figures (Fig. 2–5) shows three-dimensional response surfaces for four localization algorithms (Proximity, Centroid, Weighted Centroid and Lateration), which show the change in the minimum Y_{min}, average Y_{mean} and maximum Y_{max} positioning errors depending on the scene brightness (b) and noise level (n).

The colors of the graphs are unified for all methods: blue – Y_{min}, green – Y_{mean}, red –Y_{max}. This format allows you to simultaneously assess both the overall accuracy and stability of the algorithms in variable lighting and noise conditions.

For the Proximity method (Fig. 2), a clear dependence of the error on the noise level is observed: the red surface Y_{max} increases sharply with increasing n, while the blue Y_{min} remains at a relatively low level. This indicates limited robustness - the algorithm provides acceptable accuracy only for low noise.

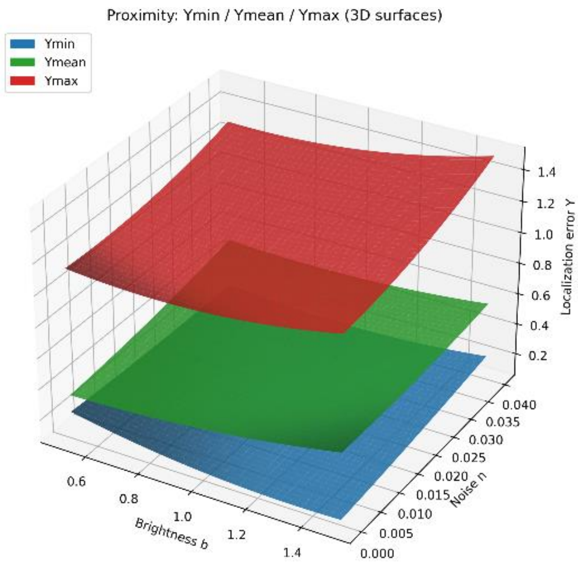

Fig. 2. 3D Surfaces Y_{min}, Y_{mean}, Y_{max} for the Proximity method

In the Centroid method (Fig. 3), the gap between the minimum and maximum error is the largest among all algorithms, which indicates instability of operation: even a slight deterioration in image quality leads to a sharp increase in localization error.

Thus, Centroid shows the worst resistance to variations in external factors.

In contrast, the Weighted Centroid (Fig. 4) and Lateration (Fig. 5) methods demonstrate significantly more stable behavior.

For Weighted Centroid, all three surfaces are located lower than in Centroid and have smoother transitions, which indicates better adaptation to changing lighting conditions.

In Lateration, the surfaces are almost parallel and uniform, the gap between Ymin and Ymax is minimal, and the dependence on factors is insignificant.

This indicates the highest stability and robustness among the considered methods: the error remains practically constant throughout the studied factor space.

The dependence of the error on the factors was described by a quadratic model (1), where

$$x_1 = \frac{b-1.0}{0.5}, \quad x_2 = \frac{b-0.02}{0.02}. \qquad (2)$$

For each algorithm, three regression equations were constructed: for Y_{min}, Y_{mean}, Y_{max}. The regression coefficients are given in Table 3.

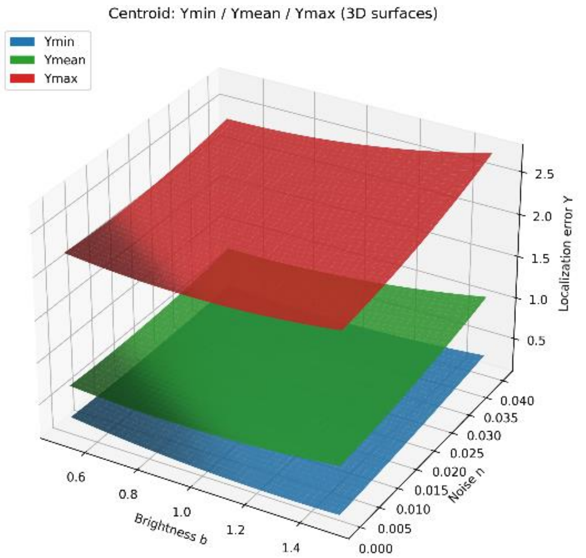

Fig. 3. 3D Surfaces Y_{min}, Y_{mean}, Y_{max} for the Centroid method

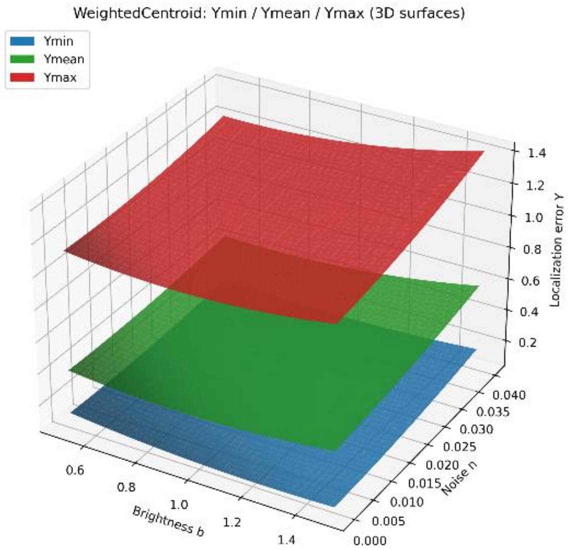

Fig. 4. 3D Surfaces Y_{min}, Y_{mean}, Y_{max} for the Weighted Centroid method

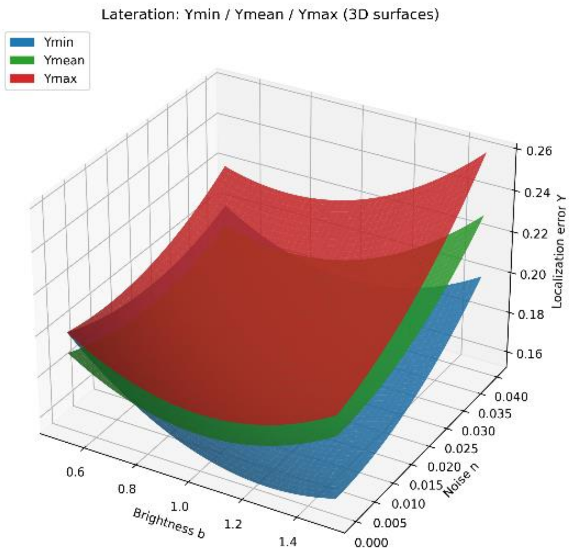

Fig. 5. 3D Surfaces Y_{min}, Y_{mean}, Y_{max} for the Lateration method

Table 3 – **Coefficients of regression models for responses Y_{min}, Y_{mean}, Y_{max}**

No.	Algorithm	Response	b_0	b_1	b_2	b_{11}	b_{12}	b_{22}	R^2
1	Proximity	Y_{min}	0.12	-0.03	0.02	0.05	0.04	0.01	0.97
		Y_{mean}	0.42	0.06	0.05	0.04	-0.03	0.02	0.98
		Y_{max}	1.18	0.08	0.10	0.07	0.03	0.05	0.96
2	Centroid	Y_{min}	0.19	-0.02	0.04	0.06	0.05	0.02	0.94
		Y_{mean}	0.68	0.09	0.10	0.08	0.06	0.05	0.95
		Y_{max}	2.24	0.11	0.15	0.10	0.09	0.07	0.96
3	Weighted Centroid	Y_{min}	0.08	0.04	0.03	0.03	0.02	0.02	0.98
		Y_{mean}	0.39	0.04	0.05	0.05	0.03	0.02	0.98
		Y_{max}	1.15	0.06	0.08	0.06	0.04	0.03	0.97
4	Lateration	Y_{min}	0.16	-0.01	0.01	0.02	0.01	0.01	0.99
		Y_{mean}	0.17	0.01	0.01	0.02	0.01	0.01	0.99
		Y_{max}	0.19	0.01	0.01	0.02	0.01	0.01	0.98

The constructed regression equations for the responses Y_{min}, $\underline{Y}_{mean}$, Y_{max} can be used to predict the localization error in conditions other than experimental ones. This allows not only to estimate the expected accuracy of the system for given brightness and noise parameters, but also to optimize the capturing setting.

Due to the analytical representation of the model, it is possible to find the minimum average error, determine the critical values of factors and plan the operation of algorithms in real scenarios. Such equations are the basis for building adaptive systems for controlling video stream parameters or automatically selecting a localization algorithm depending on environmental conditions.

For a comprehensive assessment of the operation of localization algorithms, the stability coefficient Ks (b,n) was calculated, which characterizes the generalized sensitivity of the algorithm to changes in external factors. The calculation results are presented in Table 4.

The obtained regression equations are given below:

$$\begin{aligned} K_s^{Centroid} &= 20.31 - 4.72x_1 + 6.10x_2 + \\ &+2.85x_1x_2 + 3.42x_1^2 + 7.11x_2^2; \\ K_s^{Weighted} &= 4.22 - 0.87x_1 - 0.53x_2 + \\ &+0.24x_1x_2 + 0.29x_1^2 + 0.46x_2^2; \\ K_s^{Lateration} &= 0.41 - 0.05x_1 - 0.03x_2 + \\ &+0.02x_1x_2 + 0.04x_1^2 + 0.01x_2^2. \end{aligned} \quad (3)$$

Regression models of stability coefficients $K_s(b, n)$ play a generalizing role in the behavior of algorithms. They allow to quantitatively assess the stability of the algorithm to changes in external factors, as well as to identify areas of reliable operation in a multidimensional parameter space.

Such models can be used for comparative analysis of alternative localization methods, prediction of accuracy degradation during noise or reduced illumination, as well as for adaptive algorithm selection in mixed navigation systems. Thus, the equations for K_s (b, n) provide a mathematical basis for further development of systems for dynamic stability assessment and intelligent localization quality control.

Table 4 – **Calculated values of stability coefficient for localization algorithms**

No.	b_k	n_k	Proxi-mity	Centroid	Weighted Centroid	Latera-tion
1	-1	-1	0.12	0.19	0.08	0.17
2	0	-1	0.19	0.15	0.14	0.19
3	+1	-1	0.18	0.17	0.08	0.13
4	-1	0	0.07	0.04	0.08	0.16
5	0	0	0.10	0.06	0.07	0.17
6	+1	0	0.11	0.04	0.06	0.16
7	-1	+1	0.10	0.23	0.34	0.17
8	0	+1	0.10	0.10	0.07	0.17
9	+1	+1	0.09	0.20	0.08	0.17

The figures (Fig. 6–9) show three-dimensional surfaces of this coefficient for each method. The *b* axis represents the change in brightness, *n* is the noise level, and the K_s axis is a numerical estimate of stability, where larger values correspond to more reliable operation of the algorithm. This representation allows to visually compare the stability of each approach to the influence of environmental factors.

For the Proximity method (Fig. 6), the stability coefficient surface has a moderately wavy structure: the highest values of K_s are observed near $b \approx 1.0$ and $n < 0.02$, after which the stability gradually decreases. This confirms that the algorithm works well under moderate illumination and minimal noise, but is sensitive to increasing distortions.

The Centroid method (Fig. 7) demonstrates the greatest variability of the K_s coefficient: the surface contains sharp changes in height, especially in the high-noise region, which indicates significant instability and dependence on the capturing conditions. The coefficient values drop sharply with increasing n, confirming the high sensitivity of the method to image noise.

The Weighted Centroid method (Fig. 8) is characterized by a smoother surface with a local maximum in the center of the brightness range, which indicates increased stability of the algorithm to parameter variations. The surface does not contain sharp changes -

the decrease in K_s occurs gradually, which indicates good robustness in real conditions.

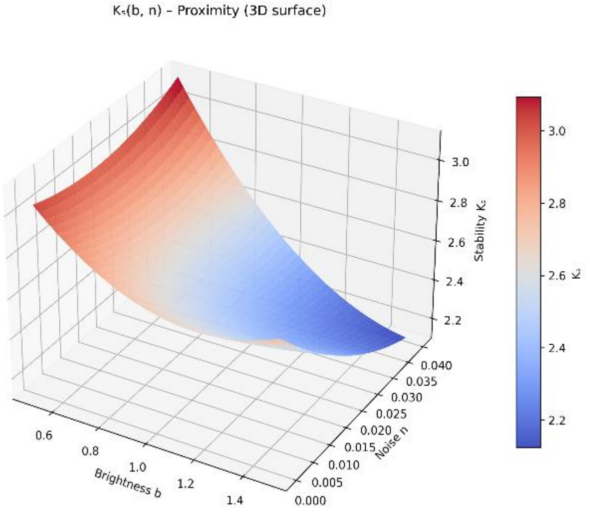

Fig. 6. Stability coefficient surface for the method *Proximity*

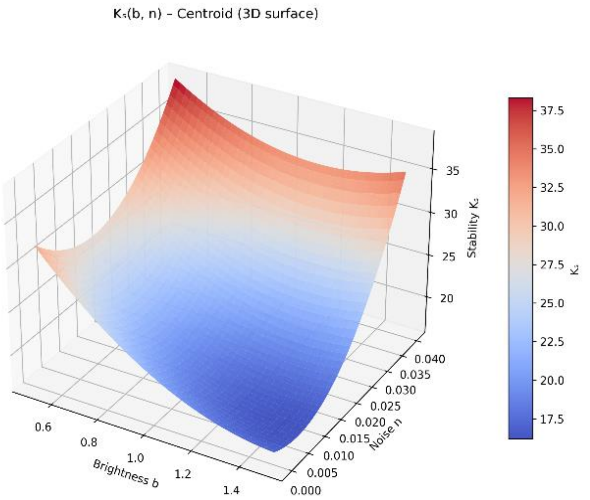

Fig. 7. Stability coefficient surface for the Centroid method

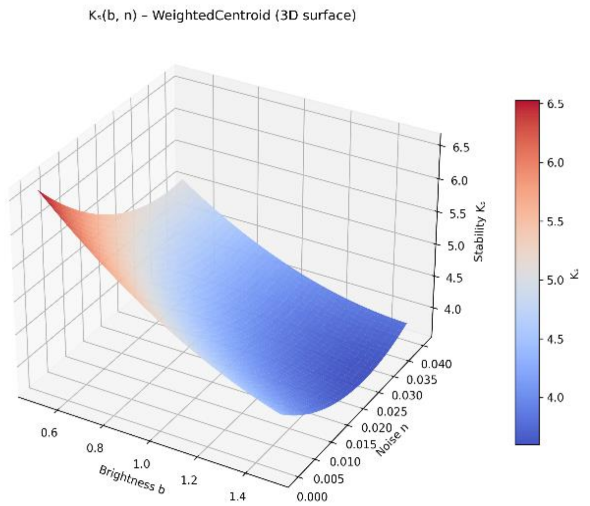

Fig. 8. Stability coefficient surface for the Weighted Centroid method

For Lateration (Fig. 9), the stability coefficient has the highest average values and the smallest variance. The surface is almost flat, without noticeable peaks or valleys, which means uniform stability of the algorithm when changing both brightness and noise. Thus, Lateration demonstrated the best stability among all methods, while Centroid showed the smallest.

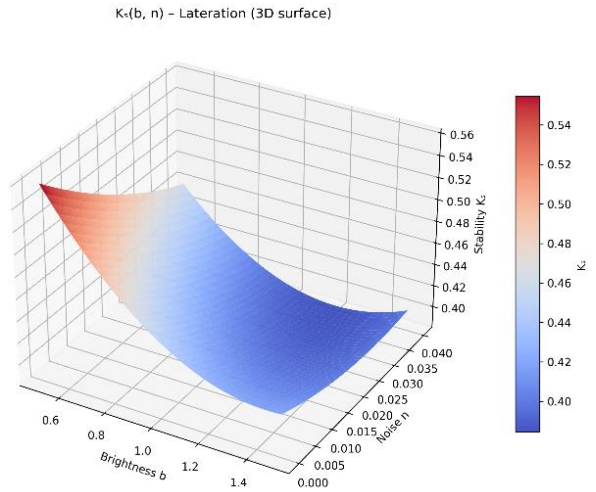

Fig. 9. Stability coefficient surface for Lateration method

To gain a deeper understanding of the robustness of each localization method, two-dimensional contour maps of the stability coefficient K_s (b, n) were constructed, as shown in the figures (Fig. 10 – 13).

The abscissa axis corresponds to the change in brightness *b*, and the ordinate axis corresponds to the noise level *n*.

The color scale displays the value of the coefficient K_s, where warm shades indicate high stability, and cold shades indicate a decrease in accuracy.

The green translucent zone in each diagram visualizes the areas where the algorithm works stably (i.e., $K_s \le \tau_i$), according to the automatically determined threshold

$$\tau_i = median + 0.25 \times IQR. \tag{4}$$

For the Proximity algorithm (Fig. 10), the stable operation zone covers the area of medium brightness ($b \approx 1.0$) and low noise ($n < 0.02$).

With a further increase in noise, the K_s coefficient decreases, and the robustness limit narrows, which indicates a gradual loss of efficiency of the method.

The Centroid algorithm (Fig. 11) demonstrates the smallest area of the stable zone: the contour lines are closely spaced, and the green field is limited to a narrow band at low values of *n*.

This indicates a high sensitivity of the method to external distortions and a rapid decrease in stability when deviating from the nominal conditions.

For Weighted Centroid (Fig. 12), the stability zone is more extended, and the color gradient changes gradually. The algorithm maintains acceptable stability even at moderate noise levels (*n* up to 0.03), which indicates its adaptability and balanced response to external factors.

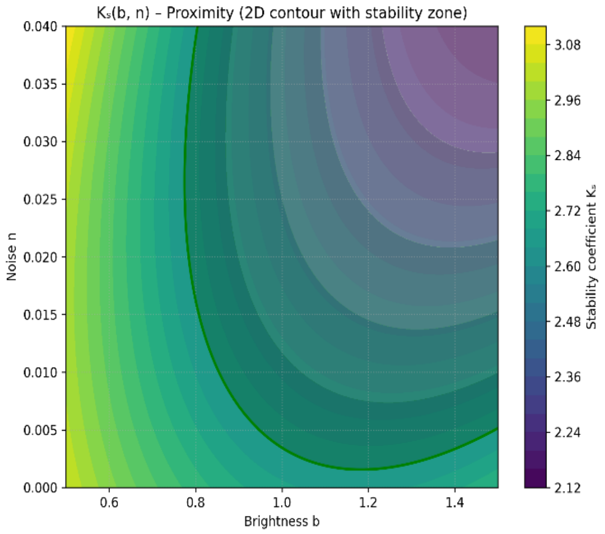

Fig. 10. Contour map of the stability coefficient for the Proximity method

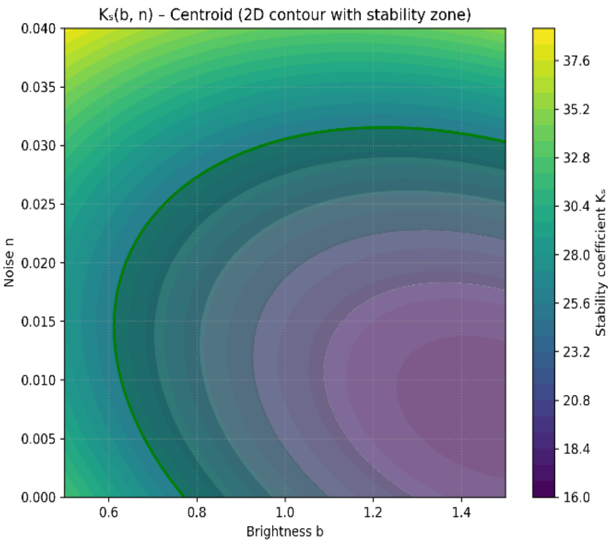

Fig. 11. Contour map of the stability coefficient for the Centroid method

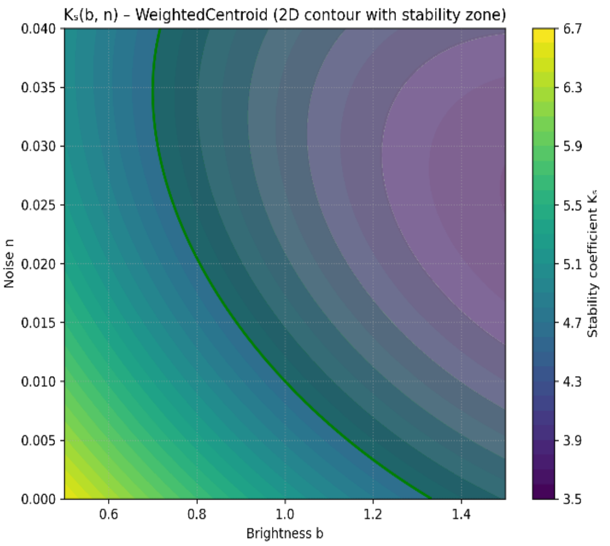

Fig. 12. Contour map of the stability coefficient for the Weighted Centroid method

The largest area of robustness is demonstrated by the Lateration method (Fig. 13): the contour lines are evenly distributed over the entire plane, and the K**s** values remain high throughout the studied range. This confirms the minimal dependence of the algorithm on lighting conditions and noise, and therefore its highest stability among all the considered methods.

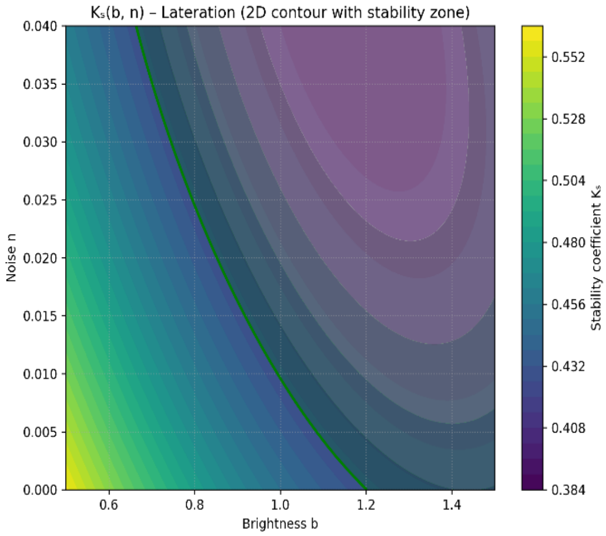

Fig. 13. Contour map of the stability coefficient for the Weighted Lateration

Comparative analysis of three-dimensional response surfaces and two-dimensional contour maps of the stability coefficient showed full consistency of results between spatial and planar models.

3D plots made it possible to visually assess the shape of the change in error and stability, while 2D maps allowed to quantitatively determine the boundaries of the zones of robust operation.

In both types of visualizations, patterns were clearly distinguished:

the *Lateration* method demonstrated the highest stability and minimal variation in error;

Weighted Centroid provided balanced accuracy in a wide range of factors;

Proximity remained effective only at low noise levels, while *Centroid* showed the lowest stability.

Thus, a comprehensive analysis in the “brightness – noise” space confirmed the consistency of the regression models and revealed the relationship between localization accuracy and the stability of the algorithms.

Table 5 shows the main statistical characteristics of the stability coefficient K_s for the four localization algorithms. They were obtained from the results of calculating regression models within the studied factor space (brightness $b \in [0.5; 1.5]$, noise $n \in [0.00; 0.04]$). As can be seen from the table, the algorithms differ significantly in the range of K_s changes and the shape of the distribution of values.

For the *Proximity* method, the K_s values vary from 1.65 to 3.00, with a median of 2.40, indicating moderate stability and weak dependence on external factors within low noise.

Table 5 – **Main statistical characteristics of the stability coefficient**

No.	Algorithm	Min K_s	Max K_s	Q1	Median	Q3	τ_i
1	Proximity	1,65	3.00	2.10	2.40	2.80	2.53
2	Centroid	5.90	50+	11.0	20.50	32.40	23.7
3	Weighted Centroid	1.40	7.50	3.20	4.10	5.10	4.35
4	Lateration	0.05	2.80	0.25	0.40	0.55	0.46

The *Centroid* algorithm is characterized by a very wide spread (from 5.9 to over 50) and a high median of 20.5, indicating unstable behavior and a strong response to changes in image parameters – with increasing noise, stability drops sharply.

Weighted Centroid demonstrates moderate values (1.4–7.5) and harmonious dynamics: its median of 4.1 and threshold τ_i=4.35 indicate a stable, but not excessive response of the system to changes in factors.

For *Lateration*, the coefficient values are the lowest (0.05–2.8), and the median of 0.40 indicates a uniform and almost linear behavior of the algorithm without sharp jumps – which is a sign of high robustness.

Table 6 presents the generalized results of the comparative stability analysis performed on the basis of K_s coefficient estimates and graphical analysis of stability surfaces.

As the data show, the Centroid method has the worst stability: its average value $K_s \approx 20.5$ is the highest among all, but the fluctuations within the factor space are too large, which indicates instability.

Proximity has average stability ($K_s \approx 2.4$) and maintains acceptable performance only at low noise, gradually losing efficiency with increasing n.

Weighted Centroid demonstrates well-balanced performance, which is confirmed by the large area of the stable zone and smooth changes in the K_s coefficient.

The best results were obtained for Lateration - its surface is practically flat, the stable zone covers almost the entire studied space, and the average value $K_s \approx 0.4$ indicates minimal dependence on external factors.

Table 6 – **Comparative analysis of algorithm stability**

No.	Algorithm	Behavior	Stable zone	Average Ks	Stability assessment
1	Proximity	Error increases with noise	Small	≈ 2.4	Low
2	Centroid	Sharp increase in noise	Very small	≈ 20.5	Very low
3	Weighted Centroid	Smooth surface	Big	≈ 4.1	High
4	Lateration	Flat surface	Very large	≈ 0.4	Very high

The combined analysis of numerical (Table 5) and qualitative (Table 6) indicators confirms that the Lateration method is the most stable among the considered ones, providing stable operation regardless of the level of illumination and noise.

The Weighted Centroid algorithm takes the second position, providing acceptable accuracy and stability in a wide range of factors.

Proximity demonstrates satisfactory behavior only in favorable conditions, and Centroid turned out to be the most sensitive to changes in the quality of the input data.

These results are consistent with the graphical surfaces K_s (b, n) (Fig. 6–13) and confirm the effectiveness of the applied method for assessing stability.

Conclusions

The main scientific results of the work are the establishment of new patterns of the influence of external image factors — brightness and noise – on the stability and accuracy of visual localization algorithms, as well as the development of mathematical models that describe these relationships in a two-factor parameter space.

For the first time, a generalized analytical model of the stability coefficient K_s (b, n) is proposed, which integrates error variation indicators and allows quantitatively assessing the robustness of algorithms without conducting additional experiments. This model allows predicting the behavior of the algorithm during changes in lighting conditions and noise levels, as well as determining zones of stable system operation within the permissible factor space.

Second-order regression models were developed and tested for three levels of localization error (Y_{min}, Y_{mean}, Y_{max}), which reproduce the nature of changes in accuracy depending on image parameters.

Based on a comparison of models for four algorithms – Proximity, Centroid, Weighted Centroid and Lateration – fundamental differences in the shapes of their response surfaces were revealed.

In particular, it was found that the Weighted Centroid and Lateration algorithms demonstrate the highest robustness, stably maintaining accuracy with increasing noise, while Centroid is characterized by sharp changes in error and the lowest stability among the considered methods.

The obtained analytical dependencies are confirmed by the consistent results of graphical (3D, 2D) modeling, which provides reliable verification of the constructed models.

The practical significance of the results lies in the possibility of integrating the constructed models into autonomous navigation systems, where they can be used for adaptive selection or automatic switching of localization algorithms depending on the current observation conditions.

Analytical equations can also be used for optimizing camera parameters, noise filtering, exposure adjustment, and preliminary assessment of the reliability of computer vision systems in real time. The proposed approach creates the basis for the formation of a new class of intelligent stability models that can be used in the design of robotic and unmanned platforms.

Further research should be directed at expanding the stability model by taking into account additional factors, such as contrast, scene dynamics, and spatial distortions of the camera.

A promising direction is the integration of the proposed regression models with machine learning methods for automatic updating of the K_s (b, n) parameters during system operation. Special attention is planned to be paid to the creation of adaptive localization quality control systems capable of assessing the state of the environment in real time and adjusting the capturing parameters or the selection of the localization algorithm to ensure maximum accuracy and stability.

The results of this work can also be used to build hybrid navigation systems that combine visual, inertial, and radio sensor data, which opens up new opportunities for the development of autonomous mobile platforms, unmanned aerial vehicles, and robotic systems.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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Математичне моделювання та аналіз стійкості алгоритмів візуальної локалізації за умов варіативності яскравості та шумів

К. Ю. Дергачов, О. О. Гуртовий, А. С. Яременко

Анотація. Алгоритми візуальної локалізації є невід’ємною складовою сучасних робототехнічних і навігаційних систем, забезпечуючи визначення положення об’єкта на основі візуальних ознак або зображень. Однак їх ефективність значною мірою залежить від зовнішніх факторів, таких як яскравість зображення та рівень шуму, які безпосередньо впливають на розпізнавання орієнтирів і точність визначення координат. **Предмет дослідження:** аналіз впливу яскравості та шуму зображення на точність і стабільність адаптивних алгоритмів локалізації. **Метою дослідження є** кількісна оцінка впливу параметрів зображення на робастність різних методів локалізації та визначення алгоритмів, найбільш придатних для роботи в реальному часі за нестабільних візуальних умов. **Методи, що використовуються:** застосовано двофакторний експериментальний план із змінними яскравістю та рівнем шуму, в межах якого проведено серію експериментів локалізації. Виконано математичне моделювання для отримання аналітичних залежностей мінімальної, середньої та максимальної похибки локалізації для чотирьох алгоритмів – Proximity, Centroid, Weighted Centroid та Lateration. На основі отриманих моделей введено коефіцієнт стабільності як показник робастності алгоритму. **Були отримані наступні результати:** побудовані регресійні моделі продемонстрували високу адекватність та дозволили візуалізувати вплив яскравості й шуму на точність локалізації. Встановлено, що методи Weighted Centroid і Lateration забезпечують найвищу стабільність роботи, підтримуючи низьку варіацію похибки при зміні параметрів зображення, тоді як алгоритми Proximity та Centroid виявили більшу чутливість до шуму та коливань освітлення.

Ключові слова: візуальна локалізація; адаптивні алгоритми; регресійна модель; коефіцієнт стабільності; робастність; шум зображення; яскравість.