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METHOD FOR CONSTRUCTING SIMILARITY CRITERIA FOR STOCHASTIC STATIONARY MATERIAL FLOWS

Abstract: The object of the study is a stationary stochastic input flow of material arriving at the input of a transport conveyor system. **The goal** of the study is to develop methods for comparing such flows based on dimensionless similarity criteria that allow classifying flows with different statistical characteristics but similar structural properties. **The results** obtained. The study proposed a canonical representation of the input stochastic flow in the form of a Fourier decomposition over a fixed time interval. Based on this representation, a technique was developed for constructing aggregated similarity criteria that take into account the mathematical expectation, standard deviation, and correlation time of the input flow. The applicability of the proposed criteria for modeling and analyzing stochastic processes in transport systems is substantiated. Two alternative methods of dimensionless representation of the flow model are introduced, each of which allows unifying the description of input implementations. A multilayer perceptron trained on the basis of experimental data is used to identify the distribution law of random components of the canonical decomposition. A comparative analysis of real and synthetic implementations of the input flow was carried out, confirming the effectiveness of the proposed approach in the task of reproducing the statistical and correlation characteristics. **Conclusion.** The developed technique allows for the classification and comparison of input material flows in transport systems, and also serves as a basis for creating a universal approach to constructing mathematical models and flow control algorithms under stochastic uncertainty.

Keywords: material flow; stochastic stationary process; canonical decomposition; dimensionless similarity criteria, Beta distribution; conveyor; transport systems.

Introduction

Design, modeling and optimization of flow parameters in conveyor-type transport systems require a deep understanding of the characteristics of the input material flows [1]. In real production conditions, such flows have a stochastic nature and non-stationary behavior over time, which significantly complicates their mathematical description [2].

Reduction in the costs of mineral extraction can be achieved largely by reducing the specific costs of transporting raw materials. One of the key ways to achieve this is to ensure uniform distribution of material along the entire transport highway, which contributes to more efficient operation of conveyor systems [3–5]. Increasing the uniformity of loading allows achieving a high level of use of transport capacity, increasing productivity and simultaneously reducing costs associated with maintenance [6–8]. At the same time, fluctuations in the input material flow, caused by a number of factors, remain a significant obstacle to the stable operation of the system. These include, first of all, the random nature of the input flow of material entering the system [9]. In addition, the operation of control subsystems, including belt speed control mechanisms [10–12] or material feed from storage bins [13], has a significant impact.

Adequate modeling of both non-stationary and stationary stochastic input material flows is necessary for developing effective algorithms for controlling the flow parameters of a transport conveyor, conducting a comparative analysis of various route configurations of material flows in branched transport systems [14, 15], and ensuring the stability and reliability of such systems [16, 17].

This work is devoted to modeling a stationary stochastic input flow of material entering the input of a transport conveyor system. The main objective of the study is to develop a methodology for comparing and classifying such flows based on dimensionless similarity criteria. The use of similarity criteria allows us to formalize the approach to comparative analysis of flows with different statistical properties and ensures the transfer of modeling results between different conveyor systems with different parameters.

The proposed approach is based on the canonical representation of a stochastic process. The input flow model is formed by means of expansion in a Fourier series on a fixed time interval, which allows us to represent the flow as a sum of harmonic components with random amplitudes and certain frequencies. Such a representation allows us to calculate the key statistical characteristics of the flow: mathematical expectation, variance, and characteristic correlation time.

The construction of a system of aggregated similarity criteria for input material flows is one of the key issues in modeling input material flows for transport conveyors. Conducting an experiment and analyzing experimental measurements of the input material flow values is insufficient to determine general statistical patterns. Ultimately, the experiment allows us to obtain a number of particular patterns between individual variables characterizing the analyzed material flow. In this regard, the problem of determining the relationship between individual groups of quantities and combining these quantities into complexes of a strictly defined type that have a clear physical meaning and characterize the statistical properties of the input material flows obtained for different conveyor systems is relevant. The construction of stable combinations of quantities essential for the analysis of input material flows provides important advantages. First of all, a reduction in the number of parameters characterizing the general properties of the input material flows is achieved. At the same time, when analyzing material flows taking into account these criteria, expressing the influence of different factors not separately but in combination, it is possible to more clearly express the internal connections characterizing the input material flow. Thus, the methodology for forming similarity criteria is an important stage in creating a universal classification system for input material flows in conveyor-type transport systems. The proposed technique opens up opportunities for increasing the accuracy of modeling, improving control algorithms and reducing the costs of additional experimental studies.

Aggregated similarity criteria for stochastic stationary input material flows

The stochastic stationary flow of material entering the input of the transport conveyor $\lambda(t)$ is characterized by the mathematical expectation m_λ, the standard deviation σ_λ, the correlation function $k_\lambda(\eta)$ with the correlation time η_λ. For the problem of classifying the input material flows represented by the values of the characteristic numbers $m_\lambda, \sigma_\lambda, \eta_\lambda$ we introduce similarity criteria. When constructing similarity criteria for input material flows, we use the π-theorem. To model the stochastic stationary input material flow, we introduce:

a) basic units of change: time $[T]$, mass $[M]$;

b) defining parameters: $m_\lambda, \sigma_\lambda, t, \eta_\lambda$.

Then each of the defining parameters P_n of the model can be expressed through the basic units of measurement:

$$[P_n] = [T]^{\alpha_n} [M]^{\beta_n}, \quad n = 1...N, \tag{1}$$

where N is the number of defining parameters of the model. The stochastic stationary flow of material entering the input of the transport conveyor will be characterized by $(N-R)$ similarity criteria:

$$\pi_i = \prod_{n=1}^{N} [P_n]^{z_{ni}}, \quad i = 1...(N-R). \tag{2}$$

where R is the number of basic units of change. To model the stochastic stationary flow of material, we use four defining parameters $N = 4$ and two basic parameters $R = 2$, respectively (Table 1).

Table 1 – **Basic and defining parameters for modeling stochastic stationary material flow**

n	Parameter, P_n	Basic units of measurement			
		time, [T]		mass, [M]	
1	m_λ	α_1	-1	β_1	1
2	σ_λ	α_2	-1	β_2	1
3	t	α_3	1	β_3	0
4	η_λ	α_4	1	β_4	0

Similarity criteria are dimensionless quantities. This circumstance allows us to write down a system of equations for forming similarity criteria π_i:

$$\pi_i = \prod_{n=1}^{N} \left([T]^{\alpha_n} [M]^{\beta_n} \right)^{z_{ni}}, \quad i = 1...(N-R). \tag{3}$$

The exponents of degree z_{ni} can be determined by solving the system of equations:

$$\sum_{n=1}^{N} \alpha_n z_{ni} = 0, \sum_{n=1}^{N} \beta_n z_{ni} = 0, \; i = 1...(N-R), \tag{4}$$

which, taking into account the values of the coefficients α_n, β_n can be represented in the form:

$$\begin{cases} -z_{1i} - z_{2i} + z_{3i} + z_{4i} = 0, \\ z_{1i} + z_{2i} = 0, \end{cases} \quad i = 1, 2. \tag{5}$$

The system of equations (5) has two degrees of freedom $(N-R) = 2$, that is, the values of the two quantities z_{ni} can take arbitrary values. The solution to the system of equations has the form:

$$\begin{cases} z_{1i} = \omega_{1i}, \\ z_{2i} = -\omega_{1i}, \\ z_{3i} = \omega_{2i}, \\ z_{4i} = -\omega_{2i}, \end{cases} \quad i = 1, 2. \tag{6}$$

Since the choice of the value of the unknown quantities does not affect the method of determining the similarity criteria for the input flow of material $\lambda(t)$, then when solving the system of equations we use a simple set of values ω_{1i}, ω_{2i}. The similarity criteria in accordance with the solution of the system of equations (6) are presented in Table 2.

Table 2 – **Aggregated similarity criteria for modeling stochastic stationary material flow**

i	ω_{1i}	ω_{2i}	z_{1i}	z_{2i}	z_{3i}	z_{4i}	Similarity criterion
1	-1	0	-1	1	0	0	$\pi_1 = m_\lambda / \sigma_\lambda$
2	0	1	0	0	1	-1	$\pi_2 = t / \eta_\lambda$

Criterion π_1 is quite common in the analysis of stochastic systems. Criterion π_2 is a time scale criterion, which is useful when considering a model in dimensionless units. Let the input material flow be determined by characteristic parameters m_λ^*, σ_λ^*, t^*, η_λ^*. Using the characteristic parameters, we introduce dimensionless parameters for modeling the input material flow are introducing:

$$\tau = \frac{t}{t^*}, \; \lambda_{in} = \frac{\lambda}{\sigma_\lambda^*}, \vartheta = \frac{\eta}{\eta_\lambda^*}, \gamma = \frac{\lambda - m_\lambda^*}{\sigma_\lambda^*}, \tag{7}$$

then the input flow of material can be represented as a combination of terms containing a centered component:

$$\gamma_{in}(\tau) = \frac{m_\lambda^*}{\sigma_\lambda^*} + \gamma(\tau) = \pi_1 + \gamma(\tau), \tag{8}$$

$$t = \eta_\lambda^* \frac{t^*}{\eta_\lambda^*} \tau = \pi_2 \eta_\lambda^* \tau, \quad (9)$$

where $\gamma(\tau)$ is a centered stochastic flow of material with statistical characteristics:

$$m = M\left[\gamma(\tau)\right] = 0,\ \sigma^2 = M\left[\gamma^2(\tau)\right] = 1, \quad (10)$$

$$k(\vartheta) = M\left[\gamma(\tau)\gamma(\tau+\vartheta)\right], \quad k(0) = \sigma^2 = 1. \quad (11)$$

The correlation function $k(\vartheta)$ is normalized. The material flows are similar if they have approximately the same value of the criterion π_1. The similarity criterion π_2, as was indicated earlier, plays the role of a scale factor. The characteristic time t^* determines the scale of visualization of the realizations of various stationary stochastic material flows. The characteristic time t^* sets the interval of decomposition of the stationary stochastic material flow by coordinate functions. When calculating the statistical characteristics (10), (11), an interval t^* of different duration can be used. However, when visualizing and comparatively analyzing, for example, the correlation function, the value of t^* should be determined such that the realizations of different material flows have the same value of the similarity criterion π_2.

The second method for selecting dimensionless parameters of the input material flow is as follows:

$$\tau = \frac{t}{t^*},\ \gamma_{in} = \frac{\lambda}{m_\lambda^*},\ \vartheta = \frac{\eta}{\eta_\lambda^*},\ \gamma = \frac{\lambda - m_\lambda^*}{m_\lambda^*}, \quad (12)$$

which corresponds to the equation for the material flow in dimensionless form:

$$\gamma_{in}(\tau) = 1 + \gamma(\tau). \quad (13)$$

The disadvantage of this approach is that the standard deviation σ^2 and, accordingly, the initial value of the correlation function $k(0) = \sigma^2$ depend on the value of the similarity criterion π_1. Unlike the first form of recording (8), where the similarity criterion is presented in explicit form and the expression is convenient for analysis, the similarity criterion is implicitly contained within function $\gamma(\tau)$. The final choice of the form of recording is determined by the formulation of the research problem.

Similarity criteria for stochastic stationary input material flows

The aggregated similarity criteria of the input material flow (Table 2) do not allow distinguishing between the material flow implementations with different distribution laws of random values of the material flow, as well as with different types of correlation function. In the presented approach with aggregated similarity criteria, the distribution law of random values of the material flow is characterized by the mathematical expectation and the standard deviation (aggregated similarity criterion 1). The correlation function is characterized by the characteristic value of the correlation time (aggregated similarity criterion 2). The first step in constructing a model from which similarity criteria can be obtained that take into account the distribution law of the random value of the input material flow and the type of correlation function is the selection of dimensionless parameters of the model. As the main dimensionless parameters of the model, we will use parameters of type (7), presenting model (8) taking into account the canonical expansion of the stochastic centered stationary input material flow by coordinate functions on the interval $\tau \in [0, \tau_d]$:

$$\gamma_{in}(\tau) = \pi_1 + \gamma(\tau), \quad \omega_j = 2\pi j / \tau_d, \quad (14)$$

$$\gamma(\tau) = \frac{\Theta_0}{2} + \sum_{j=1}^{\infty} \Theta_{cj} \cos\left(\omega_j \tau\right) + \Theta_{sj} \sin\left(\omega_j \tau\right), \quad (15)$$

where $\Theta_0, \Theta_{cj}, \Theta_{sj}$ are centered random variables for which the mathematical expectation

$$M\left[\Theta_0\right] = M\left[\Theta_{cj}\right] = M\left[\Theta_{sj}\right] = 0$$

and standard deviation

$$M\left[\Theta_0^2/4\right] = \sigma_0^2,\ M\left[\Theta_{cj}^2\right] = M\left[\Theta_{sj}^2\right] = \sigma_j^2. \quad (16)$$

Similarity criterion π_2 is a scale factor used when comparing input material flows. In experimental observation, the interval for measuring the input material flow values can be arbitrary in duration. In other words, if two material flows are similar, then it is obvious that increasing the measurement time to obtain more experimental values of the input material flow parameters should not change the value of similarity criterion π_2. In this regard, we will select the characteristic value for comparative analysis as ratio $t^* = \pi_2 \eta_\lambda^*$. The time of experimental measurements of the material flow values can be significantly greater than the characteristic value $t^* = \pi_2 \eta_\lambda^*$, but this characteristic value will be used to assess the similarity of two material flows. This characteristic value of the process time for analyzing material flows is the minimum acceptable for constructing a set of experimental data. Taking these considerations into account, the dimensionless parameters of the model can be rewritten as:

$$\tau = \frac{t}{\pi_2 \eta_\lambda},\ \ \vartheta = \frac{\eta}{\pi_2 \eta_\lambda},\ \gamma_{in} = \frac{\lambda}{\sigma_\lambda},\ \gamma = \frac{\lambda - m_\lambda}{\sigma_\lambda}, \quad (17)$$

where the characteristic values m_λ^*, σ_λ^*, η_λ^* are expressed through respectively through the mathematical expectation m_λ, the standard deviation σ_λ, and the correlation time η_λ of the analyzed material flow. In this case, the characteristic time $t^* = \pi_2 \eta_\lambda$ corresponds to the value of the dimensionless time $\tau = \tau_\vartheta = 1$. In its magnitude, the characteristic time of the process τ_ϑ is π_2 times greater than the correlation time ϑ_d. The dimensionless time of measurements τ_d of the input material flow, characterizing the interval of implementation of the random process, can differ significantly from τ_ϑ, but when constructing a comparative analysis for different material flows, time τ_ϑ is used. The value τ_ϑ is the minimum permissible interval of the duration of the stochastic material flow when analyzing the similarity of input material flows.

For the canonical decomposition of the stochastic centered ergodic stationary input material flow $\gamma(\tau)$ (15), the statistical characteristics are defined in [18] and can be represented as:

a) mathematical expectation:

$$m = M\left[\gamma(\tau)\right] = M\left[\gamma(\tau)\right] = 0; \qquad (18)$$

b) standard deviation:

$$\sigma^2 = M\left[\gamma^2(\tau)\right] = = \sum_{j=1}^{\infty} M\left[\Theta_{cj}^2 \cos^2\left(\omega_j\tau\right) + \Theta_{sj}^2 \sin^2\left(\omega_j\tau\right)\right] = = \sum_{j=1}^{\infty} \cos^2\left(\omega_j\tau\right) M\left[\Theta_{cj}^2\right] + \sin^2\left(\omega_j\tau\right) M\left[\Theta_{sj}^2\right] = = \sum_{j=1}^{\infty}\left(\cos^2\left(\omega_j\tau\right) + \sin^2\left(\omega_j\tau\right)\right)\sigma_j^2 = \sum_{j=1}^{\infty}\sigma_j^2. \qquad (19)$$

c) correlation function

$$k(\vartheta) = M\left[\gamma(\tau)\gamma(\tau+\vartheta)\right] = \sum_{j=1}^{\infty}\cos\left(\omega_j\vartheta\right)\sigma_j^2. \qquad (20)$$

Let the stochastic input flow of material be represented by N realizations $\gamma_n(\tau)$ of the random process $\gamma(\tau)$, $n = 1..N$. Let us expand the realization $\gamma_n(\tau)$ of the stochastic flow of material $\gamma(\tau)$ (15) on the interval $\tau \in \left[0, \tau_\vartheta\right]$ ($\tau_\vartheta = 1$) into a Fourier series:

$$\gamma_n(\tau) = \frac{\theta_{n0}}{2} + \sum_{j=1}^{\infty}\theta_{cnj}\cos\left(\frac{2\pi j}{\tau_\vartheta}\tau\right) + \theta_{snj}\sin\left(\frac{2\pi j}{\tau_\vartheta}\tau\right), \qquad (21)$$

where the quantities θ_{0n}, θ_{cnj}, θ_{snj} are constant expansion coefficients:

$$\theta_{n0} = \frac{2}{\tau_\vartheta}\int_0^{\tau_\vartheta}\gamma_n(\tau)d\tau, \quad j = 1,2,3, \qquad (22)$$

$$\theta_{cnj} = \frac{2}{\tau_\vartheta}\int_0^{\tau_\vartheta}\gamma_n(\tau)\cos\left(\frac{2\pi j}{\tau_\vartheta}\tau\right)d\tau, \qquad (23)$$

$$\theta_{snj} = \frac{2}{\tau_\vartheta}\int_0^{\tau_\vartheta}\gamma_n(\tau)\sin\left(\frac{2\pi j}{\tau_\vartheta}\tau\right)d\tau. \qquad (24)$$

Then the mathematical expectation, standard deviation and correlation function of the input material flow can be calculated from N realization of the random process $\gamma(\tau)$:

$$m = \frac{1}{N}\sum_{n=1}^{N}\int_0^1\gamma_n(\tau)d\tau = \frac{1}{N}\sum_{n=1}^{N}\frac{\theta_{0n}}{2}; \qquad (25)$$

$$\sigma^2 = \frac{1}{N}\sum_{n=1}^{N}\int_0^1\gamma_n^2(\tau)d\tau = = \sum_{n=1}^{N}\frac{\theta_{0n}^2}{4N} + \sum_{j=1}^{\infty}\frac{1}{N}\sum_{n=1}^{N}\frac{\theta_{cnj}^2 + \theta_{snj}^2}{2}; \qquad (26)$$

$$k(\vartheta) = \frac{1}{N}\sum_{n=1}^{N}\int_0^1\gamma_n(\tau)\gamma_n(\tau+\vartheta)d\tau = = \sum_{n=1}^{N}\frac{\theta_{0n}^2}{4N} + \sum_{j=1}^{\infty}\cos\left(\omega_j\vartheta\right)\sum_{n=1}^{N}\left(\frac{\theta_{cnj}^2}{2N} + \frac{\theta_{snj}^2}{2N}\right) + + \sum_{j=1}^{\theta_{snj}\theta_{cnj}}\sin\left(\omega_j\vartheta\right)\sum_{n=1}^{N}\left(\frac{\theta_{snj}\theta_{cnj}}{2N} - \frac{\theta_{cnj}\theta_{snj}}{2N}\right) = = \sum_{n=1}^{N}\frac{\theta_{0n}^2}{N} + \sum_{j=1}^{\infty}\cos\left(\omega_j\vartheta\right)\sum_{n=1}^{N}\left(\frac{\theta_{cnj}^2}{2N} + \frac{\theta_{snj}^2}{2N}\right), \qquad (27)$$

where

$$\int_{-1}^{1}\begin{pmatrix}\cos\left(\omega_j\tau\right)\times \\ \times\cos\left(\omega_i\left(\tau+\vartheta\right)\right)\end{pmatrix}d\vartheta = \begin{cases}\cos\left(\omega_j\vartheta\right)/2, & j = i; \\ 0, & j \ne i;\end{cases} \qquad (28)$$

$$\int_{-1}^{1}\cos\left(\omega_j\tau\right)\cos\left(\omega_i\left(\tau+\vartheta\right)\right)d\vartheta = \begin{cases}\cos\left(\omega_j\vartheta\right)/2, & j = i; \\ 0, & j \ne i;\end{cases}$$

$$\int_{-1}^{1}\sin\left(\omega_j\tau\right)\sin\left(\omega_i\left(\tau+\vartheta\right)\right)d\vartheta = \begin{cases}-\cos\left(\omega_j\vartheta\right)/2, & j = i; \\ 0, & j \ne i;\end{cases}$$

$$\int_{-1}^{1}\sin\left(\omega_j\tau\right)\cos\left(\omega_i\left(\tau+\vartheta\right)\right)d\vartheta = \begin{cases}-\sin\left(\omega_j\vartheta\right)/2, & j = i; \\ 0, & j \ne i;\end{cases}$$

$$\int_{-1}^{1}\cos\left(\omega_j\tau\right)\sin\left(\omega_i\left(\tau+\vartheta\right)\right)d\vartheta = \begin{cases}\sin\left(\omega_j\vartheta\right)/2, & j = i; \\ 0, & j \ne i.\end{cases}$$

Taking into account that

$$M\left[\frac{\Theta_0^2}{4}\right] = \sigma_0^2 \cong \frac{1}{N}\sum_{n=1}^{N}\frac{\theta_{0n}^2}{4}, \qquad (29)$$

$$M\left[\Theta_{cj}^2\right] = \sigma_j^2 \cong \frac{1}{N}\sum_{n=1}^{N}\theta_{cnj}^2,$$

$$M\left[\Theta_{sj}^2\right] = \sigma_j^2 \cong \frac{1}{N}\sum_{n=1}^{N}\theta_{snj}^2,$$

expressions for the mathematical expectation (26) and the correlation function (27) can be represented in the following form:

$$\sigma^2 = \sigma_0^2 + \sum_{j=1}^{\infty}\sigma_j^2, \qquad (30)$$

$$k(\vartheta)=\sigma_0^2+\sum_{j=1}^{\infty}\sigma_j^2\cos\left(\frac{2\pi j}{\tau_\vartheta}\vartheta\right). \quad (31)$$

The obtained formulas allow calculating the standard deviation and the correlation function of the stochastic stationary material flow based on N experimental realizations of the random process. The obtained results are the basis for constructing similarity criteria for stochastic material flows, taking into account the type of the correlation function. In order to simplify the methodology for forming similarity criteria, further constructions are carried out for stationary ergodic material flows.

Such material flows are characterized by a canonical decomposition of the form [18]:

$$\gamma(\tau)=\sum_{j=1}^{\infty}\Theta_{cj}\cos\left(\frac{2\pi j}{\tau_d}\tau\right)+\Theta_{sj}\sin\left(\frac{2\pi j}{\tau_d}\tau\right). \quad (32)$$

The main simplification is not the very fact of the absence of term $\Theta_0/2$, which actually simplifies the form of expressions (30), (31), but the fact that the statistical characteristics of the input material flow:

$$\sigma^2=\sum_{j=1}^{\infty}\sigma_j^2,\quad k(\vartheta)=\sum_{j=1}^{\infty}\sigma_j^2\cos\left(\frac{2\pi j}{\tau_\vartheta}\vartheta\right), \quad (33)$$

can be calculated on the basis of only one, sufficiently extended realization of the input material flow:

$$\gamma_1(\tau)=\sum_{j=1}^{\infty}\theta_{c1j}\cos\left(\frac{2\pi j}{\tau_\vartheta}\tau\right)+\theta_{s1j}\sin\left(\frac{2\pi j}{\tau_\vartheta}\tau\right), \quad (34)$$

where the standard deviation of the stochastic material flow is expressed through the expansion coefficients θ_{c1j}, θ_{s1j}:

$$\sigma_j^2\approx\left(\theta_{c1j}^2+\theta_{s1j}^2\right)/2. \quad (35)$$

The values σ_j^2 allow us to uniquely determine the standard deviation σ^2 and identify the type of correlation function $k(\vartheta)$ (33). The final step is to determine the distribution law of random values of the input material flow.

The canonical decomposition of the stochastic material flow (32) contains information only that the random variables Θ_{cj}, Θ_{sj} satisfy the relations:

$$M\left[\Theta_{cj}\right]=M\left[\Theta_{sj}\right]=0,$$

$$M\left[\Theta_{cj}^2\right]=M\left[\Theta_{sj}^2\right]=\sigma_j^2, \quad (36)$$

which allow us to calculate the correlation function of the input material flow, but do not allow us to determine the distribution law of random values of the input material flow.

In this regard, the next step is to determine the distribution law of random values of the input material flow. Using the affine transformation, we represent the centered random variables Θ_{cj}, Θ_{sj} as:

$$\Theta=\Omega-m_\Omega,\quad M[\Omega]=m_\Omega, \quad (37)$$

where Ω is a random variable that has a Beta distribution with parameters α,β Beta functions $B(\alpha,\beta)$:

$$f(\Omega)=\frac{\Omega^{\alpha-1}(1-\Omega)^{\beta-1}}{B(\alpha,\beta)}, \quad (38)$$

$$B(\alpha,\beta)=\int_0^1\Omega^{\alpha-1}(1-\Omega)^{\beta-1}\,d\Omega., \quad (39)$$

The mathematical expectation m_Ω and variance σ_Ω^2 of a random variable Ω, for the case of Beta distribution, are determined by the expressions:

$$m_\Omega=\frac{\alpha}{\alpha+\beta},\quad \sigma_\Omega^2=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \quad (40)$$

Expressing $\beta=\alpha(1-m_\Omega)/m_\Omega$, from the first equation and substituting it into the second equation, we obtain the solution to the system of equations (40):

$$\begin{cases}\alpha=\left(m_\Omega^2(1-m_\Omega)-m_\Omega\sigma_\Omega^2\right)/\sigma_\Omega,\\ \beta=\left(m_\Omega(1-m_\Omega)^2-(1-m_\Omega)\sigma_\Omega^2\right)/\sigma_\Omega^2.\end{cases} \quad (41)$$

This solution allows us to find the parameters α,β Beta function $B(\alpha,\beta)$ for the distribution of the random variable Ω. The value σ_Ω^2 is known for the random variable $\Theta\in\{\Theta_{cj},\Theta_{sj}\}$, determined by the value σ_j^2 (35). Thus, the problem of determining the distribution law of the values of the input material flow is reduced to selecting the values m_Ω to determine the distribution law of the random variable Ω. To determine the unknown values m_Ω each of the random variables Θ_{cj}, Θ_{sj} a multilayer perceptron is used.

The general scheme for calculating the coefficients $m_{\Omega cj}$, $m_{\Omega sj}$ is shown in Fig. 1.

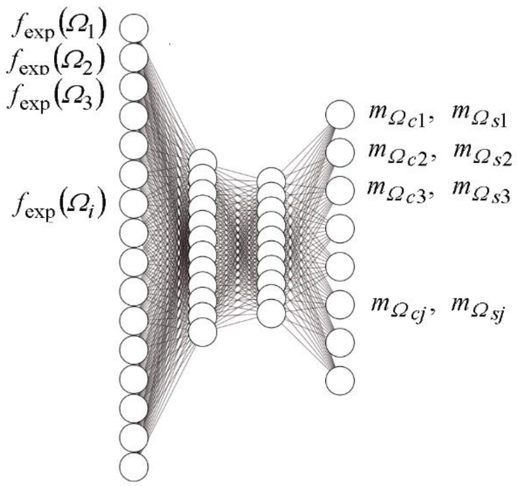


Fig. 1. Model for calculating the values of mathematical expectations $m_{\Omega cj}$, $m_{\Omega sj}$ (multilayer perceptron)

The coefficients of mathematical expectation $m_{\Omega cj}$, $m_{\Omega sj}$, calculated in this way together with the values of the standard deviations σ_j, are used to construct similarity criteria reflecting the structure of the input material flows.

When calculating mathematical expectations m_Ω, the following restrictions are introduced:

$$m_{\Omega cj} > 0, \quad m_{\Omega sj} > 0. \tag{42}$$

The input parameters of the neural network are the density of the distribution function of the values of the input material flow, constructed on the basis of experimental data. The output parameters are the values of the mathematical expectations m_Ω of random variables Θ_{cj}, Θ_{sj}. This choice of input and output nodes is justified by the optimal solution to determine for each set of experimental values that form the distribution density $f_{\exp}(\Omega)$ of the values of the input material flow, the values of the mathematical expectations that are used both to construct the theoretical distribution density of the values of the input material flow and to calculate the similarity criteria of the input material flows. An integral of the form is chosen as the loss function:

$$\int_0^\infty \left(f_{\exp}(\Omega) - f(\Omega) \right)^2 d\Omega, \tag{43}$$

where $f(\Omega)$, $f_{\exp}(\Omega)$ is the theoretical and experimental density distribution of the random variable Ω, respectively.

Models based on neural networks are widely used to determine defects in elements of transport conveyors [19, 20].

There are also quite a few models for predicting deterministic values of flow parameters of a transport system, used to design effective control systems for flow parameters of a transport system [21–23]. However, this approach to constructing similarity criteria for input material flows is used for the first time.

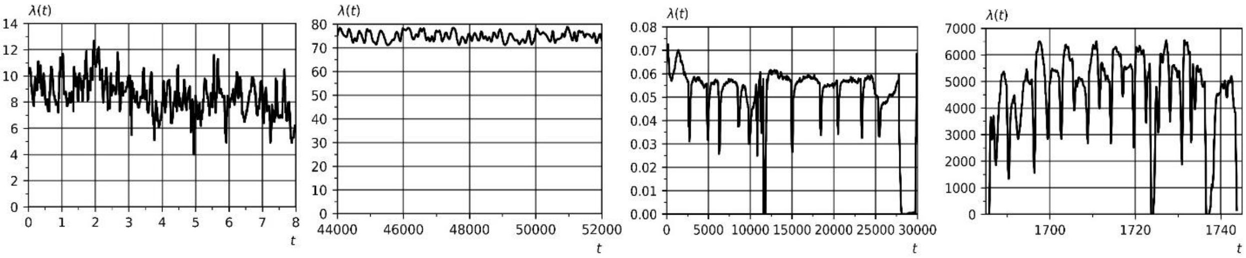
Analysis of results

To demonstrate the methodology for comparative analysis of stochastic stationary ergodic material flows based on the similarity criteria developed in the present work, experimental implementations of material flows presented in the research papers [24–27] are used, Fig. 2.

When conducting the comparative analysis, each of the material flows is designated A-flow, B-flow, C-flow, D-flow, respectively. Partial statistical analysis of the A-flow and C-flow material flows is carried out in [28, 18] when constructing approximation methods for the input material flows.

The assumption of ergodicity of the input material flows is introduced, as indicated in the theoretical part of the work, due to the fact that a single experimental implementation for each of the analyzed material flows is used to construct similarity criteria based on statistical characteristics.

The model of each of the analyzed stochastic material flows is reduced to the form (8) using dimensionless parameters (16). The values of the similarity criteria of the input material flows are given in Table 1.



a – A-flow [18, 24] b – B-flow [25] c – C-flow [26, 28] d – D-flow [27]

Fig. 2: Experimental implementations of input material flows

Table 1 – **Comparative analysis of experimental realization of the input material flow**

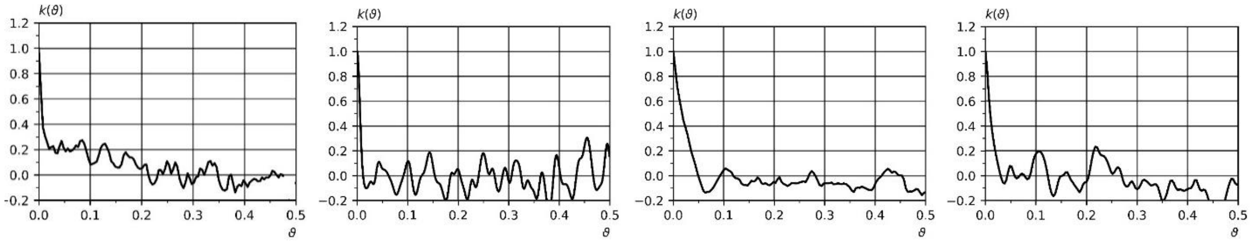
Parameter	**Experimental realization of the input material flow**			
	A-flow [18, 24]	**B-flow [25]**	**C-flow [26, 28]**	**D-flow [27]**
Mathematical expectation m_λ	8.5114	74.8540	0.0511	4663.60
Standard deviation σ_λ	1.3180	1.8278	0.0153	1469.80
$\min[\lambda(t)]$	4.0000	71.2120	0.0	0.0
$\max[\lambda(t)]$	12.7000	78.8220	0.0727	6542.80
Aggregate similarity criterion π_1	6.5	41.0	3.3	3.2
Aggregate similarity criterion π_2	100	77	44	77

The table does not provide the units of measurement of the material flows A-flow, B-flow, C-flow, D-flow due to the fact that the similarity criteria are relative dimensionless quantities. If necessary, the units of measurement can be found in the works [24–27].

The aggregated similarity criterion π_1 is obtained on the basis of the values of mathematical expectation m_λ and standard deviation σ_λ.

The material flows C-flow and D-flow are similar in terms of the aggregated similarity criterion π_1. The aggregated similarity criterion π_1 is involved in the first term of expression (14), which is a deterministic value.

To calculate the aggregated similarity criterion π_2, the characteristic correlation time of the input material flow was determined from the correlation function graph (Fig. 3).



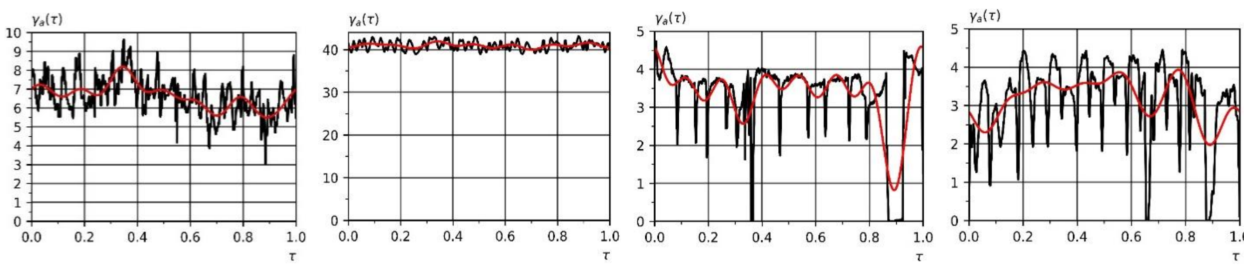
a – A-flow [18, 24] b – B-flow [25] c – C-flow [26, 28] d – D-flow [27]

Fig. 3: Input material flow correlation functions

The criterion was calculated as the ratio of the time interval during which the experimental measurements were carried out to the correlation time of the input material flow values. This interval is used to expand the input material flow realizations into a Fourier series in accordance with the canonical representation of the input material flow (15). The aggregated similarity criterion π_2 is a scale factor when performing a comparative analysis of the input material flows. When performing a comparative analysis of the input material flows shown in Fig. 2, we will select the value $\pi_2 = 77$ as the base value of the aggregated similarity criterion π_2, which is associated with the values of the aggregated similarity criterion π_2 of the B-flow and D-flow material flows. In this regard, we will reduce the observation interval of the A-flow material flow to a value at which the value of the aggregated similarity criterion π_2 for this input material flow reaches the base value. Of course, when the length of the time interval for experimental measurements is reduced, the accuracy in calculating the statistical characteristics of the input material flow will decrease. In this regard, initially, the implementation of the input material flow should be of such a time duration that will ensure the required accuracy when conducting a comparative analysis. For the C-flow material flow, it is necessary to increase the measurement interval to a value at which the basic value of the aggregated similarity criterion π_2 can be obtained. One way to solve this problem is to approximate the value of the C-flow material flow on the added interval. It should be understood that this approach leads to a distortion of the statistical characteristics of the input C-flow material flow. For a refined analysis, it is necessary to implement a stochastic input C-flow material flow with the required length of the time interval for conducting experimental measurements.

Let us assign a characteristic time for measurements to each of the analyzed material flows. This time is determined by the value of similarity criterion $\pi_2 = 77$. Taking this condition into account, let us reconstruct the dimensionless realizations of the input material flows A-flow, B-flow, C-flow, D-flow.

The transformed material flows with the required characteristic observation time for the analysis in accordance with the basic value of the aggregated similarity criterion π_2, as well as the approximations of the realizations for these input material flows are shown in Fig. 4.



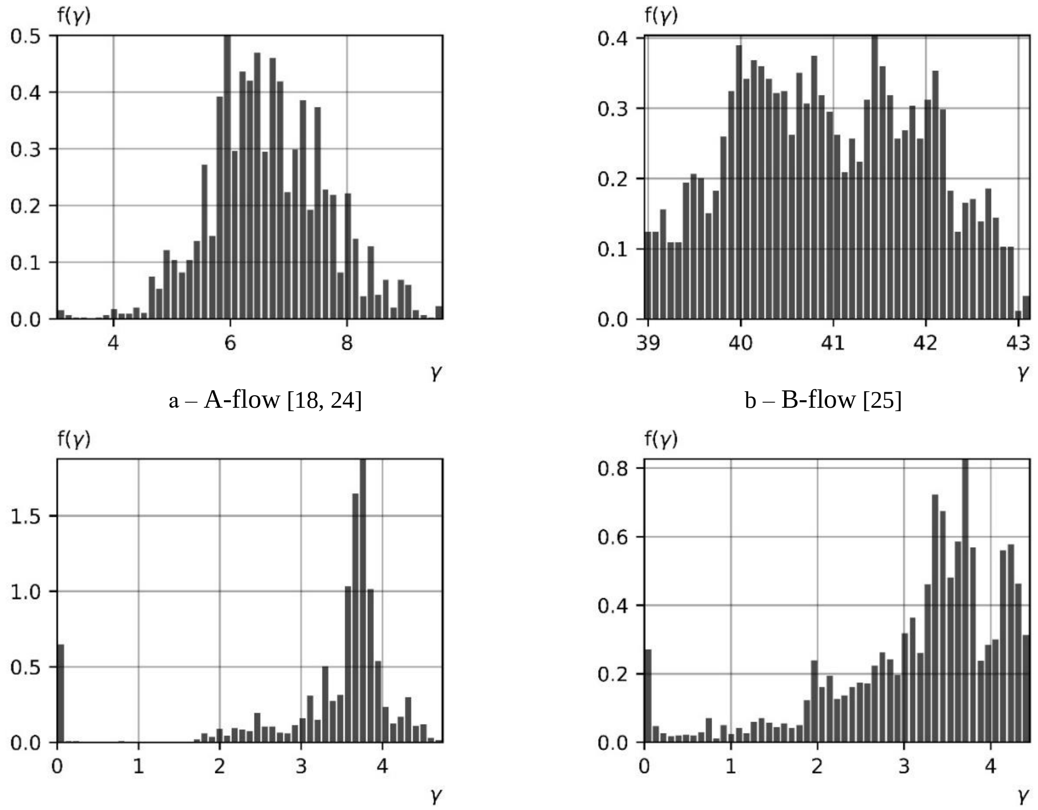
a – A-flow [18, 24] b – B-flow [25] c – C-flow [26, 28] d – D-flow [27]

Fig. 4: Dimensionless realizations and approximations of realizations for input material flows

When approximating the realizations of the input material flows, 8 terms of the canonical decomposition of the stochastic material flow (32) were taken. In this study, a small number of terms of the canonical decomposition were used in order to simplify the qualitative analysis of the input material flows. The scale of the value scale $\gamma_1(\tau)$(34) correlates with the value of the aggregated similarity criterion π_1.

The distribution density functions of the material flow values, represented by a single realization of each of the analyzed material flows, are given in Fig. 5. The coefficients of the canonical expansion θ_{c1j}, θ_{s1j} of the realizations of the input material flows, allow us to calculate the values of the standard deviation σ_j for the random variables Θ_{cj}, Θ_{sj}.

Taking into account that $\sigma_{\Omega j} = \sigma_j$ the last step of the presented methodology is to determine the mathematical expectation $m_{\Omega j}$ (40) of the random variables Ω*j*.



a – A-flow [18, 24] b – B-flow [25]

c – C-flow [26, 28] d – D-flow [27]

Fig. 5: Density distribution functions of the values of the realizations of input material flows

Thus, for the canonical representation of the stochastic input material flow, the distribution laws of the random variables, represented in the form of the Beta distribution with the distribution coefficients (41), are determined.

The values of mathematical expectations of random variables included in the canonical decomposition of the material flow are calculated using a multilayer perceptron trained on the basis of experimentally obtained distribution densities of the input material flow values (Fig. 5). For each implementation of the input flow, the parameters α, β Beta of the function B(α, β) for the distribution of the random variable Ω are calculated, which allows us to construct an approximated model of the distribution of the values of random amplitudes in the decomposition (32). Thus, the approach presented in the work allows us to: normalize flows with different scales of the magnitude of experimentally measured parameters; estimate the degree of similarity of flows taking into account their stochastic nature; approximate missing sections of implementations with a limited length of the experimental interval; determine the type of the distribution law of random variables included in the flow structure. The use of aggregated similarity criteria and dimensionless modeling allows us to move from particular empirical observations to a generalized and portable scalable model suitable for various types of conveyor systems.

This opens up opportunities for unifying algorithms for analyzing and managing material flows, as well as for constructing generators of stochastic stationary material flows in models of branched transport systems.

Conclusions

In this paper, a method for forming dimensionless similarity criteria for stationary stochastic input material flows entering conveyor-type transport systems is proposed and substantiated. The proposed approach is based on representing the flow as a canonical decomposition performed using the Fourier transform on a limited time interval. This representation allowed us to reduce the process under study to a set of deterministic and random parameters for which a system of aggregated dimensionless characteristics was constructed. The developed system of similarity criteria makes it possible to classify and compare various implementations of stochastic input flows taking into account their statistical (mathematical expectation, dispersion, correlation time) and structural properties. Two alternative methods for normalizing the material flow are presented and analyzed, each of which can be used depending on the modeling task.

One of the significant advantages of the proposed approach is its suitability for reproducing input flows based on a limited set of experimental data. This is achieved by using a multilayer perceptron to restore the distribution laws of random amplitude coefficients, which allows for a reliable reconstruction of the flow structure when generating new implementations. The comparative analysis of experimental implementations of the input flow confirms the effectiveness of the proposed similarity criteria for assessing the degree of structural similarity of flows.

The results obtained can be used in developing systems for optimal control of flow parameters of conveyor systems.

A promising direction for further research is to expand the proposed approach to non-stationary processes and adapt the similarity criteria for multidimensional or branching material supply systems.

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Метод побудови критеріїв подібності стохастичних стаціонарних потоків матеріалу

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Анотація. Об'єкт дослідження – стаціонарний стохастичний вхідний потік матеріалу, що надходить на вхід транспортної конвеєрної системи. **Мета дослідження.** розробка методів порівняння таких потоків на основі безрозмірних критеріїв подібності, що дозволяють класифікувати потоки з різними статистичними характеристиками, але схожими структурними властивостями. **Отримані результати.** У рамках дослідження запропоновано канонічне подання вхідного стохастичного потоку у вигляді Фур'є-розкладання на фіксованому часовому інтервалі. На основі цього уявлення розроблено методику побудови агрегованих критеріїв подібності, що враховують математичне очікування, середньоквадратичне відхилення та час кореляції вхідного потоку. Обґрунтовано застосовність запропонованих критеріїв для моделювання та аналізу стохастичних процесів у транспортних системах. Введено два альтернативні способи безрозмірного представлення моделі потоку, кожен з яких дозволяє уніфікувати опис вхідних реалізацій. Для ідентифікації закону розподілу випадкових складових канонічного розкладання використано багатошаровий персептрон, який навчається на основі експериментальних даних. Проведено порівняльний аналіз реальних та синтетичних реалізацій вхідного потоку, що підтверджує ефективність запропонованого підходу в задачі відтворення статистичних та кореляційних характеристик. **Висновки.** Розроблена методика дозволяє проводити класифікацію та порівняння вхідних потоків матеріалу в транспортних системах, а також є основою для створення універсального підходу до побудови математичних моделей та алгоритмів управління потоками в умовах стохастичної невизначеності.

Ключові слова: потік матеріалу; стохастичний стаціонарний процес; канонічний розклад; безрозмірні критерії подібності, бета-розподіл; конвеєр; транспортні системи.