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INPUT MATERIAL FLOW VALUES GENERATOR OF A CONVEYOR WITH A GIVEN CORRELATION FUNCTION AND DISTRIBUTION LAW

Abstract. The object of this study is a stationary stochastic input flow of material arriving at the input of an industrial conveyor transport system. **The goal** of this research is to develop a universal, statistically mathematical model of the input flow of materials, fully identifiable from a single long-term experimental implementation, as well as to create a multi-level system of dimensionless stochastic similarity criteria, enabling the objective classification and comparison of heterogeneous flows with similar structural properties. **The results** obtained. A simplified canonical decomposition of a stationary ergodic process with a minimum number of random coefficients is proposed, reproducing the specified mathematical expectation, variance, correlation function, and one-dimensional probability density of flow values. Analytical expressions are derived for approximating the distribution density of random coefficients with guaranteed fulfillment of the conditions of centering, normalization, and non-negativity. A multilevel system of stochastic similarity criteria is developed, including aggregated dimensionless criteria, a functional similarity criterion based on a normalized autocorrelation function, and a functional criterion based on quantile-quantile diagrams. A dimensionless flow normalization method is proposed, ensuring model transferability between conveyor systems differing by orders of magnitude in throughput and time scales. Using six independent long-term implementations of real conveyor systems in the mining and processing industries, the accuracy of the developed stochastic input flow generator using an analytical approximation of random coefficients is demonstrated. **Conclusion.** The developed methodology enables the classification and comparison of material input flows in transport systems and serves as the basis for a universal approach to constructing mathematical models and flow control algorithms under stochastic uncertainty.

Keywords: material flow; stochastic stationary process; canonical decomposition; dimensionless similarity criteria; conveyor; transport systems.

Introduction

Modern industrial transport and processing systems are characterized by a high degree of complexity and the stochastic nature of the processes that determine their operation. One of the key elements of such systems is conveyor transport lines, which ensure the continuous supply and redistribution of material flows. Their operational efficiency directly depends on the nature and structure of the input material flow, which in real conditions is stochastic due to fluctuations in the productivity of previous process units, uneven supply, and variations in the physical and mechanical properties of the transported material.

Reducing overall costs in the extraction and processing of minerals is largely determined by the optimization of transport processes, in particular, by reducing the specific costs of moving raw materials [1]. One of the most common ways to improve efficiency is to ensure uniform distribution of material along the conveyor line, which facilitates more efficient use of transport capacity and increases system productivity [2–4]. Improving the uniformity of conveyor loading not only increases productivity but also reduces operating costs associated with equipment wear and maintenance [5–7]. The stable operation of transport systems depends largely on the nature of the incoming material flow, which is affected by stochastic feed fluctuations, uneven supply from adjacent process units, and control features.

However, fluctuations in the incoming material flow, caused by a number of factors, remain a significant obstacle to stable system operation. These include, first and foremost, the stochastic nature of the incoming material flow entering the system [8]. Furthermore, the operation of control subsystems, including belt speed control mechanisms [9–11] or material feed from storage bins [12], has a significant impact. Accurate modeling of both stationary and non-stationary stochastic input material flows is critical for developing effective strategies to control the flow characteristics of conveyor lines, performing comparative evaluations of alternative routing schemes in extensive transportation networks [13, 14], and improving the overall stability and reliability of such systems [15–17].

The design and subsequent tuning of the characteristics of such systems require a thorough analysis of the statistical patterns of input material flows [18]. In real-world conditions, these flows exhibit significant stochastic fluctuations and temporal variability, which creates significant obstacles to the development of their mathematical descriptions. The presence of correlated random factors, as well as the heterogeneity and variability of flow properties, necessitates the development of more universal and accurate modeling methods that take into account the actual mechanisms for generating stochastic input material flows in conveyor transport systems [19].

Control subsystems also play an important role, including belt speed control mechanisms [9, 10, 11, 20] and systems for metering material feed from storage bins [12]. These factors create a complex dynamic flow structure, which introduces additional fluctuations and complicates the problem of maintaining a stable transport mode. Adequately describing the behaviour of conveyor systems requires correct modeling of the input material flow as a stochastic process with specified statistical characteristics. This requires not only determining the mathematical expectation and variance, but also accounting for correlations between successive flow values and the shape of its distribution. Traditional models based on the assumption of normality or independence of random effects do not reflect the real characteristics of stochastic flows, leading to errors in the analysis of the dynamic and energy parameters of transport systems.

In recent years, there has been growing interest in the use of stochastic modeling methods that allow the statistical properties of real material flows to be reproduced based on experimental data. However, the practical application of such models requires taking into account the limited scope of observations and preserving physically meaningful statistical characteristics. A promising solution to this problem is the use of a canonical decomposition of a stochastic process into orthogonal coordinate functions, which allows the input flow to be formed as a superposition of deterministic and stochastic components. This approach enables direct comparison of the model's statistical characteristics with experimental data and allows for the inclusion of aggregated similarity criteria linking the mathematical expectation, standard deviation, correlation time, and characteristic time of flow property change.

This paper proposes a model of a stochastic stationary input material flow constructed using the canonical decomposition of a stochastic process and an approximation of the distribution density of its random components. Analytical relationships for determining the canonical decomposition coefficients that ensure the centering and non-negativity of the distribution density are considered. It is shown that a limited representation of the distribution density using a small number of terms ensures satisfactory accuracy while preserving the physical meaning of the model. In recent years, the application of artificial neural networks has been rapidly developing, both for diagnosing the condition of conveyor belts and rollers [21, 22, 23, 24], and for improving the efficiency and stability of mining process control [25], forecasting the flow parameters of multi-section conveyor systems [26], and constructing intelligent control systems based on embedded artificial intelligence [1, 22, 27]. This confirms the high potential of neural network methods for solving the problem of accurately approximating the distribution density of random coefficients in the canonical decomposition of a stochastic input flow.

To improve the accuracy of approximation for complex distributions, a neural network approach is discussed, based on the use of a multilayer perceptron for calculating mathematical expectations and approximation parameters based on experimental data. The developed model enables the generation of stochastic input material flows with specified statistical characteristics, a correlation function, and an approximate distribution density of values, which makes it suitable for use in the analysis and optimization of flow parameters of conveyor systems. The proposed approach facilitates a more accurate reproduction of real-world operating conditions of transport lines, as well as the creation of digital twins of technological processes that take into account the stochastic nature of material flows.

The aim of this article is to develop and theoretically substantiate a model of a stochastic stationary input material flow based on the canonical decomposition of a stochastic process, with an analytical determination of the parameters for approximating the distribution density of random coefficients. The proposed model is intended to create a generator of input material flows that reproduces a given correlation function and the distribution density of values corresponding to experimental data.

The results of this article can be used to improve the accuracy of modeling, forecasting, and optimization of the operating modes of transport and processing systems.

Statement of the problem

Based on an analysis of modern industrial transport and technological systems characterized by a highly stochastic input material flows, this article formulates the problem of developing a mathematical model of a stationary stochastic material flow entering a conveyor line. The model must adequately reflect the physical mechanisms of fluctuation formation caused by uneven raw material supply from previous process links, the presence of correlated random disturbances, and the influence of control subsystems (belt speed control, metered feed from storage bins). A key requirement is to ensure accurate reproduction of a complete set of statistical characteristics – the mathematical expectation, variance, correlation function, and distribution density function – based on a limited set of experimental realizations.

Formally, the problem is reduced to constructing a canonical decomposition of a stationary ergodic process.

The problem statement includes solving the following interrelated subproblems:

a) determining the statistical characteristics of the input material flow based on experimental realizations;

b) evaluation of aggregated similarity criteria for the input material flow;

c) construction of a simplified canonical decomposition of the stochastic input material flow;

d) analytical approximation of the distribution density of random coefficients in the canonical decomposition of the stochastic input material flow;

d) development of a generator of realizations of the stochastic input material flow.

The solution to this problem will ensure the creation of a universal tool for stochastic modeling of input flows, applicable to the optimization of conveyor system operating modes, taking into account real-world stochasticity.

Model of stochastic stationary input material flow

The key parameters in modeling a stochastic stationary flow of material $\lambda(t)$ arriving at the input of a transport conveyor at time $t \in [0, t_d]$ are the mathematical expectation m_λ, the standard deviation σ_λ, the correlation time η_λ of the input material flow values, and the characteristic time t_λ, which determines the scale of change in the statistical properties of the material flow. Taking into account the similarity criteria for a stochastic stationary material flow:

$$\pi_1 = \frac{m_\lambda}{\sigma_\lambda}, \quad \pi_2 = \frac{t_\lambda}{\eta_\lambda}, \quad t_\lambda \le t_d, \qquad (1)$$

the model of a stochastic stationary flow of material, taking into account the canonical expansion in the coordinate functions on the interval $\tau \in [0, \tau_\vartheta], \tau_\vartheta \le \tau_d$ can be represented in the following form [28]:

$$\gamma_{in}(\tau) = \pi_1 + \gamma(\tau), \quad \omega_j = 2\pi j/\tau_\vartheta, \quad \tau_\vartheta \le \tau_d, \qquad (2)$$

$$\gamma(\tau) = \frac{\Theta_0}{2} + \sum_{j=1}^{\infty} \Theta_{cj} \cos(\omega_j \tau) + \Theta_{sj} \sin(\omega_j \tau), \qquad (3)$$

with dimensionless parameters:

$$\tau = \frac{t}{\pi_2 \eta_\lambda}, \quad \tau_d = \frac{t_d}{t_\lambda}, \quad \tau_\vartheta = \frac{t_\lambda}{t_\lambda} = 1, \quad \vartheta = \frac{\eta}{\pi_2 \eta_\lambda}, \qquad (4)$$

$$\gamma_{in} = \frac{\lambda}{\sigma_\lambda}, \quad \gamma = \frac{\lambda - M[\lambda]}{\sigma_\lambda} = \frac{\lambda - m_\lambda}{\sigma_\lambda}, \qquad (5)$$

$$M[\gamma(\tau)] = 0, \quad M[\gamma^2(\tau)] = 1. \qquad (6)$$

The coefficients of the canonical decomposition Θ_0, Θ_{cj}, Θ_{sj} are centered random variables with the mathematical expectation:

$$M[\Theta_0] = M[\Theta_{cj}] = M[\Theta_{sj}] = 0, \qquad (7)$$

and the standard deviation:

$$M[\Theta_0^2/4] = \sigma_0^2, \; M[\Theta_{cj}^2] = M[\Theta_{sj}^2] = \sigma_j^2. \qquad (8)$$

The model of a stochastic stationary material flow (2) is represented by the sum of a deterministic term π_1 and a stochastic term $\gamma(\tau)$. The aggregated similarity criteria of the input material flow (1) determine the relationship between the general statistical characteristics m_λ, σ_λ, η_λ without directly answering the questions of which distribution function and correlation function represent the input material flow. The similarity criterion π_2 is a time scaling factor that determines the characteristic time interval for comparative analysis as a ratio $t_\lambda = \pi_2 \eta_\lambda$. When modeling a stochastic stationary input material flow, a good choice is the value $\pi_2 > 10$. If possible, the maximum permissible value of the similarity criterion π_2 should be selected, which allows the use of a rich set of experimental data for the analysis. In the comparative analysis of stochastic stationary material flows, the characteristic time for each n-th material flow is selected based on condition $t_{\lambda n} = \pi_2 \eta_{\lambda n}$, where the similarity criterion value π_2 is the same for all material flows and is the maximum possible value provided condition $t_{\lambda n} \le t_{dn}$, $n = 1..N$ is met. In some cases, assumption $t_{\lambda n} > t_{dn}$, may be adopted, whereby additional data are added to time interval $t \in [t_{dn}, t_{\lambda n}]$ through extrapolation or other methods.

In the analysis of stochastic stationary input material flows, the similarity criterion value $\pi_2 \sim 100$ [28] was used. A lower similarity criterion value could have been chosen in this article. However, this would have reduced the volume of experimental data and, accordingly, the accuracy of the comparative analysis. The statistical characteristics of the canonical decomposition of the stochastic centered stationary input material flow $\gamma(\tau)$ (2), taking into account notation (8), are defined in [28, 29] and can be represented as:

a) mathematical expectation:

$$m = M[\gamma(\tau)] = M[\Theta_0/2] + \sum_{j=1}^{\infty} M[\Theta_{cj}]\cos(\omega_j \tau) + M[\Theta_{sj}]\sin(\omega_j \tau) = 0; \qquad (9)$$

b) standard deviation:

$$\sigma^2 = M[\gamma^2(\tau)] = M[(\Theta_0/2)^2] + \sum_{j=1}^{\infty} \cos^2(\omega_j \tau) M[\Theta_{cj}^2] + \sin^2(\omega_j \tau) M[\Theta_{sj}^2] = \sigma_0^2 + \sum_{j=1}^{\infty} \sigma_j^2; \qquad (10)$$

c) correlation function

$$k(\vartheta) = M[\gamma(\tau)\gamma(\tau + \vartheta)] = \sigma_0^2 + \sum_{j=1}^{\infty} \cos(\omega_j \vartheta) \sigma_j^2. \qquad (11)$$

It is assumed that N realizations of the random process $\gamma(\tau)$, $n = 1..N$ is available for constructing a model of the stochastic input material flow. Each realization $\gamma_n(\tau)$ of the stochastic process over interval $\tau \in [0, \tau_\vartheta]$, $\tau_\vartheta = 1$ is expanded into a Fourier series:

$$\gamma_n(\tau) = \frac{\theta_{n0}}{2} + \sum_{j=1}^{\infty} \theta_{cnj} \cos(\omega_j \tau) + \theta_{snj} \sin(\omega_j \tau), \quad \omega_j = 2\pi j, \qquad (12)$$

with constant expansion coefficients:

$$\theta_{n0} = 2\int_0^1 \gamma_n(\tau) d\tau; \qquad (13)$$

$$\theta_{cnj} = 2\int_0^1 \gamma_n(\tau)\cos(\omega_j \tau) d\tau; \quad \theta_{snj} = 2\int_0^1 \gamma_n(\tau)\sin(\omega_j \tau) d\tau. \qquad (14)$$

Mathematical expectation, standard deviation, and correlation function of the input flow of material can be calculated from N realizations of the random process $\gamma(\tau)$ [28]:

$$m = \frac{1}{N}\sum_{n=1}^{N}\int_0^1 \gamma_n(\tau) d\tau = \frac{1}{N}\sum_{n=1}^{N} \frac{\theta_{0n}}{2}, \qquad (15)$$

$$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N}\int_0^1 \gamma_n^2(\tau) d\tau = \sigma_0^2 + \sum_{j=1}^{\infty} \sigma_j^2, \qquad (16)$$

$$k(\vartheta)=\frac{1}{N}\sum_{n=1}^{N}\int_0^1\gamma_n(\tau)\gamma_n(\tau+\vartheta)d\tau = \\ =\sigma_0^2+\sum_{j=1}^{\infty}\sigma_j^2\cos(\omega_j\vartheta), \quad (17)$$

where $$M\left[\frac{\Theta_0^2}{4}\right]=\sigma_0^2\cong\frac{1}{N}\sum_{n=1}^{N}\frac{\theta_{0n}^2}{4},$$

$$M\left[\Theta_{cj}^2\right]=\sigma_j^2\cong\frac{1}{N}\sum_{n=1}^{N}\theta_{cnj}^2, \quad (18)$$

$$M\left[\Theta_{sj}^2\right]=\sigma_j^2\cong\frac{1}{N}\sum_{n=1}^{N}\theta_{snj}^2.$$

The obtained formulas allow to calculate the standard deviation and the correlation function of the stochastic stationary flow of material according to N experimental realizations of the random process. If the material flow is a stationary and ergodic process, then its canonical decomposition has the form [28]:

$$\gamma(\tau)=\sum_{j=1}^{\infty}\Theta_{cj}\cos(\omega_j\tau)+\sum_{j=1}^{\infty}\Theta_{sj}\sin(\omega_j\tau), \\ \omega_j=2\pi j, \quad (19)$$

The key simplification in the analysis of such a process is connected not so much with the absence of the term $\Theta_0/2$, but with the fact that the statistical characteristics of the input flow of material: mathematical expectation (15), dispersion (16) and correlation function (17) can be estimated directly from one realization of the random process:

$$\gamma_1(\tau)=\sum_{j=1}^{\infty}\theta_{c1j}\cos(\omega_j\tau)+\theta_{s1j}\sin(\omega_j\tau), \quad (20)$$

$$m=\int_0^1\gamma_1(\tau)d\tau=0; \\ \sigma^2=\int_0^1\gamma_1^2(\tau)d\tau=\sum_{j=1}^{\infty}\frac{\theta_{c1j}^2+\theta_{s1j}^2}{2}, \quad (21)$$

$$k(\vartheta)=\int_0^1\gamma_1(\tau)\gamma_1(\tau+\vartheta)d\tau= \\ =\sum_{j=1}^{\infty}\left(\left(\theta_{c1j}^2+\theta_{s1j}^2\right)/2\right)\cdot\cos(\omega_j\vartheta), \quad (22)$$

where $$M\left[\gamma^2(\tau)\right]=\sigma^2=\sum_{j=1}^{\infty}\sigma_j^2=1, \quad (23)$$

$$\left(\theta_{c1j}^2+\theta_{s1j}^2\right)/2=const=\sigma_j^2. \quad (24)$$

Such an approach becomes possible due to ergodicity, which ensures the equivalence of temporal averaging and averaging over the ensemble of realizations. Restrictions (24) are imposed on coefficients Θ_{cj}, Θ_{sj} of the canonical expansion (19). If it is assumed that the coefficient Θ_{cj} is a random variable, then the coefficient Θ_{sj} is expressed through the random variable Θ_{cj} in accordance with equation (24). Constraints (24) significantly simplify the type of canonical expansion of the initial expression for a random process, allow the transition from the canonical expansion (19) to the canonical expansion:

$$\gamma(\tau)=\sum_{j=1}^{\infty}A_j\cos(\omega_j\tau-\Phi_j),\ \omega_j=2\pi j, \quad (25)$$

for the construction of which transformations are used [30]:

$$\Theta_{cj}=A_j\cos(\Phi_j);\quad \Theta_{sj}=A_j\sin(\Phi_j); \\ \Theta_{sj}/\Theta_{cj}=tg(\Phi_j), \quad (26)$$

$$\Theta_{cj}^2+\Theta_{sj}^2=A_j^2\cos^2(\Phi_j)+A_j^2\sin^2(\Phi_j)=A_j^2, \quad (27)$$

$$A_j\cos(\Phi_j)\cos(\omega_j\tau)+A_j\sin(\Phi_j)\sin(\omega_j\tau)= \\ =A_j\cos(\omega_j\tau-\Phi_j), \quad (28)$$

where Φ_j is a random variable taking a value from the interval $[0,2\pi]$. Taking into account constraint (24), the canonical expansion (25) is represented as:

$$\gamma(\tau)=\sqrt{2}\sum_{j=1}^{\infty}\sigma_j\cos(\omega_j\tau-\Phi_j),\ \Phi_j\in[0,2\pi]. \quad (29)$$

Material flow $\gamma(\tau)$ is a centered random process with statistical characteristics (9)–(11). Taking this into account, we present expressions for statistical characteristics in the form:

$$=M\left[\sum_{j=1}^{\infty}A_j\cos(\omega_j\tau-\Phi_j)\right]= \\ \sum_{j=1}^{\infty}A_j\cos(\omega_j\tau)M[\cos(\Phi_j)]+ \\ +\sum_{j=1}^{\infty}A_j\sin(\omega_j\tau)M[\sin(\Phi_j)]=0, \quad (30)$$

$$M[\gamma^2(\tau)]=M\left[\left(\sum_{j=1}^{\infty}A_j\cos(\omega_j\tau-\Phi_j)\right)^2\right]= \\ =\sum_{j=1}^{\infty}A_j^2M\left[\cos^2(\omega_j\tau-\Phi_j)\right]=\sum_{j=1}^{\infty}\sigma_j^2 \quad (31)$$

and we get the requirements:

$$M[\cos(\Phi_j)]=0,\qquad M[\sin(\Phi_j)]=0, \quad (32)$$

$$M[\cos(2\Phi_j)]=0,\quad M[\sin(2\Phi_j)]=0, \\ M[\sin(\Phi_j)\cos(\Phi_j)]=0. \quad (33)$$

Equation (33) is obtained from condition (31), from which follows

$$M\left[\cos^2(\omega_j\tau-\Phi_j)\right]=M\left[\left(1+\cos(2\omega_j\tau-2\Phi_j)\right)/2\right]=$$

$$=\frac{1}{2}+\frac{1}{2}\cos\left(2\omega_j\tau\right)M\left[\cos\left(2\Phi_j\right)\right]+ \\ +\frac{1}{2}\sin\left(2\omega_j\tau\right)M\left[\sin\left(2\Phi_j\right)\right]=\frac{1}{2}. \quad (34)$$

The density of the distribution of the random variable Φ_j on the interval $[0, 2\pi]$ in the form of an expansion is sought:

$$f_j(\Phi_j) = \frac{C_{co}}{2} + \sum_{n=1}^{\infty} C_{cn}\cos(n\Phi_j) + C_{sn}\sin(n\Phi_j), \quad (35)$$

with expansion coefficients:

$$C_{co} = \frac{1}{\pi}\int_0^{2\pi} f_j(\Phi)d\Phi, \quad (36)$$

$$C_{cn} = \frac{1}{\pi}\int_0^{2\pi} f_j(\Phi)\cos(n\Phi)d\Phi, \quad (37)$$

$$C_{sn} = \frac{1}{\pi}\int_0^{2\pi} f_j(\Phi)\sin(n\Phi)d\Phi. \quad (38)$$

Taking into account the condition of normalization, the density of distribution $f_j(\Phi)$ and the equation for calculating the average value of function $\rho(\Phi)$:

$$\int_0^{2\pi} f_j(\Phi)d\Phi = 1, \quad \int_0^{2\pi} f_j(\Phi)\rho(\Phi)d\Phi = M\left[\rho(\Phi_j)\right], \quad (39)$$

as well as constraints (32), (33), the first coefficients of the density expansion of distribution $f_j(\Phi)$ can be determined:

$$C_{co} = 1/\pi,$$

$$C_{c1} = \frac{M\left[\cos(\Phi)\right]}{\pi} = 0, \; C_{s1} = \frac{M\left[\sin(\Phi)\right]}{\pi} = 0, \quad (40)$$

$$C_{c2} = \frac{M\left[\cos(2\Phi)\right]}{\pi} = 0, \; C_{s2} = \frac{M\left[\sin(2\Phi)\right]}{\pi} = 0.$$

The expansion coefficients of C_{c1}, C_{s1}, C_{c2}, C_{s2} are equal to zero due to conditions (30), (31). These conditions ensure that the mathematical expectation (9) is equal to zero and the standard deviation (10) of the centered stochastic flow of material $\gamma(\tau)$ is equal to σ^2. Since the distribution density $f_j(\Phi)$ of the random variable Φ_j is for the centered stochastic flow of material $\gamma(\tau)$ this distribution density automatically satisfies the condition that the coefficients of the expansion C_{c1}, C_{s1}, C_{c2}, C_{s2} are equal to zero when it is approximated by the canonical expansion (35). Thus, the density of distribution $f_j(\Phi)$ can be approximated by the following decomposition:

$$f_j(\Phi_j) = \frac{1}{2\pi} + \sum_{n=3}^{\infty} C_{cn}\cos(n\Phi_j) + C_{sn}\sin(n\Phi_j). \quad (41)$$

The coefficients C_{cn}, C_{sn} are chosen from the condition of non-negativity of the distribution density $f_j(\Phi) \geq 0$. The accuracy of the approximation is determined by the number of terms in the expansion (35). The density of the distribution bounded by the first term in expression (40) corresponds to the uniform distribution of the random variable Φ_j:

$$f_j(\Phi_j) = 1/(2\pi). \quad (42)$$

In general, it is impossible to unambiguously restore the density of distribution $f_j(\Phi)$ from one realization of stochastic process $\gamma(\tau)$ (29). The density of the distribution of a stochastic process is determined by the superposition of the functions of random variables Φ_j. Since in one implementation of the stochastic process $\gamma(\tau)$ the variable Φ_j takes a random value φ_j, which does not depend on time, then the values of the function:

$$\gamma_j(\tau) = A_j\cos\left(\omega_j\tau - \varphi_j\right), \quad (43)$$

when decomposing the implementation of the stochastic process $\gamma(\tau)$ (25) with a uniformthe sample for time τ for the full period gives the same theoretical density of the distribution of values regardless of the value φ_j:

$$f_{\gamma_j}(\gamma_j) = 1\Big/\left(\pi\sqrt{A_j^2 - \gamma_j^2}\right), \quad \sigma_j = A_j/2. \quad (44)$$

The value of the flow of material $\gamma(\tau)$ at each moment of time τ is determined by the superposition $\gamma(\tau) = \sum_{j=1}^{\infty}\gamma_j(\tau)$ of the values of functions $\gamma_j(\tau)$. A uniform over time τ sampling of the values of these functions over a full period gives the arcsine law of the distribution density (44). The superposition of the values of the functions $\gamma_j(\tau)$ for one implementation determines the law of distribution of the random value of the flow of material $\gamma(\tau)$.

Each realization $\gamma_j(\tau)$ (43) corresponds to the value φ_j of the random variable Φ_j with the distribution density (41). Thus, the value of the random variable γ_j at each moment of time is given by the value of the random variable Φ_j. Based on the assumption that the distribution law of the values of the stochastic flow is the same for each implementation of the stochastic process $\gamma(\tau)$ and can be determined by a single implementation, let us determine the density of the distribution $f_j(\Phi_j)$ of the random variable Φ_j. The task of constructing the density of the distribution $f_j(\Phi_j)$ of the random variable Φ_j in this case is reduced to the determination of the expansion coefficients C_{cn} and C_{sn}, which can be the most optimal approximation of the experimental distribution law of the values of the input flow of the material in accordance with the given optimality criterion was obtained. The zero approximation, when the expansion (41) is represented only by the first term, corresponds to the distribution density $f_j(\Phi_j)$ of the random variable 4 in the form of the law of uniform distribution (42). The approach using the uniform distribution law of the random variable Φ_j is a simplified approach for approximating the stochastic process by the canonical representation of the form (25). In this study, we consider the approximation when the distribution function is represented by the bounded expansion (41):

$$f_j(\Phi_j)=\frac{1}{2\pi}+C_{s3}\sin(3\Phi_j)+C_{c3}\cos(3\Phi_j). \quad (45)$$

Coefficients C_{sn}, C_{cn} are determined from the system of equations:

$$\begin{cases}\varphi_{0j}=\int_0^{2\pi}\Phi f_j(\Phi)d\Phi\to\varphi_j,\\ \int_0^{2\pi}\left(\Phi-\varphi_{0j}\right)^2 f_j(\Phi)d\Phi\to 0.\end{cases} \quad (46)$$

Coefficients C_{sn}, C_{cn} must ensure that the distribution density $f_j(\Phi_j)$ is non-negative, namely, satisfy the inequality:

$$1/(2\pi)\ge\sqrt{C_{s3}^2+C_{c3}^2}, \quad (47)$$

which follows from the transformation from canonical form (19) to canonical form (25). Having calculated the first integral from the system of equations (46):

$$\begin{gathered}\Phi^2/(4\pi)+C_{s3}\left(\sin(3\Phi)/9-\Phi\cos(3\Phi)/3\right)\Big|_0^{2\pi}+\\ +C_{c3}\left(\Phi\sin(3\Phi)/3-\cos(3\Phi)/9\right)\Big|_0^{2\pi}\to\varphi_j,\end{gathered} \quad (48)$$

we obtain expressions for determining the coefficient C_{sn}:

$$\begin{gathered}\varphi_{0j}=\pi-C_{s3}\cdot(2\pi/3)\to\varphi_j,\ C_{s3}\to 3/2\cdot\left(1-\varphi_j/\pi\right),\\ 1/(2\pi)\ge|C_{s3}|,\quad \varphi_{0j}\in[\pi-1/3;\pi+1/3].\end{gathered} \quad (49)$$

The mathematical expectation of random variable Φ_j is bounded by interval $\varphi_{0j}\in[\pi-1/3;\pi+1/3]$, which is determined by the chosen approximation of the distribution density (45). By calculating the second integral from the system of equations (46):

$$\frac{(2\pi-\varphi_{0j})^3}{6\pi}+\frac{\varphi_{0j}{}^3}{6\pi}+C_{c3}\frac{4\pi}{9}+C_{s3}\frac{4\pi\varphi_{0j}-4\pi^2}{3}\to 0, \quad (50)$$

an expression for determining the coefficient $C_{c3}=0$ is obtained. For example, for a random variable with value $\varphi_j=\pi$, we obtain the expansion coefficients of the distribution function:

$$\begin{gathered}C_{s3}=0,\ C_{c3}=-\frac{3\pi}{4}\to-\frac{1}{2\pi},\ \frac{1}{2\pi}\ge|C_{s3}|,\\ f_j(\Phi_j)=\left(1/(2\pi)\right)\cdot\left(1-\cos(\Phi_j)\right).\end{gathered} \quad (51)$$

Distribution densities (42) and (51) have the same mathematical expectation value of $M[\Phi_j]=\pi$. However, as expected under conditions (46), the square of the standard deviation for the distribution density of type (51), the smallest of the possible distributions with canonical form (45), is $M\left[\left(\Phi_j-\pi\right)^2\right]=\left(3\pi^2-2\right)/9$, instead of $M\left[\left(\Phi_j-\pi\right)^2\right]=3\pi^2/9$ for the distribution density of type (42). One of the limiting cases of representing the distribution density $f_j(\Phi_j)$ of random variable Φ_j as a delta function $\delta(\Phi_j-\varphi_j)$:

$$\begin{gathered}f_j(\Phi_j)=\delta(\Phi_j-\varphi_j)=1/(2\pi)+\\ +\sum_{n=1}^{\infty}\cos(n\varphi_j)\cos(n\Phi_j)+\sin(n\varphi_j)\sin(n\Phi_j),\end{gathered} \quad (52)$$

which is nonnegative $\delta(\Phi_j-\varphi_j)\ge 0$ and has the statistical characteristics:

$$\begin{cases}\int_0^{2\pi}\delta(\Phi-\varphi_j)d\Phi\equiv 1,\\ \varphi_{0j}=\int_0^{2\pi}\Phi\delta(\Phi-\varphi_j)d\Phi\equiv\varphi_j,\\ \sigma_{\varphi_j}^2=\int_0^{2\pi}\left(\Phi-\varphi_{0j}\right)^2\delta(\Phi-\varphi_j)d\Phi\equiv 0.\end{cases} \quad (53)$$

However, since the expansion coefficients $C_{sn}=\sin(n\varphi_j)$, $C_{cn}=\cos(n\varphi_j)$ for n = 1.2 are not equal to zero, as is ensured for the distribution density (41), the transition conditions (32), (33) are not satisfied. The condition that the random variables Θ_{cj}, Θ_{sj} (26) are centered with zero mathematical expectation is also not satisfied. This limiting case corresponds to representing the material flow as a deterministic process with a constantly repeating single realization of the process. The second limiting case of representing the stochastic material flow is the approach in which random variables are represented as dependencies:

$$\begin{gathered}\Phi_j=\left(\Phi_1+\varphi_j-\varphi_1\right)\ \mathrm{mod}\ (2\pi),\\ j>1,\quad f_1(\Phi_1)=1/(2\pi).\end{gathered} \quad (54)$$

This representation is equivalent to a random phase shift of the initial realization of the stochastic material flow and can be used to model material flow in optimization problems for transport system flow parameters.

For the transformation of random variables Φ_j, canonical representation (25) reduces to the form:

$$\begin{gathered}\gamma(\tau)=A_1\cos\left(\omega_1\tau-\Phi_1\right)+\\ +\sum_{j=2}^{\infty}A_j\cos\left(\omega_j\tau-\left(\Phi_1+\varphi_j-\varphi_1\right)\right),\end{gathered} \quad (55)$$

in which the realization of the input material flow is generated by a random shift of a single experimental realization of the input material flow.

The realization thus generated has the same statistical characteristics (21)–(22) as the initial realization (20), and, in addition, the distribution function of the input material flow values exactly corresponds to the initial experimental realization, since a constant phase shift between the harmonics of the canonical expansion of the random process is ensured.

Representation (45) allows one to generate a stochastic material flow with given statistical characteristics, a correlation function, and an approximate distribution density of the input material flow values. Constructing a more accurate approximation for the distribution density requires using more terms in the expansion of the distribution density (41). This complicates the calculation of the expansion coefficients due to the imposition of the non-negativity condition on the distribution density. In this case, an alternative approach is to use a neural network to determine the coefficients of the canonical expansion: the coefficients of the canonical expansion (3) with the expansion coefficients Θ_0, Θ_{cj}, Θ_{sj}. Using an affine transformation, we represent the centered random variables Θ_{cj}, Θ_{sj} as follows:

$$\Theta = \Omega - m_\Omega, \qquad M[\Omega] = m_\Omega, \tag{56}$$

where Ω is a random variable that has a Beta distribution with parameters α, β of the Beta function $B(\alpha, \beta)$:

$$f(\Omega) = \Omega^{\alpha-1}(1-\Omega)^{\beta-1} \big/ B(\alpha,\beta),$$
$$B(\alpha,\beta) = \int_0^1 \Omega^{\alpha-1}(1-\Omega)^{\beta-1} d\Omega . \tag{57}$$

with a mean of m_Ω and a variance of σ_Ω^2 of the random variable Ω:

$$m_\Omega = \alpha \big/ (\alpha+\beta),$$
$$\sigma_\Omega^2 = \alpha\beta \Big/ \left((\alpha+\beta)^2(\alpha+\beta+1)\right). \tag{58}$$

to calculate the parameters α, β of the Beta function $B(\alpha, \beta)$:

$$\begin{cases} \alpha = \left(m_\Omega^2(1-m_\Omega) - m_\Omega\sigma_\Omega^2\right) \big/ \sigma_\Omega , \\ \beta = \left(m_\Omega(1-m_\Omega)^2 - (1-m_\Omega)\sigma_\Omega^2\right) \big/ \sigma_\Omega^2 . \end{cases} \tag{59}$$

As in the case of approximation (41), the problem of determining the distribution law of the values of the input material flow is reduced to the selection of values m_Ω to determine the distribution law of the random variable Ω. The basic diagram for calculating the coefficients $m_{\Omega cj}$, $m_{\Omega sj}$ is presented in Fig. 1.

The following constraints were introduced when calculating mathematical expectations:

$$m_{\Omega cj} > 0, \; m_{\Omega sj} > 0. \tag{60}$$

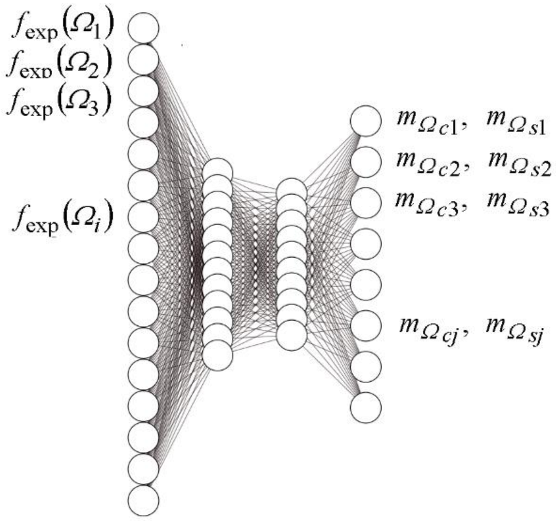


Fig. 1. Model for calculating the values of mathematical expectations $m_{\Omega cj}$, $m_{\Omega sj}$ (multilayer perceptron)

The input parameters of the neural network are the density function of the distribution of input material flow values, constructed based on experimental data. The output parameters are the expected values of random variables Θ_{cj}, Θ_{sj}. The choice of input and output nodes allows the set of experimental values that form the distribution density $f_{\exp}(\Omega)$ of input material flow values to be assigned expected values for constructing the theoretical distribution density of input material flow values. The Pearson chi-square test was used to evaluate the quality of the approximation.

This approach can be used to determine the coefficients C_{cn}, C_{sn} (41) for a large number of terms. The analytical approach demonstrated in this paper is effective for a small number of coefficients C_{cn}, C_{sn}.

Analysis of results

To qualitatively demonstrate the method for generating a stochastic flow of material entering the input of a transport conveyor, a series of computational experiments was conducted using both published experimental data and synthetic sequences generated with predefined statistical properties. Six representative material flow realizations, taken from independent studies of conveyor systems in the mining and processing industries [11, 19, 31–34], were used as input data (Fig. 2). For consistency of presentation and ease of reference in the subsequent analysis, the experimental material flow realizations are designated as A-flow, B-flow, C-flow, D-flow, E-flow, and F-flow, respectively. Previously, a fragmented statistical analysis of two flows—A-flow and C-flow—was performed in [29, 35] as part of developing methods for piecewise linear and harmonic approximation of input signals. In this paper, the ergodicity assumption is adopted for all six flows, which is justified by using a single long-term realization at each site.

This approach allows us to replace ensemble averaging with temporal averaging over a single trajectory, yielding statistically significant estimates of characteristics without the need for additional experimental data.

The first stage of the analysis involves normalizing all six experimental realizations of the input material flows, converting them to dimensionless form using parameters (4) – (5), which enables a unified comparison of disparate data.

This procedure ensures the unification of disparate data sets, eliminating the influence of the absolute scales of the input material flow values and the observation time interval for different process objects. As a result, each flow is represented as a stationary process with unit variance and a normalized time interval of 1, creating a unified coordinate space for the correct application of aggregated similarity criteria (1), comparative analysis of statistical and correlation properties, and model validation, regardless of the initial physical units and operating conditions.

Fig. 3 shows six experimental realizations of input material flows in dimensionless form (black color – 1).

For each, the following are additionally displayed:

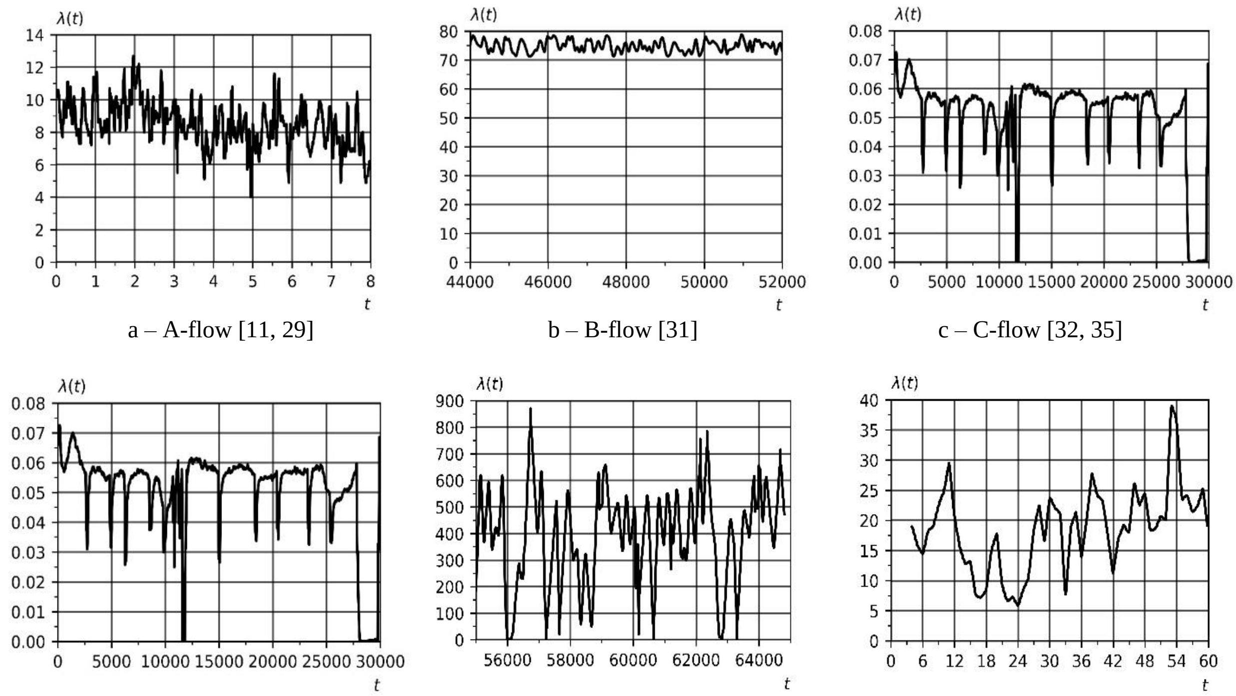
a) approximated experimental realization $\gamma_\alpha(\tau)$ (read color - 2), obtained by projecting the original signal onto an orthogonal system of coordinate functions of the canonical decomposition (25) with preservation of the first N = 32 harmonics and subsequent reconstruction of the deterministic component using formula (29). This allows for the effective suppression of additive measurement noise while preserving the key dynamic and statistical characteristics of the input material flow;

b) generated model realization $\gamma_g(\tau)$ (green color - 3), formed using the proposed generator based on a two-term analytical decomposition of the distribution density (45) with fixed values of the similarity criteria π_1, π_2 corresponding to the experimental ones. A visual comparison demonstrates the degree of qualitative correspondence between the three curves for each flow: the generated trajectories reproduce the characteristic oscillations, amplitude extremes, and temporal structure of fluctuations inherent in real data.

Table 1 contains the calculated statistical characteristics and the values of the aggregated similarity criteria (1) for all six experimental material flows in dimensionless form. A detailed description of the physical recording conditions, process flow diagrams, and operating modes corresponding to each of the flows is given in the original sources [11, 31–34]. These works also contain data on the types of transported material, the design features of the conveyor lines, and the flow measurement methods, which allows us to interpret the observed statistical differences from the standpoint of production factors.

A comparative analysis of the statistical characteristics and aggregated similarity criteria (Table 1) provides a qualitative and quantitative understanding of the degree of similarity and differences between the six experimental implementations of the input material flows.

In particular, the C- and D- flows demonstrate a high degree of similarity according to the aggregate criterion π_1. The similarity criterion allows us to consider the material flows as representative examples of one class of transport modes with moderate variability, typical for systems with buffer storage and adjustable feed.



a – A-flow [11, 29] b – B-flow [31] c – C-flow [32, 35]

d – D-flow [33] e – E-flow [34] f – F-flow [19]

Fig. 2. Experimental implementations of input material flows

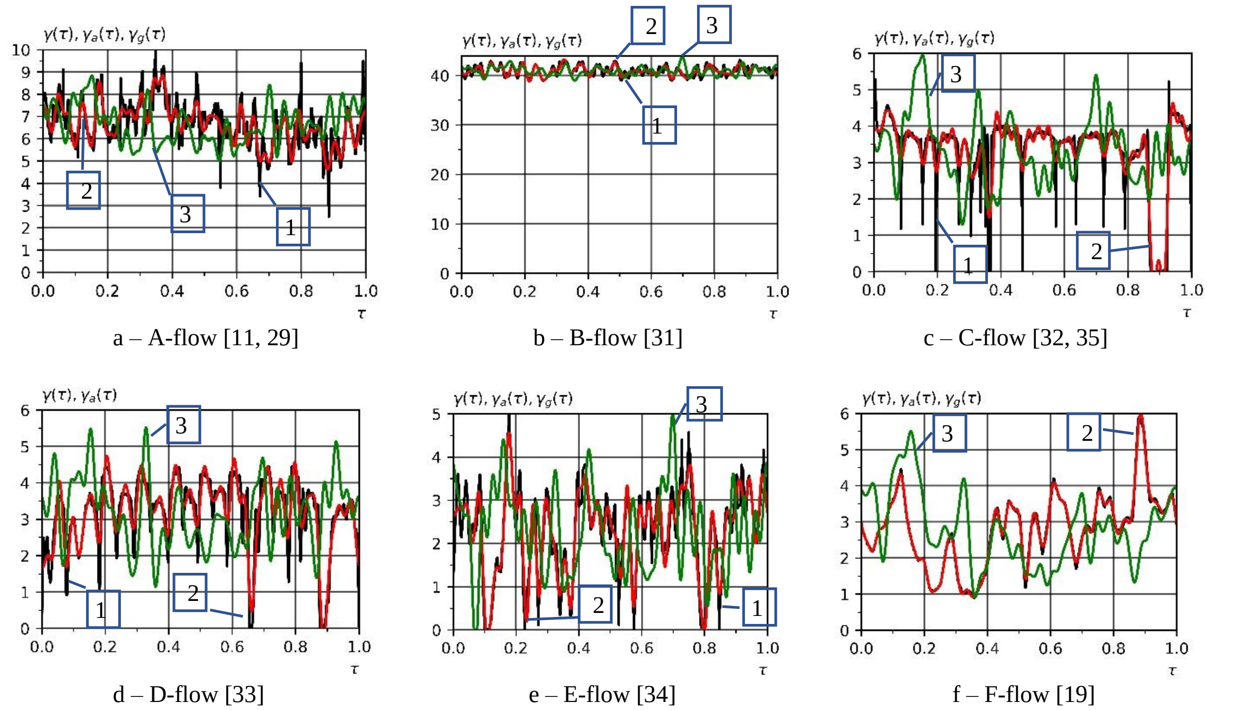


Fig. 3. Dimensionless realizations of input material flows: $\gamma(\tau)$ – experimental realization (black color – 1); $\gamma_\alpha(\tau)$ – approximated experimental realization (read color –2); $\gamma_g(\tau)$– generated realization (green color – 3)

Table 1 – **Statistical characteristics and values of similarity criteria of input material flows**

Parameter	**Experimental realization of the input material flow**					
	A-flow [11, 29]	B-flow [31]	C-flow [32, 35]	D-flow [33]	E-flow [34]	F-flow [19]
Mathematical expectation m_λ	8.51	74.85	0.05	4663.60	394.08	18.52
Standard deviation σ_λ	1.32	1.83	0.0153	1469.80	171.65	6.61
min [$\lambda(t)$]	4.00	71.21	0.0	0.0	0.0	38.98
max [$\lambda(t)$]	12.70	78.82	0.0727	6542.80	870.00	5.86
Aggregate similarity criterion π_1	6.5	41.0	3.3	3.2	2.30	2.80
Aggregate similarity criterion π_2	100	77	44	77	71	20

To quantitatively assess the temporal structure of fluctuations in each of the six streams, empirical correlation functions $k(\vartheta)$ were constructed using formula (22) based on a single implementation (Fig. 4).

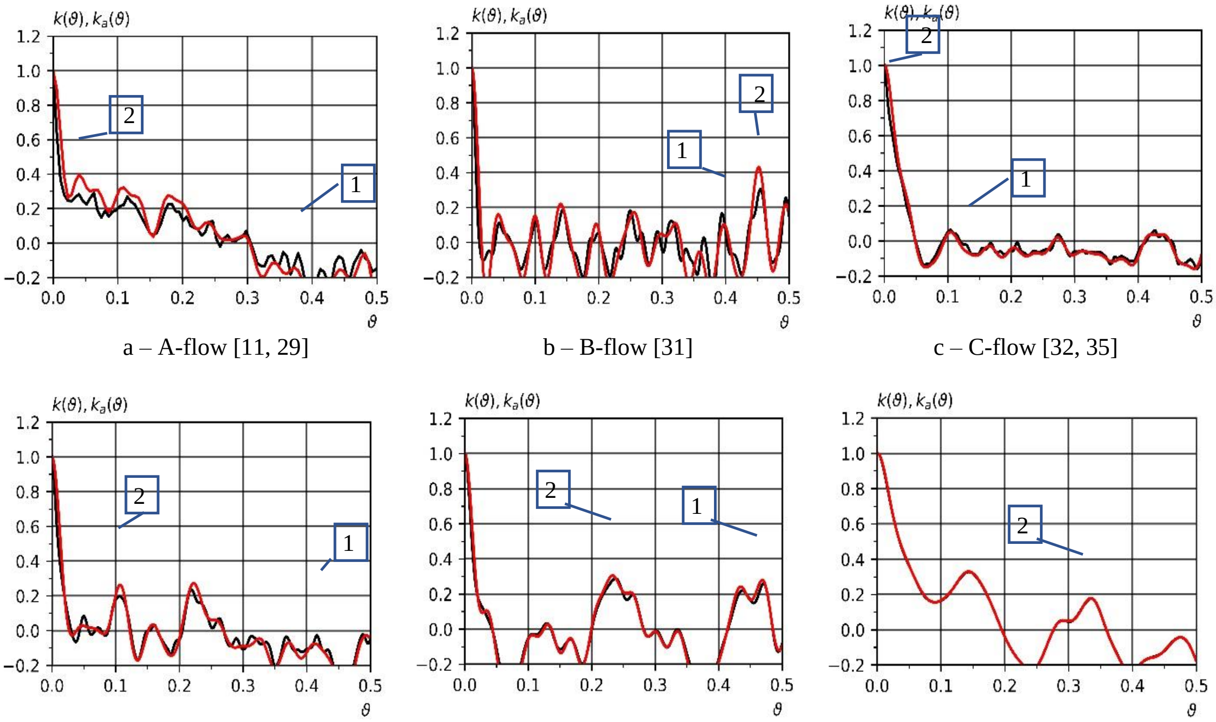
To calculate the aggregated similarity criterion π_2 the characteristic correlation time of each input material flow is determined based on the graphs of normalized correlation functions (Fig. 4). Similarity criterion π_2 is calculated as the ratio of the total experimental measurement interval during which the experimental measurements were conducted to the characteristic correlation time of the input material flow values.

This time interval is used as the decomposition period when representing each experimental realization of the input material flow as a Fourier series, in accordance with the canonical decomposition of the stochastic process described in equation (19). This interval is used to decompose the input material flow realizations into a Fourier series in accordance with the canonical representation of the input material flow (15). The aggregated similarity criterion π_2 acts as a scaling factor, ensuring the correct comparison and comparative analysis of disparate input material flows.

Normalized correlation function $k(\vartheta)$, constructed in dimensionless coordinates, serves as a visual and quantitative criterion for the similarity of input material flows. The normalized correlation function $k(\vartheta)$ integrates information about the spectral composition of the Fourier coefficients in the canonical expansion (19), (25), reflecting both the characteristic scale of correlated fluctuations and the form of oscillation damping at different time horizons. Flows with similar normalized correlation functions $\pi_3(\vartheta) = k(\vartheta)$can be classified as similar in the stochastic sense.

Thus, the coincidence of the normalized correlation functions serves as a fundamental basis for identifying the generalized distribution law of random values of the input material flow.

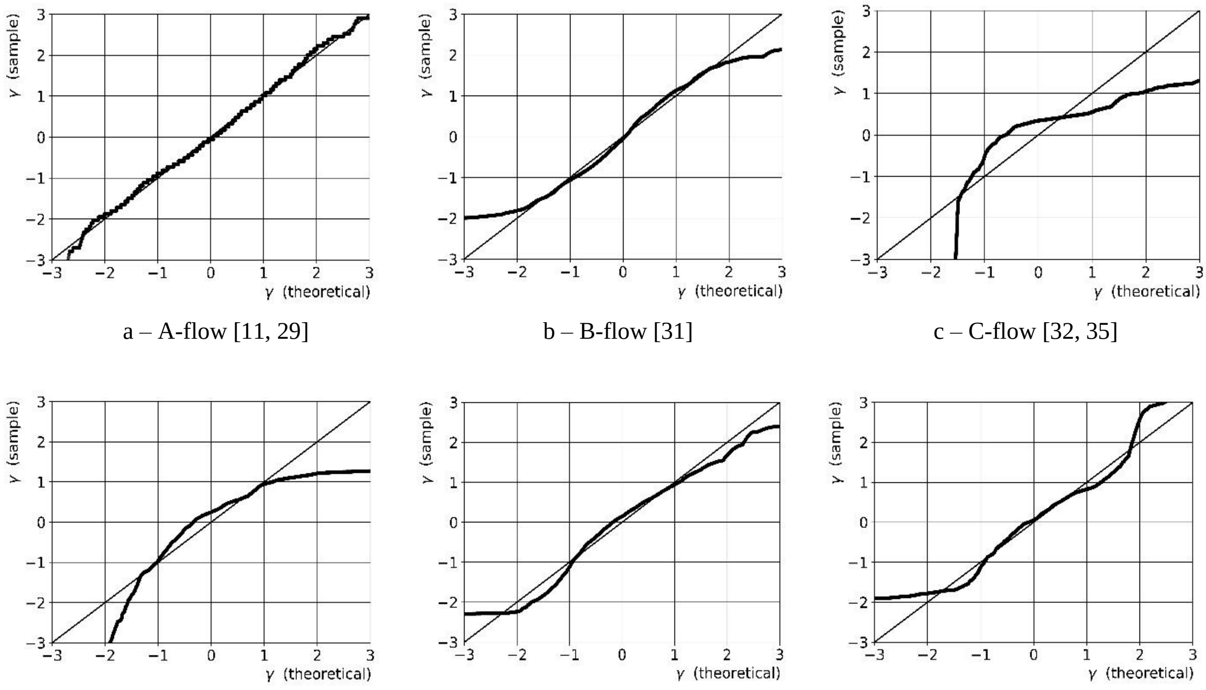
Fig. 5 shows the quantile-quantile diagrams (Q-Q plots) of all six flows (A-flow – F-flow), constructed based on the values of the dimensionless approximated realization $\gamma_\alpha(\tau)$ and the standard normal distribution, intended for visual and quantitative assessment of the degree of correspondence of the empirical distribution of the values of the dimensionless approximated realization of the input material flow to the theoretical standard normal distribution $N(0,1)$.



a – A-flow [11, 29] b – B-flow [31] c – C-flow [32, 35]

d – D-flow [33] e – E-flow [34] f – F-flow [19]

Fig. 4. Correlation functions $k(\vartheta)$ of input material flows



a – A-flow [11, 29] b – B-flow [31] c – C-flow [32, 35]

d – D-flow [33] e – E-flow [34] f – F-flow [19]

Fig. 5. Q-Q plots (Quantile-to-Quantile) for comparing the empirical distribution of values of the dimensionless approximated realization of the input material flow $\gamma_\alpha(\tau)$ with the theoretical normal distribution law

The theoretical quantiles of the normal law are plotted along the abscissa axis, and the empirical quantiles calculated from the sorted values of the approximated realization on interval $\tau \in [0,1]$ are plotted along the ordinate axis.

The bisector of type $y = x$ (thin line) serves as the base trajectory, the deviation from which indicates systematic deviations from the normal distribution law. Just as the normalized correlation function $k(\vartheta)$ serves as a functional similarity criterion $\pi_3(\vartheta)$, reflecting the spectral composition of the coefficients of the canonical expansion (19)–(25), the quantile-quantile diagram

$$\gamma_{sample}(\gamma_{theoretical})$$

serves as an independent functional similarity criterion

$$\pi_4(\gamma_{theoretical}) = \gamma_{sample}(\gamma_{theoretical}),$$

assessing the differences in the distribution laws of the random values of the input material flows. In comparative analysis, input flows should be considered stochastically similar if the following conditions are simultaneously met:

a) close values of the aggregated similarity criteria π_1, π_2;

b) similar behavior of functional similarity criterion $\pi_3(\vartheta)$ (nearly identical normalized autocorrelation functions);

c) similar behavior of functional similarity criterion

$$\pi_4(\gamma_{theoretical}),$$

which characterizes the similar nature of the deviation of the Q-Q function of type

$$\gamma_{sample}(\gamma_{theoretical})$$

from the bisector (the same bending shape, similar systematic deviations in the central region and tails).

Satisfaction of these conditions means that the flows are generated by the same statistical mechanism and allow the use of a single generalized distribution law for the random coefficients of the canonical decomposition.

In particular, the high degree of coincidence between the Q-Q diagrams of the C-flow and D-flow flows (Fig. 5) confirms the validity of applying the same distribution density approximation to them—both the analytical binomial (45) and the parametric beta distribution with identical parameter values (59). This allows us to classify these material flows into a separate class, which will enable the use of generalized models for modeling such material flows.

A similar similarity in Q-Q plots is also observed for the B-flow and E-flow pair, suggesting that these flows belong to the same subclass with pronounced heavy tails.

Thus, the combined use of the normalized autocorrelation function and Q-Q plots forms a comprehensive multi-component system of functional similarity criteria, significantly superior in information content to traditional scalar indicators and enabling an objective classification of input material flows even with limited experimental data.

Fig. 6 shows the distribution histograms of instantaneous values of the dimensionless material flow for all six studied modes (A-flow – F-flow). For each A-flow – F-flow, three histograms are given in a single coordinate field:

a) a histogram constructed directly from the original experimental realization of input material flow $\gamma(\tau)$;

b) a histogram constructed from an approximated experimental realization of input material flow $\gamma_a(\tau)$;

c) a histogram constructed from a generated realization of input material flow $\gamma_g(\tau)$.

The generation of model realizations is performed using a simplified canonical decomposition of the input material flow (25), in which the distribution density $f_j(\Phi_j)$ of random coefficient Φ_j was specified by a two-term analytical expression (45). Random values of this coefficient were generated using inverse transform sampling, which guarantees strict adherence to the target distribution law while maintaining all the required statistical properties of the process.

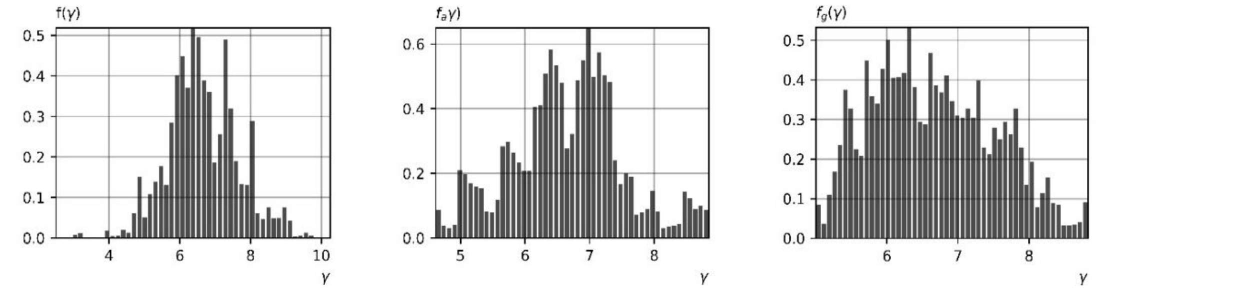
The conducted histogram analysis (Fig. 6) confirms the high efficiency of the proposed approach. As expected, a fairly close match in the distribution shape is observed between the original experimental implementation $\gamma(\tau)$ and its approximated version $\gamma_a(\tau)$, indicating the correctness of the noise filtering procedure and the selection of the number of retained harmonics of the canonical decomposition. The correlation functions of the approximated experimental implementation $\gamma_a(\tau)$ and the generated implementation $\gamma_g(\tau)$ coincide, which is a direct consequence of the orthogonality of the basis functions and strict fulfillment of the centering conditions in the decomposition (25).

The observed differences between the histograms of approximated experimental realization $\gamma_a(\tau)$ and generated realization $\gamma_g(\tau)$ are systematic and are fully explained by the chosen level of approximation of the distribution density $f_j(\Phi_j)$ of the random coefficient Φ_j.

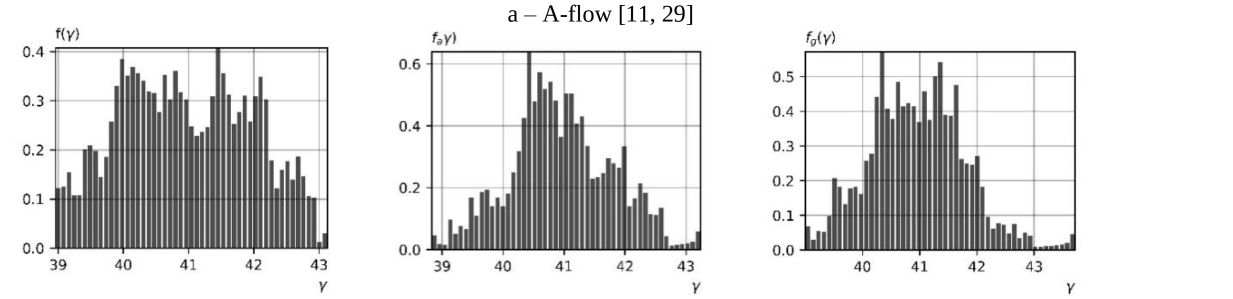
In this article, the analytical decomposition (45) is used, which, being the minimally possible non-constant representation, necessarily limits the class of reproducible distributions.

Increasing the number of terms in the decomposition or switching to more flexible parametric forms (beta distribution, mixtures of distributions, neural network approximation) leads to an asymptotic convergence of the model and empirical histograms in accordance with theoretical convergence estimates following from conditions (46).

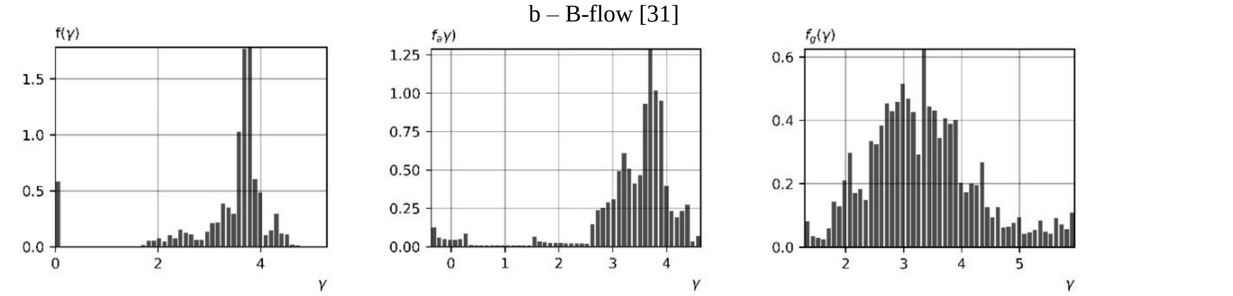
Thus, the achieved level of agreement, even with just two terms, should be considered satisfactory, especially considering that the model parameters were identified based on a single experimental realization of each flow.



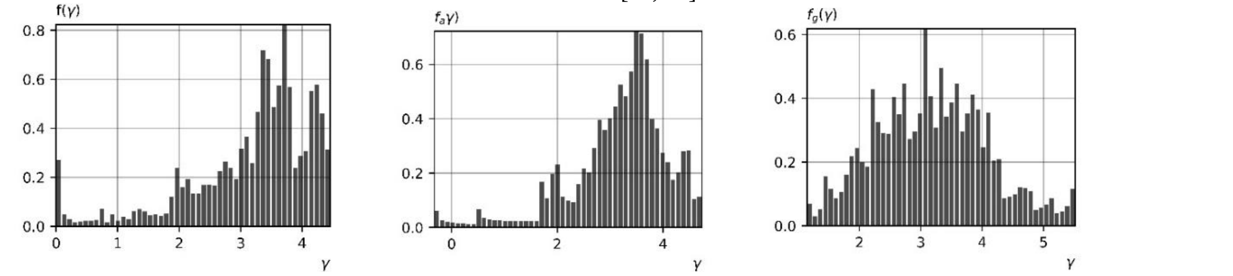
a – A-flow [11, 29]



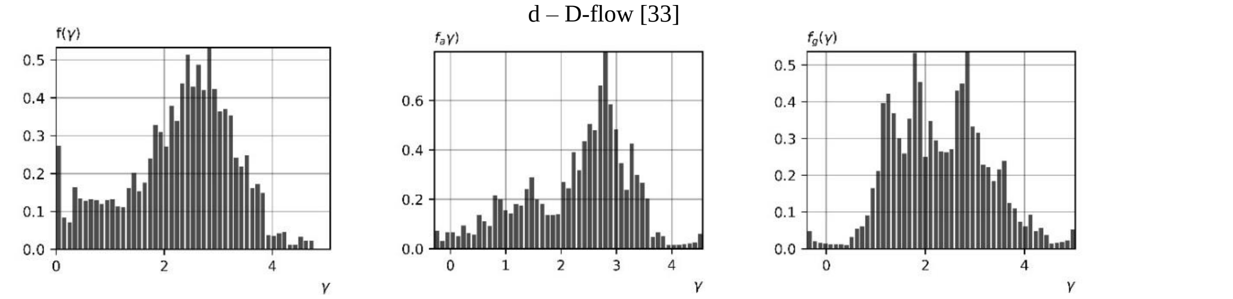
b – B-flow [31]



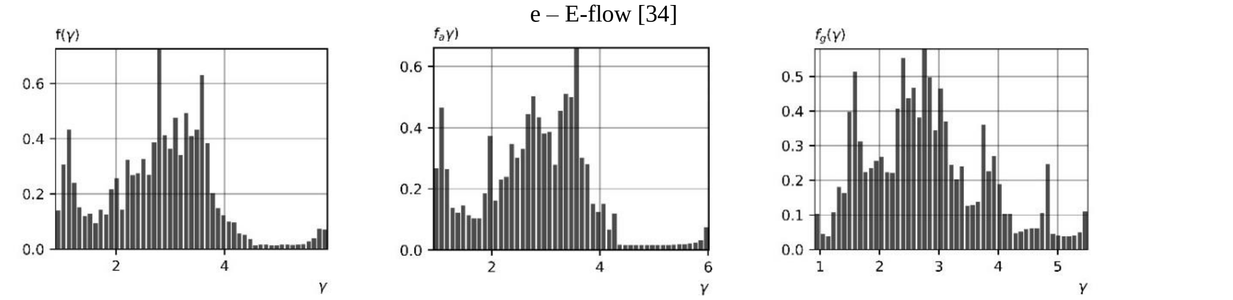
c – C-flow [32, 35]



d – D-flow [33]



e – E-flow [34]



f – F-flow [19]

Fig. 6. Histograms of the distribution of values of the input material flows: experimental implementation $\gamma(\tau)$; approximated experimental implementation $\gamma_a(\tau)$; generated implementation $\gamma_g(\tau)$

So, the methodological complex developed in this article enables the solution of a number of fundamental and applied problems.

The research includes:

1) dimensionless normalization of heterogeneous material flows differing by orders of magnitude in average productivity, variance, and characteristic time scales, enabling their direct comparison and classification;

2) quantitative assessment of the degree of stochastic similarity based on a system of aggregated scalar criteria and functional similarity criteria (focused on the normalized correlation function of the Q-Q diagram), significantly expanding traditional approaches;

3) approximation of experimental records;

4) determination of the type and parameters of the distribution law of random coefficients of the canonical expansion based on a single realization without the use of ensemble data;

5) development of a highly accurate generator of stochastic stationary input flows capable of reproducing not only the first two moments and the correlation structure, but also the full one-dimensional distribution density of instantaneous values with controlled error.

The use of dimensionless modeling in combination with a developed system of similarity criteria enables, for the first time, the transition from a set of individual, object-specific empirical observations to a universal, scalable, and transferable physical and mathematical model of a stationary stochastic input material flow, suitable for a wide range of conveyor systems, regardless of their nominal throughput, type of raw material transported, route length, and the control systems used.

Conclusion

This research addresses the pressing scientific and practical problem of creating a universal mathematical model of a stationary stochastic input material flow for industrial conveyor transport systems. A modeling approach based on a simplified canonical decomposition of a stochastic process with analytical and parametric specification of the distribution density of random coefficients is proposed and theoretically substantiated.

The key results of this study are:

1) a simplified canonical decomposition of a stationary ergodic input material flow is developed, allowing for the reproduction of the specified mathematical expectation, variance, and correlation function from a single experimental realization with a minimum number of random coefficients;

2) analytical expressions is defined for approximating the distribution density of a random coefficient with guaranteed fulfillment of the conditions of centering, normalization, and non-negativity;

3) the system of multilevel criteria for stochastic similarity of input material flows is expanded, including: the previously used aggregated scalar similarity criteria [28]; a functional similarity criterion based on a normalized correlation function; a functional similarity criterion based on quantile-quantile diagrams. The combined application of these criteria enables objective classification of heterogeneous input material flows and identification of classes of stochastically similar modes, even with only one long-term implementation.

4) based on six independent experimental implementations of real mining conveyor systems (A-F flows), the accuracy of the proposed generator of stochastic input material flows is demonstrated. Qualitative and quantitative agreement between the generated trajectories, correlation functions, and one-dimensional distribution densities with experimental data is demonstrated.

5) an effective method for dimensionless normalization and unification of heterogeneous flows differing by orders of magnitude in productivity and time scales is developed, enabling model transfer between facilities of varying capacity, configuration, and type of transported raw material.

The proposed approach enables the construction of a generator of stationary stochastic input material flow, completely determined by a single process implementation.

The results of this work provide a solid foundation for the further development of stochastic modeling methods, the synthesis of robust conveyor line control systems, and the optimization of buffer capacities and routing in extensive transport networks. The proposed model and generator can be directly used in simulation systems and digital management platforms for mining enterprises.

A promising direction for further research is to expand the proposed approach to non-stationary processes and adapt similarity criteria to multidimensional or branching material supply systems.

Conflicts of interest

The authors declare that they have no conflicts of interest in relation to the current study, including financial, personal, authorship, or any other, that could affect the study, as well as the results reported in this paper.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

References

1. Alharbi, F. and Luo, S. (2024), "A Review of Fault Detecting Devices for Belt Conveyor Idlers", *Journal of Mechanical Engineering Science and Technology* (JMEST), vol. 8(1), pp. 4–15, doi: https://doi.org/10.17977/um016v8i12024p039
2. Ji, J., Miao, C., Li, X. and Liu, Y. (2021), "Speed regulation strategy and algorithm for the variable-belt-speed energy-saving control of a belt conveyor based on the material flow rate", *PLoS ONE*, vol. 16, no. 2, e0247279, doi: https://doi.org/10.1371/journal.pone.0247279

3. Stadnik, M., Semenchenko, D., Semenchenko, A., Belytsky, P., Virych, S. and Tkachov, V. (2019), “Improving energy efficiency of coal transportation by of a combine and a mine face conveyor”, *Eastern-European Journal of Enterprise Technologies*, vol. 8(97), no. 1, pp. 60–70, doi: http://doi.org/10.15587/1729-4061.2019.156121
4. Pihnastyi, O., Khodusov, V. and Kotova, A. (2022), “The problem of combined optimal load flow control of main conveyor line“, *Acta Montanistica Slovaca*, vol. 27, no. 1, pp. 216–229, doi: https://doi.org/10.46544/AMS.v27i1.16
5. Carvalho R. and Nascimento, G. L. (2020), “A UAV-based framework for semi-automated thermographic inspection of belt conveyors in the mining industry”, *Sensors*, no. 22(8), pp. 1–19, doi: http://dx.doi.org/10.3390/s20082243
6. Lu, Ya. and Li, Q. (2019), “A regression model for prediction of idler rotational resistance on belt conveyor”, *Measurement and Control*, no. 52(5), pp. 441–448, doi: http://dx.doi.org/10.1177/0020294019840723
7. Shareef, I.R. and Hussein, H.K. (2021), “Implementation of Artificial Neural Network to Achieve Speed Control and Power Saving of a Belt Conveyor System”, *Eastern-European Journal of Enterprise Technologies*, no. 2(2(110)), pp. 44–53, doi: https://doi.org/10.15587/1729-4061.2021.224137
8. Bardzinsk, P., Walker, P. and Kawalec, W. (2018), “Simulation of random tagged ore flow through the bunker in a belt conveying system“, *International Journal of Simulation Modelling*, no. 4, pp. 597–608, doi: https://doi.org/10.2507/IJSIMM17(4)445
9. Pang, Y., Lodewijks, G. and Liu, X. (2016), “Determination of Acceleration for Belt Conveyor Speed Control in Transient Operation”, *International Journal of Engineering and Technology*, vol. 8(3), pp. 206–211, doi: http://dx.doi.org/10.7763/IJET.2016.V8.886
10. He D., Pang Yu., Lodewijks G. and Liu X. (2018), “Healthy speed control of belt conveyors on conveying bulk materials”, Powder Technology, no 327, pp. 408–419, doi: https://doi.org/10.1016/j.powtec.2018.01.002
11. Zeng, F., Yan, C., Wu, Q. and Wang, T. (2020), “Dynamic behaviour of a conveyor belt considering non-uniform bulk material distribution for speed control”, *Applied Sciences*, vol.10, no 13, 4436, doi: https://doi.org/10.3390/app10134436
12. Kiriia, R. and Shyrin, L. (2019), “Reducing the energy consumption of the conveyor transport system of mining enterprises”, *International Conference Essays of Mining Science and Practice*, vol. 109, pp. 1-8, doi: https://doi.org/10.1051/e3sconf/201910900036
13. Pihnastyi, O.M., Khodusov, V. and Kotova, A. (2023), “Mathematical model of a long-distance conveyor”, *Mining Science*, vol. 30, pp. 27-43, doi: https://doi.org/10.37190/msc233002
14. Bahrun, S., Yusoff, M.S., Said, M.S. and Hassan, A. (2022), “Coal Flowability Control on Preventing Spillage on Conveyor Belt Through Modeling and Simulation for Improving its Performance”, *J. Adv. Res. Fluid Mech. Therm. Sci*., vol. 90, pp. 130–145, doi: https://doi.org/10.37934/arfmts.90.1.130145
15. Vasic, M., Miloradovic, N. and Blagojevic, M., (2021), “Speed control high power multiple drive belt conveyors”, *Research and Development in Heavy Machinery*, vol. 27(1), pp. 9–15, doi: https://doi.org/10.5937/IMK2101009V
16. (2025), *Sustainable transformation of the mining industry*, SIEMENS, Mining industry, available at: https://xcelerator.siemens.com/global/en/industries/mining-industry.html
17. Mascenik, J. and Coranic, T. (2024), “The Influence of Changing Belt Loading Conditions on the Operational Condition of the Belt Transmission”, *Actuators*, vol. 13, 506, doi: https://doi.org/10.3390/act13120506
18. Lubenets, T.M., Koroviaka, Ye.A., Snigur, V.H., Tkachuk, A.V. and Rastsvietaiev, V.O. (2023), “Theoretical model of random freight flow distribution in the conveyor transport line of the coal mine”, *Naukovyi Visnyk Natsionalnoho Hirnychoho Universytetu*, vol. 6, pp. 12–18, doi: https://doi.org/10.33271/nvngu/2023-6/012
19. Curtis, A., and Sarc, R. (2021), ”Real-time monitoring volume flow, mass flow and shredder power consumption in mixed solid waste pro-cessing”, *Waste Management*, no 131, pp. 41–49, doi: https://doi.org/10.1016/j.wasman.2021.05.024
20. Jurdziak, L. and Bajda, M. (2025), “Balancing Load and Speed: A New Approach to Reducing Energy Use in Coal Conveyor Systems”, *Energies*, vol. 18, 4716, doi: https://doi.org/10.3390/en18174716
21. Kirjanów, A. (2016), “The possibility for adopting an artificial neural network model in the diagnostics of conveyor belt splices”, *Interdisciplinary issues in mining and geology*, vol. 6, pp. 1–11, available at: https://www.researchgate.net/publication/314952302
22. Klippel, E., Oliveira, R., Maslov, D., Bianchi, A., Silva, S. and Garrocho, C. (2021), “Embedded Edge Artificial Intelligence for Longitudinal Rip Detection in Conveyor Belt Applied at the Industrial Mining Environment”, *Proceedings of the 23rd International Conference on Enterprise Information Systems*, vol. 1, pp. 496–505, available at: https://www.scitepress.org/Papers/2021/104472
23. Lu, Y. and Li, Q. (2019), “A regression model for prediction of idler rotational resistance on belt conveyor”, *Measurement and Control*, vol. 52(5), pp. 441–448, doi: https://doi.org/10.1177/0020294019840723
24. Andrejiova, M., Grincova, A., Marasova, D. and Kimakova, Z. (2025) “Machine Learning Methods as a Tool for Analysis and Prediction of Impact Resistance of Rubber–Textile Conveyor Belts”, *Appl. Sci*., vol. 15, article number 8511, doi: https://doi.org/10.3390/app15158511
25. Wiecek, D., Burduk, A. and Kuric, I. (2019), “The use of ANN in improving efficiency and ensuring the stability of the copper ore mining process”, *Acta Montanistica Slovaca*, no 24(1), pp. 1–14, available at: https://actamont.tuke.sk/pdf/2019/n1/1wiecek.pdf
26. Pihnastyi, O. and Ivanovska, O. (2021), “Improving the Prediction Quality for a Multi-Section Transport Conveyor Model Based on a Neural Network”, *8th International Scientific Conference "Information Technology and Implementation", CEUR Workshop Proceedings*, December 2021vol. 3132, pp. 24–38, available at: https://ceur-ws.org/Vol-3132/Paper_3.pdf
27. Xinglei, L. and Hongbin, Yu. (2015), “The Design and Application of Control System Based on the BP Neural Network”, *III International Conference on Mechanical Engineering and Intelligent Systems* (ICMEIS), pp. 789-793, available at: https://www.atlantis-press.com/proceedings/icmeis-15/25837377
28. Pihnastyi, O., Korolova, Y., Kozhevnikov, G. and Burduk, A. (2025), “Method for constructing similarity criteria for stochastic stationary material flows”, *Advanced Information Systems*, vol. 9, no. 3, pp. 102-111, doi: https://doi.org/10.20998/2522-9052.2025.3.12
29. Pihnastyi, O., Usik, V., and Burduk, A. (2025), “Stationary stochastic input flow modeling of branched conveyor systems”. *Advanced Information Systems*, vol. 9 no. 2, pp. 25–35., doi: https://doi.org/10.20998/2522-9052.2025.2.04

30. Pihnastyi, O., and Kudii, D. (2023), "Development of a method for calculating statistical characteristics of the input material flow of a transport conveyor", Eastern-European Journal of Enterprise Technologies, vol. 5, no. 2(125), pp. 87–96, doi: https://doi.org/10.15587/1729-4061.2023.289931
31. Pihnastyi, O., Kozhevnikov, G., and Burduk, A. (2024), "Constructing generators of values of an input material flow of conveyor-type transport systems", *Advanced Information Systems*, vol. 8, no. 4, pp. 74–81, doi: https://doi.org/10.20998/2522-9052.2024.4.10
32. Bhadani, K., Asbjörnsson,G., Hulthén, E., Hofling, K. and Evertsson, M. (2021), "Application of Optimization Method for Calibration and Maintenance of Power-Based Belt Scale", *Minerals*, vol. 11, no. 4, doi: https://doi.org/10.3390/min11040412
33. Kawalec, W., and Król, R. (2021), "Generating of electric energy by declined overburden conveyor in continuous surface mine", *Energies*, vol. 14(13), pp. 1–11, doi: https://doi.org/10.3390/en14134030
34. Doroszuk, B., Król, R., and Wajs, J. (2021), "Simple design solution for harsh operating conditions: redesign of conveyor transfer station with reverse engineering and DEM simulations", *Energies*, vol. 14, no. 13, article number 113, doi: https://doi.org/10.3390/en14134008
35. Pihnastyi, O., Kozhevnikov, G., and Burduk, A. (2024), "Method for generating a data set for training a neural network in a transport conveyor model", *Advanced Information Systems*, vol. 8, no. 2, pp. 79–88, doi: https://doi.org/10.20998/2522-9052.2024.2.09

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Генератор значень стаціонарного вхідного потоку матеріалу для конвеєрних систем із заданою функцією кореляції та одномірним законом розподілу

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Анотація. Об'єкт дослідження – стохастичний стаціонарний вхідний потік матеріалу транспортної системи конвеєрного типу. **Предмет дослідження** - метод генерації реалізацій стаціонарного стохастичного вхідного потоку матеріалу на основі експериментальних даних. **Мета дослідження** - розробка генератора випадкових значень для побудови реалізації вхідного потоку матеріалу транспортного конвеєра, який має задані статистичні характеристики, розраховані за результатами попередньо проведених експериментальних вимірювань. **Отримані результати**. Стаціонарний стохастичний вхідний потік матеріалу представлений канонічним розкладанням як сума гармонійних коливань з випадковими амплітудами на різних невипадкових частотах. Запропоновано двоетапний підхід до формування реалізацій вхідного матеріального потоку. На першому етапі за допомогою канонічного розкладання по заданих координатних функціях апроксимується експериментальна реалізація потоку вхідного матеріалу для заданого інтервалу. На другому етапі розраховуються статистичні характеристики реалізацій вхідного матеріального потоку. Проведений аналіз показав, що застосування методу згладжування реалізацій матеріального потоку, заснованого на канонічній декомпозиції реалізацій вхідного матеріального потоку, забезпечує достатньо точне відтворення статистичних характеристик такого потоку, що важливо при проектуванні ефективних систем управління потоковими параметрами транспортної системи. Проведено порівняльний аналіз кореляційних функцій для експериментальної, апроксимованої та згенерованої реалізацій вхідного матеріального потоку. Обґрунтовано тривалість інтервалу часу, необхідного для проведення експериментальних змін потоку вхідного матеріалу. **Висновок**. Запропоновані в роботі методи генерації вхідних потоків на основі експериментальних даних дозволяють підвищити точність моделювання та керування конвеєрними системами, що в перспективі може призвести до зниження експлуатаційних витрат та підвищення продуктивності транспортних конвеєрних систем.

Ключові слова: потік матеріалу; стохастичний стаціонарний процес; канонічне розкладання; безрозмірні критерії подібності; конвеєр; транспортні системи.