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MATHEMATICAL MODEL OF INTELLIGENT UAV FLIGHT PATH PLANNING

Abstract. **The object of study** is the process of planning the UAV flight path. **The subject of the study** is a mathematical model of intelligent UAV flight path planning. **The purpose of the research** is to develop a mathematical model for intelligent planning of the flight trajectory of unmanned aerial vehicles. **Research results.** The practical use of the developed model will allow us to take into account the key stages of selection, implementation and training of the model in conditions of adaptability and reactivity of UAV movement. A distinctive feature of the model is a reasoned breakdown of the intelligent planning process into key stages. During the research process, GERT network approaches and probability theory methods are used for data analysis and modeling. Particular attention is paid to data preprocessing and model selection, which directly affects trajectory optimization and validation of the results obtained. **Conclusions.** The work confirms the need to take into account adaptability and reactivity in the context of external influences, which makes the planning process more effective in a dynamically changing environment. Experimental results show that the proposed model significantly reduces the computational complexity of planning, which in turn contributes to a higher level of safety and reliability of UAV missions. The results of the study of the mathematical model made it possible to put forward and confirm a hypothesis about the priority importance of a number of characteristics for assessing probabilistic-time characteristics. They also confirmed the importance of further research at the “Vibration and implementation of the model” stage.

Keywords: mathematical model; UAV; flight trajectory; GERT; probabilistic-temporal characteristics.

Introduction

The desire of modern society to facilitate and improve the quality of various aspects of activity has led to the widespread use of robotic devices. Among such devices, a special place is occupied by unmanned aerial vehicles (UAVs), capable of performing a number of specific tasks, such as monitoring and inspection of infrastructure, control over individual territories and objects, environmental studies, including the atmosphere, ocean, glaciers and other complex and inaccessible regions etc. One of the main requirements for such systems is high accuracy and safety of task performance.

It should be noted that intelligent support for UAV control and life support systems is one of the ways to solve the problem of meeting accuracy and safety requirements. Such support is necessary starting from the early stages of the UAV mission - the planning stages of the UAV flight path. When solving this problem, a significant problem arises of taking into account the influence of external factors. In this situation, the problem of planning the flight path of a UAV is transformed into the problem of intelligent planning of the flight path of a UAV, taking into account the influence of external factors.

The goal of intelligent UAV flight path planning, taking into account the influence of external factors, is to develop effective algorithms and strategies that will allow the UAV to adapt to changing conditions and ensure the accuracy and safety of its flight.

One of the key tasks of intelligent UAV flight path planning is modeling their movement along a given route. This task is widely used in missions of automatic and semi-automatic goal achievement, overcoming obstacles, as well as taking into account the influence of external factors and influences.

To successfully solve this problem, it is important to take into account many aspects, such as analysis of the external environment, adaptability and response of the system to changes, route optimization, management of potential conflicts, and compliance with various restrictions and requirements. The inclusion of these factors significantly complicates the process of modeling UAV movement, highlighting its importance in the context of trajectory planning.

At the same time, identifying the most significant features in modeling the movement of a UAV can significantly reduce computational costs and facilitate the analysis of input data. This, in turn, will increase the efficiency and practical value of the model in the process of intelligent flight path planning.

Literature analysis

To simulate UAV flight, researchers use various formalization and prototyping technologies.

In particular, the article [1] provides a comprehensive overview of the algorithms and techniques that are used for planning UAV routes . Methodologies are classified based on spatial worlds, planning stages, and map types. A variety of algorithms are evaluated, including random search, mining algorithms, genetic algorithms and A*. It is emphasized that if you want to approach the planning before the weeding, there are already a lot of excuses, and there are significant gaps in the real unique transition and the optimal planning of routes in the minds of shared computing resources. It’s a pity that the authors of the statistics focus on the overview, and not on specific new solutions; Do not provide empirical data about the productivity of various methods.

Article [2] looks at route planning algorithms for UAVs, which are based on bio-supported optimization algorithms. The robot looks at the problems that UAVs face, such as productivity, coverage, performance and energy efficiency, which make it difficult to effectively plan routes. The authors systematize different bioinspection algorithms, analyze their features, symptoms and limitations, and also give recommendations for future research in this galus. Unfortunately, many algorithms are tested only in 2D or static environments, which does not reflect the complexity of real 3D scenarios.

The work [3] presents the results of a study of a control system for autonomous vehicles (AVs) based on the pure tracking method. Among the positive aspects of the approach under consideration is the presence of a trajectory preview stage. In addition, the authors note the low complexity of the implemented model. At the same time, neglecting a number of important aspects of UAV maneuvering reduces the practical value of the proposed model. Moreover, restricting the simulation to motion only in a two-dimensional plane limits the applicability of this model to the development of control systems for UAVs.

In the opinion of the authors, it is possible to eliminate the indicated shortcomings by using a vikoryist approach to statistics [4]. It introduces a new algorithm that uses methods of retracing and orientation along the line of sight to improve the accuracy of following the route. The algorithm modifies the original 2D RRT algorithms for robots in 3D, which takes into account the unique dynamics of a UAV with a fixed wing and the need for an intermediate plane to simplify the process of trajectory planning. In addition, there are simulations that confirm the effectiveness and productivity of the algorithm in real minds, which reinforces the stagnation of various UAV missions in the minds of densely populated areas. Art. The use of the Rapidly-exploring Random Trees (RRT) algorithm allows you to quickly generate practically optimal trajectories in 3D space, which is critically important for real minds when it comes to deciding on boundaries. In addition, the algorithm improves the physical characteristics of the UAV, narrowing the maximum range of movement and the minimum turning radius, which ensures safe maneuver around the folding middle part. Among the shortcomings of this article, it is necessary to note the increased tension. The results show that although the algorithm runs within 10 seconds in complex scenarios , the computational complexity increases due to an increase in the number of recodes, which can be a problem for lethal devices with limited computational resources.

In the article [5], a hybrid algorithm GSPSODE is proposed, which combines spherical vector partial range optimization (SPSO) and differential evolution (DE) through the cooperative game model, for the task of planning UAV routes. Analysis of the article [5] for optimizing the planning of UAV routes) for the inspection of small pipeline corridors, showing a number of advantages (an integrated approach to modeling, high accuracy of modeling, the possibility of optimization ї way). Overall, the results of the cross-sectional analysis demonstrate that GSPSODE outperforms traditional methods (such as SPSO, DE, genetic algorithms, ant colony algorithm) for route planning. At the same time, you need to take note of the shortcomings. The pronunciation algorithm has a complex structure that complicates its adjustment and parameterization, which can lead to difficulties when adapting to different minds. It is recognized that the choice of algorithm parameters, differences in experimental data, and variability in results may negatively impact the accuracy of experimental data.

The article [6] is devoted to the problems of planning UAV trajectories in dynamic environments. Traditional methods, which are based on static analysis, may offer optimality and speed of response. The authors propose a better approach, a deeper understanding of reinforcement learning (DRL) for solving these problems, in addition to the advanced Dueling Double Deep Q Network (D3QN) algorithm. Simulations show that the new algorithm significantly outperforms traditional methods for planning trajectories in static and dynamic scenarios. At the same time, incorrectly adjusted values can indicate a decrease in productivity, which requires careful adjustment during the development process. In addition, the modeling is carried out in a two-dimensional space without changing the height, which can limit stagnation in practice.

Article [7] examines the use of UAVs for collecting data from the Internet of speech (IoT), focusing on minimizing the Age of Information (AoI). The authors propose an algorithm based on aggregation with reinforcements, a multi-agent algorithm with shadowing control (MATD3), combinations with a highly sophisticated particle swarm optimization (DP-MATD3), to optimize UAV routes. Experimental results show the reduction values in AoI metrics compared with other methods. However, the complexity of the model, energy exchange and problems with the timing of data transmission in IoT networks can negatively affect the collection of information.

The authors of the article [8] pose a similar problem. The article proposes a new structure to change the hassle of transferring IoT devices while ensuring a reliable connection. The technique includes optimization of the trivial placement of the UAV, association of the device in and tension control in the output mode. The implementation of the proposed structure can extract significant resources for the implementation and positioning of UAVs in real life. In addition, the proposed strategy may require additional time and resources to fine-tune the minds of the Swedish IoT environment.

The article [9] is devoted to trajectory planning for UAVs while collecting data from drone-less sensor measurements. The main method is to change the AoI, so that the hour has passed since the last time the data from the sensor nodes was updated. Vidillynoye, the main tractor Iї: Max-Aoi-Optimalnu, ShO ZMenseshui nativeshoi izhormatsya, I ave-aoi-Optimalnu, yak minimizu, middle-sower of the vik-format. It is also expected that different trajectories can be represented as short Hamiltonian paths in a boundary, and dynamic programming and genetic algorithms are used to find them. It is necessary to note that the implementation of both dynamic programming and genetic algorithms makes it possible to find trajectories in different minds. At the same time, genetic algorithms may require great computational effort and be difficult to implement in real systems.

The article [10] is devoted to the development of a detailed route planning algorithm for UAVs that deal with densely populated small towns, where there are important problems such as unique transmission and energy efficiency. It presents an advanced approach based on Deep Reinforcement Linear (DRL), following the similar model of Deep Reinforcement Q-Number (DDQN). The main innovations include a method of training on phased transitions, which improves the algorithm's ability to navigate in different flight scenarios, and a linear soft update strategy to ensure stable adjustments at the right time. The proposed algorithm takes into account environmental factors, especially wind drilling, in its model of the combined energy and functions of the vineyard, trying to minimize operations during the hour of work. The increased reliability and adaptability of the algorithm through the mechanism of meritocracy and detailed simulation further confirms the effectiveness of secure autonomous navigation of UAVs through folding bowls and landscapes. At the same time, it is necessary to ensure that the introduction of many new components into the algorithm (for example, meritocracy mechanism) can complicate its implementation and fine-tuning. In addition, the use of advanced approaches, such as DDQN, can extract great resources for learning, which increases the ability to calculate effort.

By statistic [11] є development of the methodology for intelligently seeing informative signs of modeling the UAV aircraft with the help of a variable autoencoder. It is practical to use a fragmented technique to enhance the most informative features of UAV rover modeling in the process of intelligent planning of the trajectory and flight path. To achieve the target, a practical set of input data of the variable autoencoder was formed and research was carried out on the butts of changing the speed and maneuvering of the UAV. The autoencoder is used to change the size of the data to 3 indices, which will allow further changes in the complexity of the divisions for their analysis and processing from different bases. In general, an analysis of the literature allowed us to conclude that it is advisable to improve UAV flight models in order to reduce their computational complexity while ensuring the necessary accuracy of the results. In this case, it is advisable to take into account factors of external influences, as well as adaptability and reactivity of movement. Thus, developing a model for intelligent flight path planning taking into account external influences is an urgent task.

Purpose and objectives of the study

The purpose of the study is to develop a mathematical model for intelligent UAV flight path planning. The practical use of the developed model will allow us to take into account the key stages of selection, implementation and training of the model in conditions of adaptability and reactivity of UAV movement. In the future, this will reduce the deviation of the UAV’s movement path from the planned trajectory when changing speed and maneuvering. To achieve the goal, the following tasks must be completed:

- create a universal model for intelligent UAV flight path planning;
- develop a mathematical model for intelligent UAV flight path planning based on the GERT network;
- conduct a study of the model in order to identify key characteristics that influence the process of intelligent planning of the UAV flight path.

Development of a model for intelligent flight path planning

The research was carried out using the mathematical apparatus of GERT networks, as well as methods of probability theory. To assess the effectiveness of the model, the basic principles of the theory of mathematical statistics were used [12, 13].

1. A universal model for intelligent UAV flight path planning. Let us imagine the process of intelligent planning of a UAV flight path as a set of stages in Fig. 1 [14].

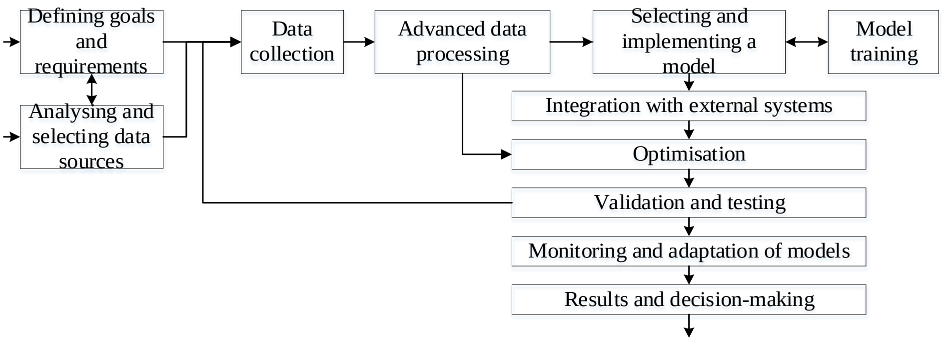

Fig. 1. Block diagram of intelligent UAV flight path planning

1. Defining goals and requirements. At this stage, it is necessary to clearly formulate the tasks and criteria that the planning system must solve.

2. Analysis and selection of data sources. Including this step will help you take a more targeted approach to data collection, choosing the most relevant and high-quality sources.

3. Data collection. At this stage, various data are collected, including the UAV's performance characteristics, geographic information, meteorological data, obstacle data and other factors that may affect the UAV's flight.

4. Pre-processing of data. At this stage, data is pre-processed to remove noise, filter and convert the

data into a format convenient for analysis. This may include geometric reconstruction of obstacles, detection of points of interest, and other processing operations.

5. Selection and implementation of models. This stage is associated with a reasoned choice of mathematical formalization technologies that will be used to plan the UAV flight path. As part of the research work, to extract the most important features and generalize the input data, it is proposed to develop a model based on the basic principles of the autoencoder theory. To reduce the dimensionality of the data and simplify the model, it is proposed to use the Bishop basis.

6. Model training. Models are trained on a training dataset to learn to identify patterns, model flight paths, and predict optimal paths for UAVs.

7. Integration with external systems. This stage is necessary to harmonize and standardize the conditions for interaction of the UAV with other systems (for example, with control systems or external sensors).

8. Optimization. The UAV flight path is optimized using trained models and taking into account various constraints such as safety, time constraints, obstacles and mission requirements.

9. Validation and testing. The resulting UAV flight trajectory is checked for compliance with the specified criteria and restrictions. The model is tested on new data to evaluate its effectiveness and accuracy.

10. Monitoring and adaptation of models. This stage is important to ensure the continued relevance and accuracy of the models under conditions of external active influences on the UAV, which is one of the key factors in the presented work.

11. Results and decision making. The obtained results are analyzed, visualized and used for decision making. This may include selecting the optimal flight path, route planning, flight control and adaptation to changing conditions.

A list of the main stages of intellectual planning was synthesized from the wikis of Dzherel [15–17].

2. Mathematical model of UAV movement in the process of intelligent flight path planning. To develop a mathematical model of UAV movement in the process of intelligent flight path planning, the GERT-network modeling approach (Graphical Evaluation and Review Technique) was chosen. Strong arguments for using this graph-based modeling approach include the following.

1. Flexibility and adaptability of data analysis under conditions of uncertainty. The GERT network approach allows stochastic and probabilistic elements to be taken into account when planning a flight path. In addition, along with the most common mathematical apparatus of probability theory and mathematical statistics, this approach can use the postulates of fuzzy mathematics. This is especially important for modeling the intelligent flight path planning processes of UAVs that may encounter uncertain conditions, such as sudden external disruptive influences (for example, changes in weather conditions), the appearance of obstacles or real-time mission changes, etc. GERT-based model , can adapt to changes in the process of task execution, increasing the accuracy and reliability of simulation results.

2. Accounting for multiple scenarios. The GERT network approach allows the simulation of various flight scenarios, taking into account the many possible branches and outcomes of events. This makes it possible to effectively analyze various trajectories and select the optimal routes to complete the mission. This factor is especially important for UAVs operating in complex and dynamic environments. However, in this context, it is necessary to take into account the fact of a significant complication of the mathematical model with a slight increase in the number of branches and nodes of the graph. It is possible to reduce the negative factor caused by the peculiarities of GERT network modeling using the GERT network simplification procedure. Such procedures are described in detail in the literature [18–20].

3. Accelerate development and testing. The use of GERT networks in modeling the flight path of UAVs helps speed up the development and testing of new control and navigation algorithms. This is due to the ability to simulate and analyze many different situations and scenarios during the design phase, reducing the need for expensive real-world testing.

4. Versatility and Scalability: The model, based on a GERT network approach, can be scaled and adapted for different types of UAVs and different missions, from reconnaissance to cargo delivery. This makes it a universal tool for developing intelligent flight control systems.

Thus, the use of GERT network technology when modeling the movement of a UAV in the process of intelligent flight path planning makes it possible to comprehensively take into account the probabilistic and time parameters of the flight task and analyze various interference scenarios.

In Fig. 2 shows a generalized GERT network illustrating the process of intelligent planning of a UAV flight path , in accordance with the block diagram (Fig. 1). In Fig. 2 state S1 forms the starting point of the GERT network, where the tasks and criteria that the planning system must solve are clearly formulated. In the model, this node sets the initial parameters and success criteria for subsequent stages. State S2 in the GERT model is represented as a decision node where the most relevant and high-quality data sources are selected. Branching (2-3) at this stage mathematically formalizes the data collection scenario, depending on the availability and quality of the analyzed sources. State S3 is represented as a node in the GERT network, where the necessary data on the UAV characteristics and external influence factors are collected. State S4 in the GERT model is represented by a node in which filtering, data transformation and other operations take place to ensure the data is ready for further analysis. This condition can be the result of operations such as noise removal, obstacle reconstruction, etc. In the GERT network, the S5 state is modeled as a decision-making node, where mathematical models and technologies are selected to formalize trajectory planning. Branches at this stage may reflect different approaches and their possible results. State S6 is the result of an iterative process (cycle) in the GERT network, where models are trained based on available data to learn how to predict the most efficient trajectories for UAVs.

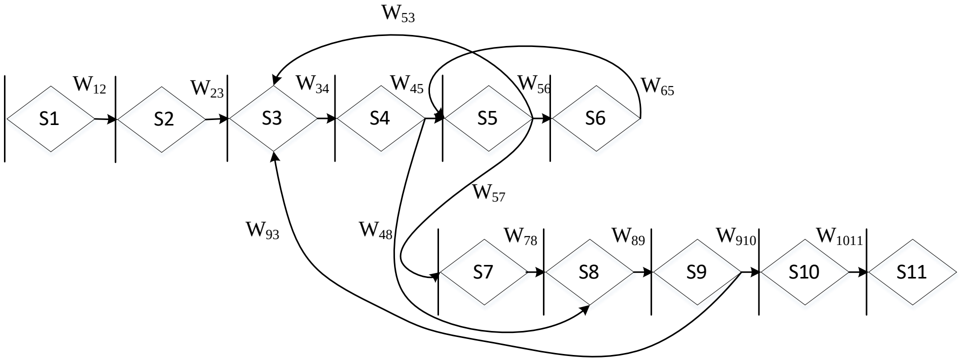

Fig. 2. Generalized diagram of the GERT network for the process of intelligent UAV flight path planning

Iterations of the process can continue until the required level of accuracy is achieved.

State S7 in the GERT model is presented as the result of a sequential process associated with synchronizing the UAV with other systems. State S8 is represented by a node that describes the result of optimizing the flight path taking into account the previously introduced restrictions. State S9 formalizes the result of checking the compliance of the resulting trajectory with the specified criteria. Branches can take into account successful testing (branch 9-10) or the need to return to previous stages (branch 9-3) to adjust the model. In a GERT network, state S10 is the result of an iterative process to ensure models are up-to-date and accurate under changing conditions. That is, the result of adaptation processes based on the data received. The final state S11 indicates the result of the trajectory selection, which in the future may become the source of the control strategy itself. The equivalent *W*-function of the process of intelligent planning of the UAV flight path can be represented as the expression:

$$W_E(s) = \\ = \frac{\begin{pmatrix} W_{12}W_{23}W_{34}W_{45}W_{56}W_{65}W_{57}W_{78}W_{89}W_{910}W_{1011} + \\ +W_{12}W_{23}W_{34}W_{48}W_{89}W_{910}W_{1011} + \\ +W_{12}W_{23}W_{34}W_{45}W_{57}W_{78}W_{89}W_{910}W_{1011} \end{pmatrix}}{1-\begin{pmatrix} W_{12}W_{23}W_{34}W_{45}W_{53} + \\ +W_{12}W_{23}W_{34}W_{45}W_{57}W_{78}W_{89}W_{93} + \\ +W_{12}W_{23}W_{34}W_{45}W_{56}W_{65}W_{57}W_{78}W_{89}W_{93} \end{pmatrix}}. \quad (1)$$

It should be noted that the study of such an equivalent W -function seems to be a rather complex process. At the same time, the transformation and finding the roots of the resulting equations is also characterized by an unreasonably large number of operations. Therefore, in order to simplify expression (1), we will use the rules of equivalent transformation described in [20, 21]. Let us transform the generalized diagram (Fig. 2) into a simplified diagram of the GERT network of the process of intelligent planning of the UAV flight path in Fig. 3.

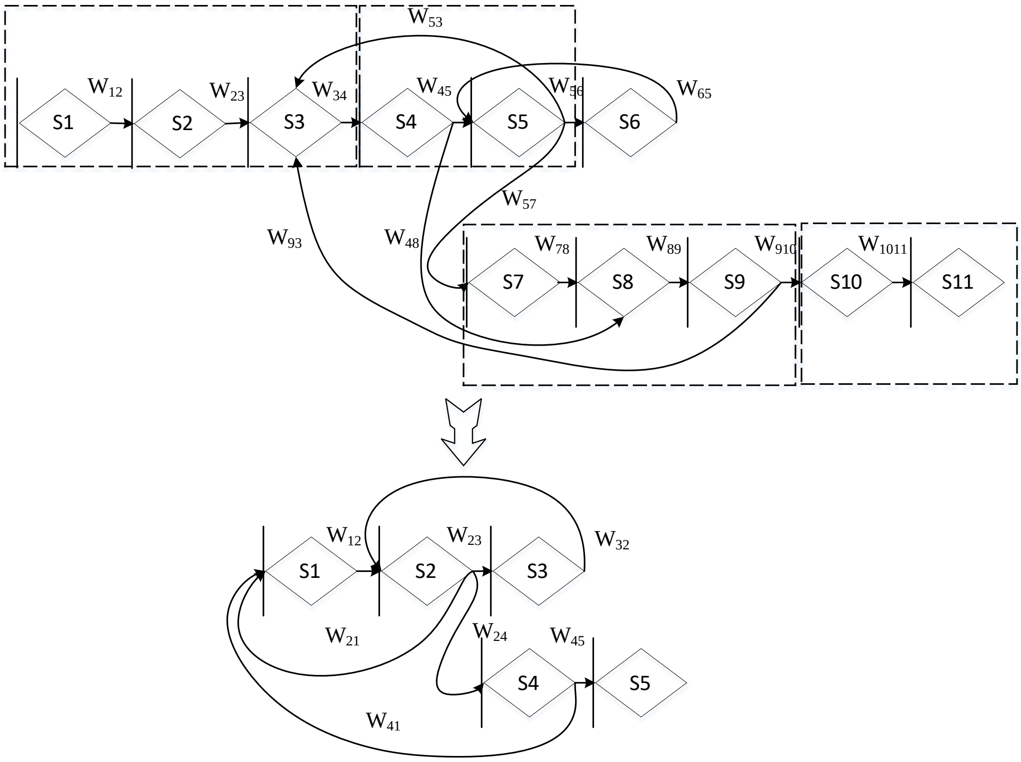

Fig. 3. Simplified diagram of the GERT network for the process of intelligent UAV flight path planning

The stages of analysis, selection of sources and data collection (states 1-3) can be combined into one node in the GERT network, which will be responsible for preparing and obtaining the necessary data. This combination makes sense because these steps are closely related to each other and are often performed sequentially without the need for detailed separation.

The combination of data preprocessing and model selection is due to the fact that in most practical cases, data preprocessing is closely related to model selection, since the format and quality of the data directly influence the choice of suitable modeling methods and technologies [22, 23].

The stages of model training, trajectory optimization, and validation and testing can be considered as part of a single optimization process. In the GERT network, they can be represented as one node in which iterative work on training models with subsequent optimization based on the results obtained takes place.

Also, the stages of monitoring and adaptation, as well as decision making, can be combined into a single process of quality control and adaptation. In the GERT network, this node will be responsible for checking models and their further adaptation based on external conditions and new data.

For a simplified scheme, the equivalent W -function of the process of intelligent planning of the UAV flight path can be represented as the expression:

$$W_E(s) = \frac{W_{12}W_{23}W_{32}W_{24}W_{45}}{1-\left(W_{12}W_{23}W_{32}W_{24}W_{41}+W_{12}W_{21}\right)}. \quad (2)$$

Let us note that already at the stage of forming the equivalent W -function, the complexity of the analytical expression has significantly decreased compared to expression 1.

Let's find an analytical relationship for calculating and studying the equivalent W -function of the process of intelligent planning of the UAV flight path .

To describe the characteristics of the GERT network branches of the process of intelligent UAV flight path planning, the generating function of moments of exponential distribution is used. This rationale has several key aspects.

1. The main task of the developed mathematical model is the final formalization in the form of analytical relationships between the time and probabilistic indicators of intelligent planning of the UAV flight path. The exponential distribution is often used to model the waiting time between random events in systems where events occur independently of each other, such as in processes associated with the movement of a UAV (for example, the time until the next disturbance appears or the reaction time to an external influence). In the context of a GERT network, exponential distribution can accurately model branch timing that reflects random delays or deviations.

2. The exponential distribution has a simple form of moment generating function. This simplicity allows the moment generating function to be easily integrated into the GERT network structure, which simplifies the calculation of branch characteristics such as average execution time, variance, and passage probability.

3. The exponential distribution has the important property of being "memoryless", which means that the future behavior of a process does not depend on how much time has already passed. This property is ideal for modeling situations where the probability of an event at the next step does not depend on the past, which simplifies the analysis and calculation of characteristics in GERT networks.

4. In UAV trajectory planning systems, stochastic processes are often encountered, such as the influence of external factors or variability in the operation of sensors. The generating function of the moments of the exponential distribution allows us to take into account such processes, providing a reliable estimate of the probabilities and timing characteristics of the branches of the GERT network, which is important for accurate prediction and adaptation of the trajectory.

In accordance with expression 2., as well as the research results, we present the characteristics of the branches and distribution parameters in the form of a Table 1.

Table 1 – **Characteristics of the branches of the process model for intelligent planning of the UAV flight path**

No.	Branch	W - function	Probability	Generating function of moments
1	(1,2)	W_{12}	p_1	$\lambda_1/(\lambda_1-s)$
2	(2.1)	W_{21}	p_2	$\lambda_2/(\lambda_2-s)$
3	(2.3)	W_{23}	p_3	$\lambda_3/(\lambda_3-s)$
4	(2.4)	W_{24}	p_4	$\lambda_4/(\lambda_4-s)$
5	(3.2)	W_{32}	p_3	$\lambda_3/(\lambda_3-s)$
6	(4.5)	W_{45}	p_5	$\lambda_5/(\lambda_5-s)$
7	(4.1)	W_{41}	p_6	$\lambda_6/(\lambda_6-s)$

For the characteristics of the branches of the simplified GERT network scheme, we will estimate the equivalent W -function of the process of intelligent planning of the UAV flight path . We describe *the* W -function of each branch as a product:

$$W_{ij} = p_k \times \lambda_m/(\lambda_m - s), \quad (3)$$

where i,j are the numbers of GERT network nodes; k is the probability index of the corresponding transition of the GERT network; m is the index of the generating function of the moments of the corresponding transition of the GERT network.

Using expressions 2 and 3 we get

$$W_E(s) = \frac{\dfrac{p_1 p_3^2 p_4 p_5 \lambda_1 \lambda_3^2 \lambda_4 \lambda_5}{(\lambda_1-s)(\lambda_3-s)^2(\lambda_4-s)(\lambda_5-s)}}{1-\dfrac{\left(\begin{array}{c} p_1 p_3^2 p_4 p_6 \lambda_1 \lambda_3^2 \lambda_4 \lambda_6 (\lambda_2-s)+ \\ +p_1 p_2 \lambda_1 \lambda_2 (\lambda_3-s)^2 (\lambda_4-s)(\lambda_6-s) \end{array}\right)}{(\lambda_1-s)(\lambda_2-s)(\lambda_3-s)^2(\lambda_4-s)(\lambda_6-s)}}. \quad (4)$$

Using standard mathematical transformations and operations, one can obtain the following expression for calculating the equivalent W -function of the process of intelligent UAV flight path planning .

$$W_E(s)=\frac{ws^2-rs+u}{(\lambda_5-s)\left(as^6+ds^5+fs^4+gs^3+hs^2+ks+q\right)}, \quad (5)$$

where $r=-p_1p_3^2p_4p_5\lambda_1\lambda_3^2\lambda_4\lambda_5(\lambda_2+\lambda_6)$;

$$q=\lambda_1\lambda_2\lambda_3{}^2\lambda_4\lambda_6\left(1-p_1p_3{}^2p_4p_6-p_1p_2-p_1p_2\lambda_3\right);$$

$$k=\lambda_1\begin{pmatrix}p_1p_3{}^2p_4p_6\lambda_3{}^2\lambda_4\lambda_6+p_1p_{23}{}^2\lambda_4\lambda_6+\\+p_1p_2\lambda_2\lambda_3{}^3\lambda_4+p_1p_2\lambda_2\lambda_3{}^3\lambda_6-\lambda_2\lambda_3{}^2\lambda_6-\\-\lambda_2\lambda_3\lambda_4\lambda_6+3p_1p_2\lambda_2\lambda_3{}^2\lambda_4\lambda_6-\lambda_2\lambda_3{}^2\lambda_4\end{pmatrix};$$

$$h=\lambda_1\begin{pmatrix}\lambda_2\lambda_3\lambda_4+\lambda_2\lambda_3\lambda_6+\lambda_2\lambda_4\lambda_6-3p_1p_2\lambda_2\lambda_3{}^2\lambda_4-\\-3p_1p_2\lambda_2\lambda_3{}^2\lambda_6-p_1p_2\lambda_2\lambda_3{}^3-3p_1p_2\lambda_2\lambda_3\lambda_4\lambda_6\end{pmatrix};$$

$$f=\lambda_1\lambda_2\left(\lambda_3\lambda_4\lambda_6-3p_1p_2\lambda_3-3p_1p_2\lambda_4-p_1p_2\lambda_6\right);$$

$$d=\lambda_1\lambda_2\left(p_1p_2-\lambda_3\lambda_4-\lambda_3\lambda_6\right);\ a=\lambda_1\lambda_2\lambda_3\lambda_4\lambda_6;$$

$$u=p_1p_3^2p_4p_5\lambda_1\lambda_2\lambda_3^2\lambda_4\lambda_5\lambda_6;\ w=p_1p_3^2p_4p_5\lambda_1\lambda_3^2\lambda_4\lambda_5.$$

Performing the complex transformation $z=-s$, we obtain

$$\Psi(z)=\frac{-wz^2+rz+u}{(\lambda_5+z)\left(-az^6-dz^5-fz^4-gz^3-hz^2-kz-q\right)}; \quad (6)$$

At this stage of modeling, it is necessary to determine the key characteristic of the study. In the context of the stated objective of the study, one of the key characteristics of the quality assessment was proposed to be a random value of the time of intelligent planning of the UAV flight path.

Then, the probability distribution density of the time of intelligent planning of the UAV flight path

$$\phi(x)=(2\pi i)^{-1}\times$$
$$\times\int_{-i\infty}^{i\infty}\frac{\exp(zx)\cdot\left(-wz^2+rz+u\right)}{(\lambda_5+z)\left(-az^6-dvz^5-fz^4-gz^3-hz^2-kz-q\right)}dz, \quad (7)$$

where integration is performed along the Bromwich contour.

The method of integration depends on whether the function has $\Psi(z)$ only simple poles, or poles of some order. In the case where the function $\Psi(z)$ has only simple poles, the expression $\Psi(z)e^{zx}$ can be represented as

$$e^{zx}\Psi(z)=\frac{e^{zx}\left(-wz^2+rz+u\right)}{\begin{pmatrix}c_7z^7+c_6z^6+c_5z^5+c_4z^4+\\+c_3z^3+c_2z^2+c_1z+c_0\end{pmatrix}}=\frac{\alpha(z)}{\beta(z)}, \quad (8)$$

where $c_7=\lambda_1\lambda_2\lambda_3\lambda_4\lambda_6$; $c_1=p_1p_2\lambda_1\lambda_2\lambda_3^3(\lambda_4+\lambda_6)$;

$$c_6=\lambda_1\lambda_2\left(\lambda_3\lambda_4\lambda_5\lambda_6-\lambda_3\lambda_4-\lambda_3\lambda_6+p_1p_2\right);$$

$$c_5=\lambda_1\lambda_2\begin{pmatrix}\lambda_3\lambda_4\lambda_5-\lambda_3\lambda_4-\lambda_3\lambda_6+\\+\lambda_3\lambda_5\lambda_6-p_1p_2\lambda_4-p_1p_2\lambda_6\end{pmatrix};$$

$$c_4=\lambda_1\lambda_2\times$$
$$\times\begin{pmatrix}p_1p_2\lambda_4\lambda_6+3p_1p_2\lambda_3\lambda_4-3p_1p_2\lambda_3\lambda_6+\\+3p_1p_2\lambda_3^2+\lambda_5\lambda_6+\lambda_5\lambda_4-\lambda_4\lambda_6-\lambda_3\lambda_6-\lambda_3\lambda_4\end{pmatrix};$$

$$c_3=\lambda_1\lambda_2\begin{pmatrix}\lambda_3\lambda_4+\lambda_3\lambda_6+\lambda_4\lambda_6-\lambda_5\lambda_4\lambda_6-\\-3p_1p_2\lambda_3^2\lambda_4-3p_1p_2\lambda_3^2\lambda_6+\\+3p_1p_2\lambda_3\lambda_4+3p_1p_2\lambda_3\lambda_6+p_1p_2\lambda_4\lambda_6\end{pmatrix};$$

$$c_2=\lambda_1\times$$
$$\times\begin{pmatrix}3p_1p_2\lambda_2\lambda_3^2\lambda_4\lambda_6-\lambda_2\lambda_3^2\lambda_4-\lambda_2\lambda_3^2\lambda_6-\lambda_2\lambda_3\lambda_4\lambda_6-\\-p_1p_2\lambda_2\lambda_3^3+p_1p_2\lambda_2\lambda_3^3\lambda_4+p_1p_2\lambda_2\lambda_3^3\lambda_6-\\-p_1p_3^2p_4p_6\lambda_3^2\lambda_4\lambda_6-3p_1p_2\lambda_2\lambda_3\lambda_4\lambda_6+3p_1p_2\lambda_2\lambda_3^2\end{pmatrix};$$

$$c_0=p_1p_2\lambda_1\lambda_2\lambda_4\lambda_6\left(-2\lambda_5-1\right).$$

Then, the probability distribution density of the time of intelligent planning of the UAV flight path

$$\phi(x)=\sum_{k=1}^{7}\mathrm{Re}\,s\left[e^{zx}\Psi(z)\right]=\sum_{k=1}^{7}\mu(z_k)/\psi(z_k)=$$
$$=\sum_{k=1}^{7}\frac{e^{zx}\left(-wz^2+rz+u\right)}{7c_7z^6-6c_6z^5-5c_5z^4-4c_4z^3-3c_3z^2-2c_2z-c_1}. \quad (9)$$

Function $\Psi(z)$ other than the simple poles determined by the roots of the equation

$$-az^6-dz^5-fz^4-gz^3-hz^2-kz-q=0,$$

has a pole $z=-\lambda_5$.

Expression (6) is a fractional rational function with respect to z with a degree of denominator greater than the degree of the numerator, therefore the conditions of the Jordan lemma are satisfied for it. The polynomial $-az^6-dz^5-fz^4-gz^3-hz^2-kz-q$ is a sixth-degree polynomial, and its roots are poles of the second kind. You can find the roots of a polynomial numerically or analytically. Solving this polynomial can be difficult in general and often requires the use of numerical methods such as Newton's method, Cardano's formulas, polynomial solving algorithms, or specialized software packages for finding the roots of polynomials.

Thus, a mathematical model for intelligent UAV flight path planning has been developed. A distinctive feature of the model is a reasoned breakdown of the intelligent planning process into key stages. This made it possible to reduce the computational complexity of the formalized model by almost 4 times.

In order to evaluate the random characteristics of the time of intelligent planning of the UAV flight path, we will conduct a study of the developed mathematical model.

3. Study of a mathematical model of UAV movement in the process of intelligent flight path planning. To study the developed mathematical model of UAV movement in the process of intelligent flight path planning, we will use the basic principles of probability theory and mathematical statistics, as well as mathematical modeling tools built into the syntax of the Python language [24].

In this work, the authors used the standard NumPy library to find the roots of the polynomial. And according to the code written in Python

```
import numpy as np
coefficients = [-a, -d, -f, -g, -h, -k, -q]
roots = np.roots(coefficients)
print ( "Roots of polynomial:" , roots)
```

For the practical case, when $p_1 = 0.9$; $p_2 = 0.3$; $p_3 = 0.9$; $p_4 = 0.9$; $p_6 = 0.3$; $\lambda_1 = 1$; $\lambda_2 = 0.1$; $\lambda_3 = 1.1$; $\lambda_4 = 1$; $\lambda_6 = 0.1$ the roots of the polynomial are equal to the following values.

9.96192177+0.j; -1.80821007+0.j;
0.38332198+0.j; 0.12004973+0.38468719j;
0.12004973-0.38468719j; -0.23167859+0.j.

The presence of such a large number of polynomial roots is one of the disadvantages of GERT modeling, which cannot be avoided even if the GERT scheme is simplified. However, taking this negative factor into account during research makes it possible to reduce its influence on the final result of the modeling. To do this, you first need to determine how the function Ψ(z) depends on z.

3.1. Analysis of the dependence *Ψ(z)*. From expression 5 it is clear that the numerator $-wz^2 + rz + u$ is a polynomial of the second degree with respect to z . It determines how the numerator of the function Ψ(z) changes depending on z , as well as the values of the characteristics of the probabilities p of transitions in the GERT circuit and the intensities of transitions λ. In the denominator of the expression 5 the first factor is $(\lambda_5 + z)$ – linear polynomial with respect to z. The second factor $-az^6 - dz^5 - fz^4 - gz^3 - hz^2 - kz - q$ is a sixth-degree polynomial in z , which contains all powers of z from 6 to 0.

The complete function *Ψ(z)* is a fractional expression in which the numerator and denominator are polynomials in z . Therefore, the function *Ψ(z)* clearly depends on z as follows:

– the denominator of the polynomial determines the poles of the function, that is, the values *of z* at which the denominator is equal to zero;

– changes in z affect the value of the function in both the numerator and denominator. Near the poles, the function may tend to infinity or to other “non-typical” values, depending on the order of the pole.

When studying the function *Ψ(z),* several factors must be taken into account to select values *of z*.

1. Values of z at which the denominator of the function is equal to zero are poles. These points should be avoided when interpolating or other numerical calculations, as the function tends to infinity or uncertainty.

2. If a function has multiple poles, attention must be paid to *z values* close to these poles to avoid numerical instabilities.

3. It is necessary to pay attention to points where the denominator becomes very small, since the function may have sudden changes.

Thus, the study of the function *Ψ(z)* , as well as the normalization of the study results near the poles, is a necessary modeling task. To confirm the existing problem, we demonstrate the graph of the function *Ψ(z)* in Fig. 4. Graph Fig. 4 confirms the fact that near the poles the function can tend to infinity or to other “non-typical” values, depending on the order of the pole. Based on this, attention should be paid to the issue of normalizing the function.

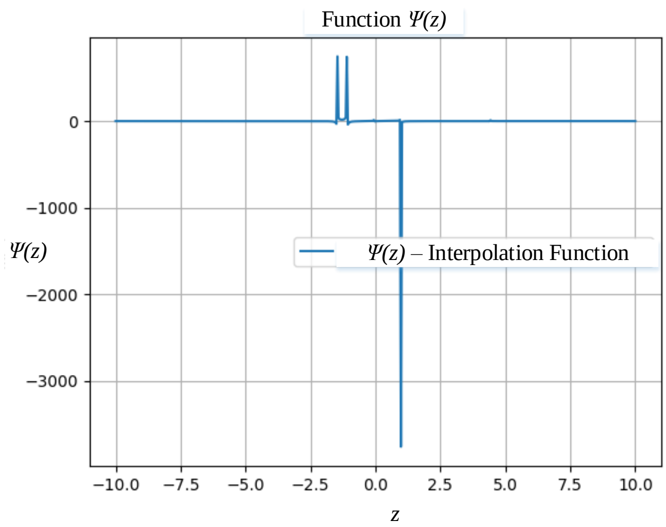

Fig. 4. Graph of function Ψ(z)

3.2. Study of the probability density distribution $\phi(x)$ of time for intelligent UAV flight path planning. Let's find the probability distribution density of the time of intelligent planning of the UAV flight path *ϕ(x)* with the following parameters of the model branches:

$p_1 = 0.9$; $p_2 = 0.3$; $p_3 = 0.9$; $p_4 = 0.9$; $p_5 = 0.6$; $p_6 = 0.3$;

$\lambda_1 = 1$; $\lambda_2 = 0.1$; $\lambda_3 = 1.1$; $\lambda_4 = 1$; $\lambda_5 = 1.1$; $\lambda_6 = 0.1$.

The modeling characteristics are synthesized empirically using sources [25–27]. Having solved the equation in the denominator of (5) using the Vieta-

Cardano method, we determined that the function $\Phi(z)$ has simple poles:

$$z_1=-1{,}1\ ,\ z_2=9{,}96192177\ ,\ z_3=-1{,}80821007\ ,$$
$$z_4=0{,}38332198\quad z_5=-0{,}23167859\ .$$

In addition to real roots, there are two complex conjugate roots: $z_6=0{,}12004973+i\cdot 0{,}38468719$ and $z_7=0{,}12004973-i\cdot 0{,}38468719$.

In accordance with expression 8 $\phi(x)$ can be represented as

$$\sum_{n=1}^{7}\operatorname{Re}s\left[e^{zx}\Psi(z)\right]=$$
$$=\frac{e^{(a+bi)_n x}\left(-(a+bi)^2 w+(a+bi)\ \ r+u\right)}{\left(\begin{array}{l}7c_7(a+bi)^6-6c_6(a+bi)^5-5c_5(a+bi)^4-\\-4c_4(a+bi)^3-3c_3(a+bi)^2-2c_2(a+bi)-c_1\end{array}\right)}+ \quad (10)$$
$$+\frac{e^{(a-bi)_n x}\left(-(a-bi)^2 w+(a-bi)\ \ r+u\right)}{\left(\begin{array}{l}7c_7(a-bi)^6-6c_6(a-bi)^5-5c_5(a-bi)^4-\\-4c_4(a-bi)^3-3c_3(a-bi)^2-2c_2(a-bi)-c_1\end{array}\right)}.$$

Sum of values of any fractional rational function

$$f(z)=\frac{d_m z^m+\ldots+d_1 z+d_0}{\ell_m z^m+\ldots+\ell_1 z+\ell_0},\ \ d_m\neq 0\ ,\ \ell_m\neq 0\ ,$$

taken at the values of complex conjugate arguments, can be presented in the form $\frac{\tau+i\beta}{\wp+i\delta}+\frac{\tau-i\beta}{\wp-i\delta}$, where $\tau,\beta,\wp,\delta$ are some coefficients.

Then, using Euler's formulas, we get

$$\sum_{n=1}^{7}\operatorname{Re}s\left[e^{zx}\Psi(z)\right]=e^{(a+bi)}\frac{\tau+i\beta}{\wp+i\delta}+e^{(a-bi)}\frac{\tau-i\beta}{\wp-i\delta}= \quad (11)$$
$$=\frac{2e^{ax}}{\wp^2+\delta^2}\left[(\tau\wp+\delta\beta)\cos(bx)+(\tau\wp-\delta\beta)\sin(bx)\right].$$

This expression is convenient for further mathematical operations performed when analyzing the processes of intelligent planning of the UAV flight path.

Let's use expression 10 when emulating a mathematical model in the Python development environment. The result of the emulation in the form of a normalized graph of the product $\Psi(z)e^{zx}$ for various values *of* x is presented in Fig. 5.

The normalized function e^{zx} $\Psi(z)$ can be interpreted as the probability density distribution of the time of intelligent planning of the UAV flight path. This means that it shows the probability with which a random variable (for example, the time before the occurrence of some event) will take a value in a specific time interval z.

On the graphs at point z close to 4.45 the function takes on high values. This indicates that the probability of the event occurring during this time is maximum.

The integral below the graph over a specific time interval (for example, from 4.44 to 4.8) represents the probability that an event will occur in that time interval.

Let us evaluate the degree of influence of the characteristics associated with the state " Vibir and implementation of the model" on the overall result of the simulation, and therefore on the random value of the time of intelligent planning of the UAV flight path [28].

In Fig. 6 shows a graph illustrating the influence of the probability p_3 of being in the "Vibir and implement the model" state on the values of the normalized function e^{zx} $\Psi(z)$.

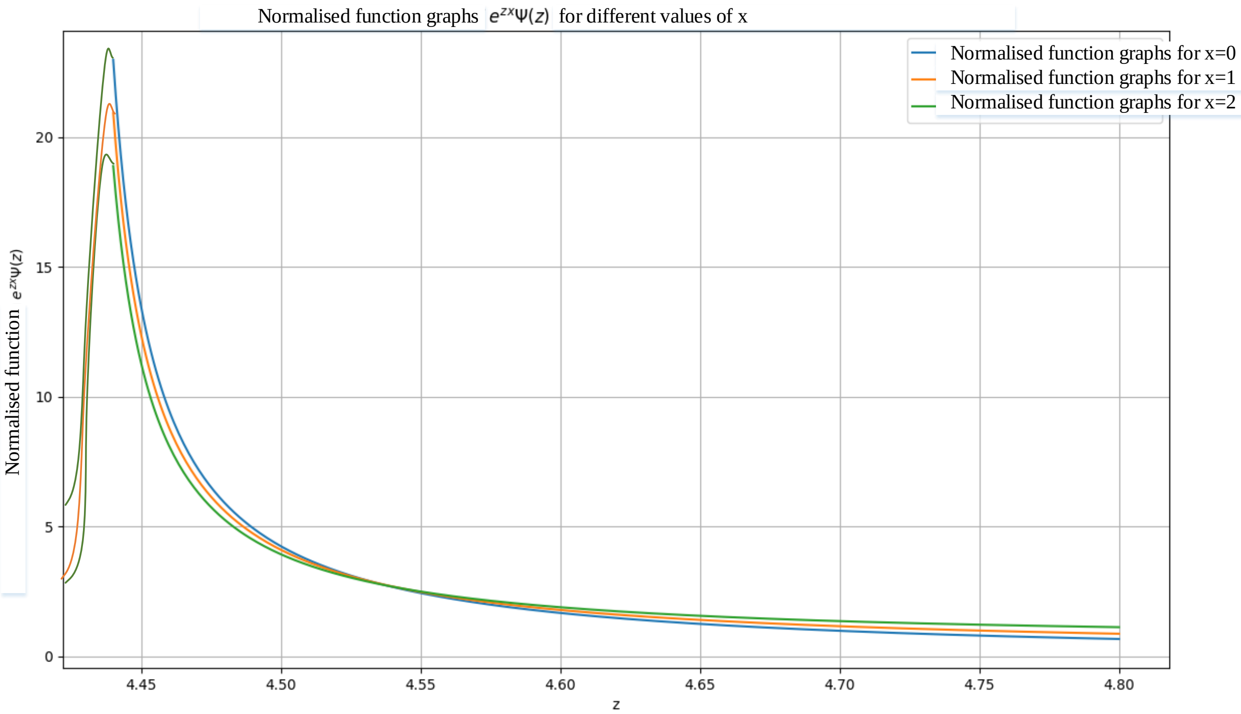

Fig. 5. Normalized graph of the product $\Psi(z)e^{zx}$ for various values of x

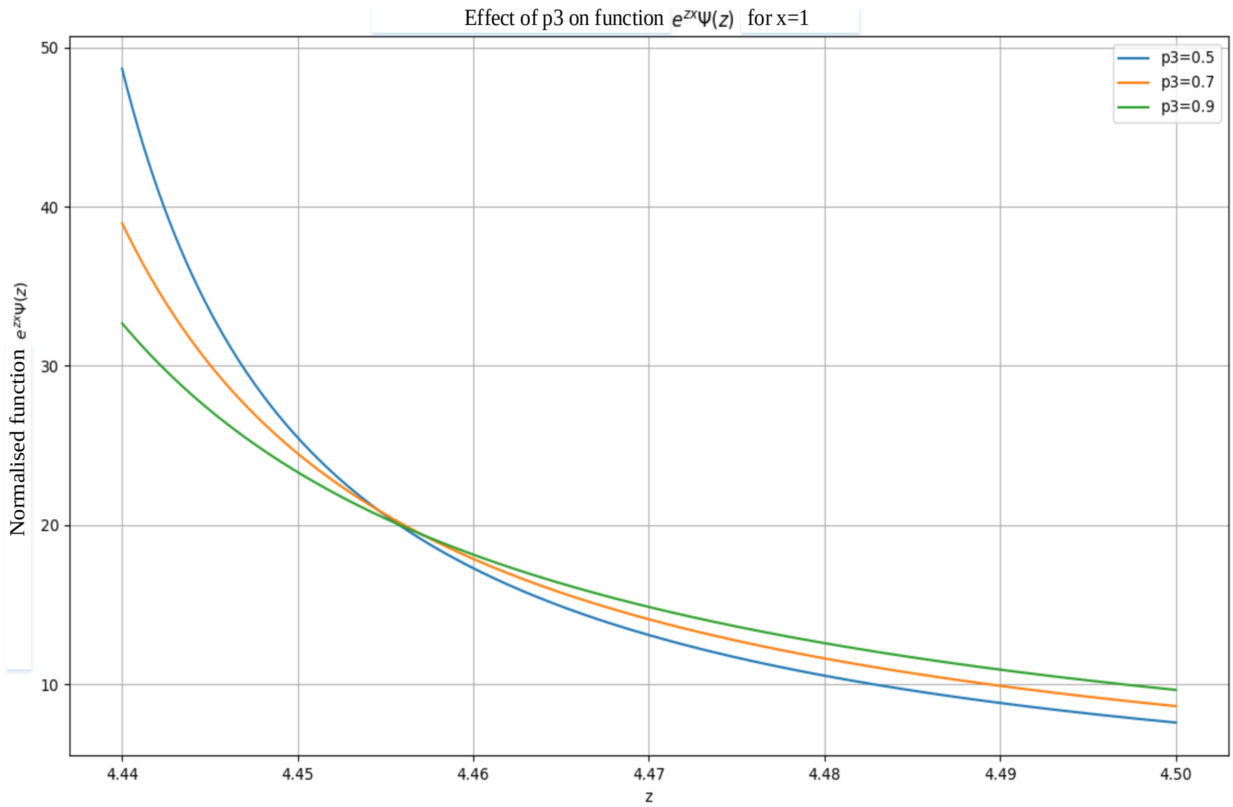

Fig. 6. Graph illustrating the influence of the probability p_3 of being in the state "Vibir and implementation of the model" on the values of the normalized function $e^{zx}\Psi(z)$

As can be seen from the graph, an increase in the probability value leads to an increase in the maximum value of the function $e^{zx}\Psi(z)$.

We investigate the degree of influence of characteristics λ on the random time variable of intelligent planning of the UAV flight path. To do this, we choose the value of the integral of the function as an indicator of influence $e^{zx}\Psi(z)$ when changing each parameter.

In Fig. 7 The dependence of the integral of the function on changes in the value of the characteristic is presented λ_3.

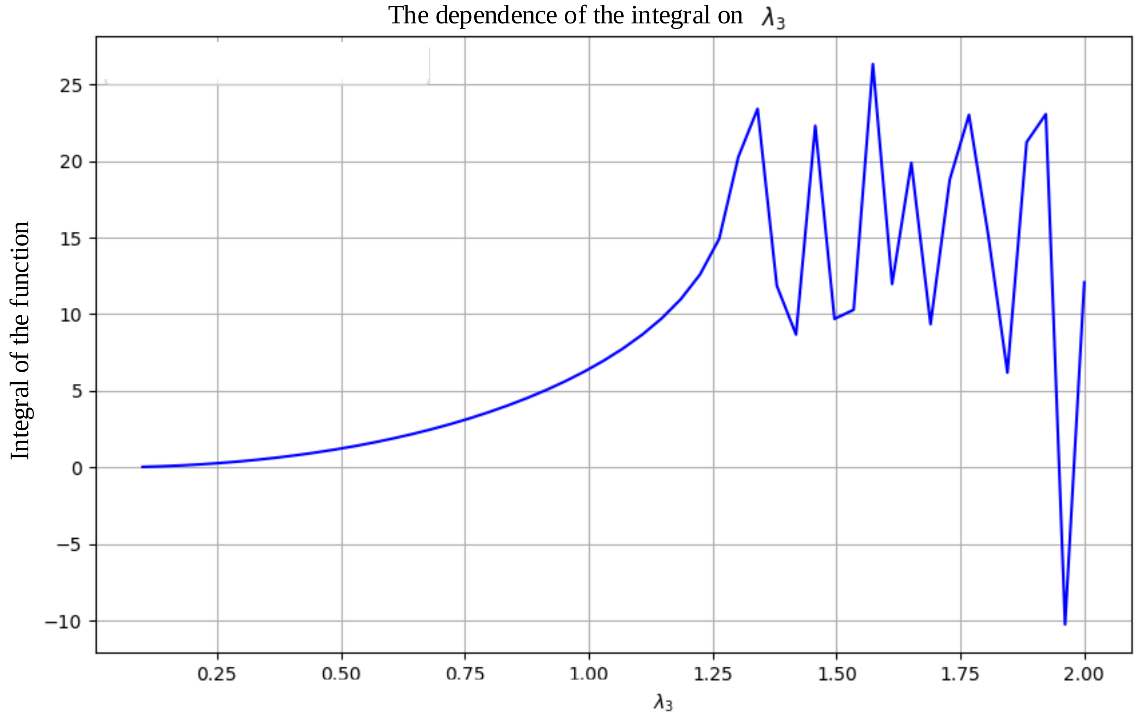

Fig. 7. Dependence of the integral of a function on changes in the value of the characteristic λ_3

One of the ways to assess the influence of characteristics on a random variable of the time of intelligent planning of a UAV flight path, the authors have chosen to compare the amplitude of changesΔ integral of a function .

It should be noted that this choice is not often used by authors.

However, the effectiveness of this assessment method has been demonstrated experimentally. The reliability of the estimate using the Pearson Chi-square test [29] reaches 0.95.

The values of the amplitude of changes Δ for various intensities are presented λ (table 2).

As can be seen from the results presented in table. 2 the largest "contribution" to the total value of the random variable of time for intelligent planning of the UAV flight path is made by values λ_3 And λ_4.

This fact indicates a significant impact on the total time of intelligent planning of the UAV flight path of the " Vibir and implement the model" stage.

And, accordingly, the need to pay attention to this particular stage in order to increase the efficiency of the process of intelligent planning of the UAV flight path.

Table 2 – **Values of the amplitude of changes Δ for various intensities λ**

λ	**Original integral**	**Modified integral**	**Change in the integral when λ *changes* by 0.1**
λ_1	0.091004334	0.091004334	-1.387778781e-17
λ_2		0.062027140	-0.028977193
λ_3		0.125357451	0.0343531172
λ_4		0.049693643	-0.041310690
λ_5		0.097544073	0.0065397392
λ_6		0.077550038	0.013454296

We will carry out similar studies for the probabilities of transitions from state to state p. The research results are presented in Table 3.

As can be seen from the results of table. 3, the hypothesis about the significant impact on the total time of intelligent planning of the UAV flight path of the " Vibir and implement the model" stage is confirmed.

Thus, the results of the study of the mathematical model of intelligent UAV flight path planning made it possible to put forward and confirm the hypothesis about the priority importance of characteristics λ_3, λ_4, p_3 and p_4 for assessing probabilistic-time characteristics.

Accordingly, they confirmed the importance of further research at the "Vibration and implementation of the model" stage.

Table 3 – **Values of the amplitude of changes Δf or various probabilities *p***

λ	**Original integral**	**Modified integral**	**Change in the integral when λ changes by 0.1**
p_1	0.062233480	0.014176750	-0.048056729
p_2		0.041192482	-0.0210409982
p_3		-0.182962236	-0.245195716
p_4		-0.029758623	-0.091992103
p_5		0.069148310	0.006914830
p_6		-0.026782761	-0.089016241

Conclusions

Thus, a mathematical model of intelligent UAV flight planning has been developed, which differs from the known ones by reasoned division of the intelligent planning process into key stages. The choice of the GERT network approach made it possible to effectively structure the process of intelligent planning and take into account factors of changes in the external environment.

During the study, a universal model of intelligent UAV flight path planning was developed, which made it possible to illustrate the main stages of the process under consideration.

Breaking down the intelligent planning process into key steps significantly reduces the computational complexity of the model, improving its practical applicability by more than 4 times. This allows you to process data more efficiently and make decisions faster in dynamic conditions.

The study revealed the critical importance of the "Selection and implementation of models" stage for assessing probabilistic-time characteristics, which confirms the need for its further analysis. This part of the planning process proved to be key to achieving time targets.

The analysis of probability dependencies when changing parameters λ and p confirmed the hypothesis about the significant influence of certain characteristics on the total planning time, which emphasizes the need for careful tuning of models and adaptation to external conditions.

It is important to emphasize that continuous monitoring and adaptation of models in response to external influences is necessary to maintain the relevance and accuracy of the planning process, which is one of the key factors for successful UAV flight control.

In the future, the results obtained require mandatory validation and testing to assess the effectiveness and accuracy of the models, which will improve the accuracy of adaptation to changing conditions during missions.

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Математична модель інтелектуального планування траєкторії польоту БПЛА

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Анотація. Об'єктом дослідження є процес планування траєкторії польоту БПЛА. **Предметом дослідження** є математична модель планування траєкторії польоту інтелектуального БПЛА. Мета дослідження – розробка математичної моделі для інтелектуального планування траєкторії польоту безпілотних літальних апаратів. **Результати дослідження.** Практичне використання розробленої моделі дозволить врахувати ключові етапи вибору, реалізації та навчання моделі в умовах адаптивності та реактивності руху БПЛА. Відмінною рисою моделі є аргументоване розбиття процесу інтелектуального планування на ключові етапи. У процесі дослідження застосовуються GERT-мережові підходи та методи теорії ймовірностей для аналізу даних та моделювання. Особлива увага приділяється попередньої обробки даних та вибору моделей, що безпосередньо впливає на оптимізацію траєкторії та валідацію отриманих результатів. **Висновки.** Робота підтверджує необхідність урахування адаптивності та реактивності в контексті зовнішніх впливів, що робить процес планування більш ефективним в умовах динамічного середовища. Експериментальні результати показують, що запропонована модель суттєво знижує обчислювальну складність планування, що сприяє більш високому рівню безпеки і надійності виконання місій БПЛА. Результати дослідження математичної моделі дозволили висунути та підтвердити гіпотезу про пріоритетну важливість низки характеристик для оцінки імовірнісно-часових характеристик. Також підтвердили важливість подальшого дослідження етапу «Вибір та реалізація моделі».

Ключові слова: математична модель; БПЛА; траєкторія польоту; GERT; імовірнісно-часові характеристики.