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# DEVELOPMENT OF A METHOD FOR ASSESSING THE ADEQUACY OF A COMPUTER SYSTEM MODEL BASED ON PETRI NETS

**Abstract. Topicality.** The purpose of modeling any system using a Petri net is to study the behavior of the modeled system based on the analysis of the defined properties of the Petri net. Therefore, it is necessary to develop a method for assessing the adequacy of the model, based on the assessment of the degree of its correspondence to the behavior of the system. **The object of research** is the behavior of a system model built using a Petri net. **The subject of the research** is the value of the deviation of the simulated processes from the real values. **The goal of the research** is to develop a method for assessing the adequacy of the description of the dynamics of the researched process in a model of a computer system based on Petri nets. **Results obtained.** A mathematical model is built, which is determined by the number of system states. In the model, options for the analysis of the state trace of the system are considered. Analysis of the adequacy of the synthesized model and its refinement are carried out according to the developed iterative algorithm. The option of testing the hypothesis regarding the Markov nature of the processes of changing system states is considered in detail. For this, appropriate statistical criteria are proposed. Considered example of evaluation of a given path of states. To test the proposed method, a study of the management algorithm of the metropolis's transport system was conducted. The simulation results practically coincided with the real results. **Conclusions.** The developed method makes it possible to assess the adequacy of the model based on Petri nets with accuracy to the entered assumptions. The method allows timely background history of dynamic processes and justify the choice of its length. The method also allows reducing the possibility of an irrational increase in the size of the synthesized model.

**Keywords:** computer system, Petri net; model; adequacy; Markov process; system state; statistical criterion; control of the transport system.

## Introduction

One of the main requirements for models is their adequacy to real systems [1]. Adequacy is understood as the ability to correctly predict various properties of processes that correspond to reality [2]. The functioning processes of a real computer traffic management system cannot be described in detail and completely [3]. This is due to the significant complexity of traffic processes [4]. Any model, due to its formalization, is adequate to the real process only under certain conditions. Increasing the degree of adequacy can be achieved through the use of different levels of detail [5, 6]. These levels depend on the characteristics of the structural and functional organization of the system and the objectives of the study. It is also necessary to take into account additional factors that influence the process under study. In addition, it is necessary to refine the model during system design [7].

**Analysis of literary sources.** Questions of the adequacy of computer system models are considered in many works.

In the works [8, 9] the emphasis is placed on reliability issues. Articles [10, 11] are focused on modelling distributed systems. Articles [12, 13] discuss models of specific systems. In [14, 15], fragments of data warehouses are modelled. Work [16] is focused on technological processes. Articles [17, 18] model smart networks. Works [19, 20] are focused on issues of access confidentiality. The article [21] discusses models of processes occurring in social networks. Works [22, 23] are focused on processes in IoT systems. Articles [24, 25] model production processes. The works of [27, 28] are focused on AI and learning issues.

However, all the reviewed works do not fully take into account the features of Petri nets. The purpose of modeling a system with a Petri net is to study the behavior of the modeled system based on the analysis of certain properties of Petri nets [28]. Finding a specific property is formulated as a corresponding analysis problem. Let us consider the most important properties of Petri nets necessary for a complete analysis of systems [29].

A Petri net is secure if all its positions are secure.

For modelling resource distribution systems, the conservation property is important. In this case, the markers in the positions of the Petri net can be interpreted as resources, the quantity of which must remain unchanged during operation.

A Petri net is called stable if all its transitions are stable. The persistence property is important for detecting conflicts between processes when the running of one process affects another process.

Therefore, a method for assessing the adequacy of a model is needed, based on assessing the degree of compliance with the behaviour of the system. The purpose of the article is to develop a method for assessing the adequacy of describing the dynamics of the process under study in a computer system model, based on Petri nets.

## Theoretical research

It is assumed that the synthesized model of the computer system has a tunable parameter: the number of system states $M$. This parameter is determined by the duration of the history $D$. The history is determined by the number of transitions taken into account in the system states. Using the hypothesis that the processes under study are Markovian ($D = 0$) leads to obtaining the most compact stochastic model, similar in its capabilities. Considering the length of the history of the entire trace leads to the most accurate, but overly cumbersome tracing model.

 

The upper limit of the possible number of system states is calculated as

$$M^{+} = C_N^D \cdot D! = N!/(N-D)!\,.$$

With a large number of analyzed events, the possible number of system states increases rapidly with increasing D. Thus, when modelling a set of programs, it is necessary to solve the problem of choosing a specific value for the duration of the taken into account prehistory of the development of processes. In addition, it is necessary to assess the reliability of the representation of the system behaviour by the trace network model.

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To solve the problem, as an initial assumption about the nature of the analyzed process, we put forward a hypothesis about the possibility of its representation by an event graph. To do this, we use the set

$$S_0 = \left\{ s_0^{(0)}, s_1^{(0)}, ..., s_n^{(0)} \right\}$$

of possible states of the system, which coincides with the set $T$ of registered events ($D = 0$).

Subsequent analysis of the adequacy of the synthesized model and its refinement is proposed to be carried out using the following iterative algorithm:

***Step 1.*** Testing the hypothesis "The process of system transitions according to states from $SD$ is Markovian."

***Step 2.*** The hypothesis is accepted - the end of the algorithm. The hypothesis is rejected – go to point 3.

***Step 3.*** Increase $D$ by one.

***Step 4.*** Determination of the set $SD$ of new system states determined by the history of the process.

***Step 5.*** The dimension of the model exceeds the specified limit - the end of the algorithm, otherwise – go to step 1.

Despite the property of the absence of aftereffects, the hypothesis about the Markovian behaviour of the system cannot be reduced to the hypothesis of randomness of the sequences of observed states of the system.

For example, for the system model shown in Fig. 1, the trace of its behaviour can look like:

$$\Im = (s_0, s_2, s_0, s_2, s_0, s_1, s_0, s_2, s_0, s_1, s_0, s_1, s_0, \ldots).$$

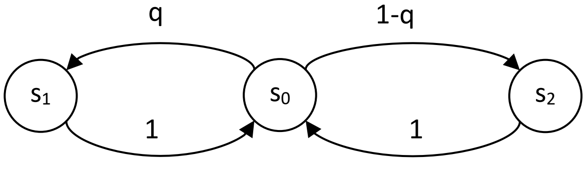


**Fig. 1.** An example of a behavioural model of a system described by a Markov chain

Without applying any criterion, it is obvious that the appearance of state $s_0$ in the trace is not accidental.

Therefore, it is proposed to test the hypothesis about the Markovian nature of the system transition processes using the theoretical value of the probability of the system reaching a certain selected state in 1, 2, ..., $n$ steps. This probability corresponds to the Markov model and is compared with the experimental value of the frequency of such events, which is calculated from the trace data.

Let the event graph of the network model be a Markov chain without absorbing states. For this network model, we determine the $P_{ij}^{(n)}$ probability of the system transitioning from the $i$-th to the j-th state in $n$ steps ($n \geq 1$).

Let us introduce the generating function

$$\tilde{P}_{ij}(s) = \sum_{n=0}^{\infty} P_{ij}^{(n)} s^n, \quad |s| < 1. \qquad (1)$$

If the $Q = \|q_{ij}\|$ is matrix of transition probabilities of a Markov chain has dimension $\rho$, then, multiplying both sides of the equality by $sq_{ki}$ and summing over $i = \overline{1, \rho}$, we obtain the relation

$$s\sum_{i=1}^{\rho} q_{ki}\tilde{P}_{ij}(s) = = \sum_{n=0}^{\infty}\sum_{i=1}^{\rho} q_{ki}P_{ij}^{(n)}s^{n+1} = \tilde{P}_{kj}(s) - P_{kj}^{(0)}, \qquad (2)$$

which defines systems of equations of the form

$$\tilde{P}_{kj}(s) - s\sum_{i=1}^{\rho} q_{ki}\tilde{P}_{ij}(s) = P_{kj}^{(0)}, \quad k = \overline{1, \rho}, \quad j = \overline{1, \rho}. \qquad (3)$$

The solutions of system (3) for fixed $k$ and $s$ are functions of the form

$$\tilde{P}_{ij}(s) = G_{ij}(s) / D(s)\,. \qquad (4)$$

Functions (4) can be expanded into simple fractions [30]:

$$\tilde{P}_{ij}(s) = \sum_{\lambda=1}^{\rho} \frac{g_{ij}^{(\lambda)}}{1 - s \cdot r_\lambda}\,, \qquad (5)$$

where $r_\lambda$ is are non-zero characteristic numbers (or eigenvalues) of the $Q$ matrix of transition probabilities; $g_{ij}^{(\lambda)}$ is some coefficients.

Let $h$ be an arbitrary left eigenvector of the $Q$ matrix, i.e.

$$hQ = r_\lambda h\,. \qquad (6)$$

Then the sum of the elements of the vectors of the left and right sides of the equality is equal to

$$\sum_{j=1}^{\rho}\sum_{i=1}^{\rho} h_i q_{ij} = \sum_{j=1}^{\rho} h_i \sum_{i=1}^{\rho} q_{ij} = \sum_{j=1}^{\rho} h_i = r_\lambda \sum_{j=1}^{\rho} h_i\,. \qquad (7)$$

Assuming that all $h_i$ are positive, $r_\lambda = 1$ determines the spectral radius of the $Q$ matrix.

Therefore, all eigenvalues of the $Q$ matrix satisfy the inequality $|r_\lambda| \le 1$. Therefore,

$$\frac{1}{1 - s \cdot r_\lambda} = \sum_{i=0}^{\infty} s^n r_\lambda^n , \quad (8)$$

and from (5) it follows

$$P_{ij}^{(n)} = \sum_{\lambda=1}^{\rho} g_{ij}^{(\lambda)} r_\lambda^n . \quad (9)$$

To find the values $g_{ij}^{(\lambda)}$ the coefficients, we use the fact that

$$P_{ij}^{(n+1)} = \sum_{\lambda=1}^{\rho} g_{ij}^{(\lambda)} r_\lambda^{n+1} = \sum_{k=1}^{\rho} q_{ik} P_{kj}^{(n)} = = \sum_{k=1}^{\rho} q_{ik} \sum_{\lambda=1}^{\rho} g_{kj}^{(\lambda)} r_\lambda^n = \sum_{\lambda=1}^{\rho} r_\lambda^n \sum_{k=1}^{\rho} q_{ik} g_{kj}^{(\lambda)}, \quad (10)$$

therefore,

$$g_{ij}^{(\lambda)} r_\lambda = \sum_{k=1}^{\rho} q_{ik} g_{kj}^{(\lambda)} . \quad (11)$$

On the other side,

$$P_{ij}^{(n+1)} = \sum_{\lambda=1}^{\rho} g_{ij}^{(\lambda)} r_\lambda^{n+1} = \sum_{k=1}^{\rho} P_{ik}^{(n)} q_{kj} = = \sum_{k=1}^{\rho} \sum_{\lambda=1}^{\rho} g_{ik}^{(\lambda)} r_\lambda^n q_{kj} = \sum_{\lambda=1}^{\rho} r_\lambda^n \sum_{k=1}^{\rho} g_{ik}^{(\lambda)} q_{kj}, \quad (12)$$

therefore,

$$g_{ij}^{(\lambda)} r_\lambda = \sum_{k=1}^{\rho} g_{ik}^{(\lambda)} q_{kj} . \quad (13)$$

In matrix form, expressions (11) and (13) can be written as

$$G^{(\lambda)} Q = r_\lambda G^{(\lambda)}, \quad Q G^{(\lambda)} = r_\lambda G^{(\lambda)} . \quad (14)$$

Thus, the columns of the $G^{(\lambda)}$ matrix are the right eigenvectors of the $Q$ matrix and are determined by $r = r_\lambda$ non-zero solutions $x_i^{(\lambda)}$ of the system of equations

$$\sum_{k=1}^{\rho} q_{ik} x_k - r \cdot x_i = 0; \quad i = \overline{1, \rho} , \quad (15)$$

and the rows are left eigen vectors determined by non-zero solutions $y_j^{(\lambda)}$ of the system

$$\sum_{k=1}^{\rho} y_k q_{kj} - r \cdot y_j = 0; \quad j = \overline{1, \rho} . \quad (16)$$

Then, up to a constant factor $C^{(\lambda)}$:

$$g_{ij}^{(\lambda)} = C^{(\lambda)} x_i^{(\lambda)} y_j^{(\lambda)} , \quad (17)$$

and the direct value $C^{(\lambda)}$ can be found based on the property of biorthonormality of the left and right eigenvectors:

$$C^{(\lambda)} \cdot \sum_{k=1}^{\rho} x_k^{(\lambda)} y_k^{(\lambda)} = 1 . \quad (18)$$

Thus, using the considered method for Markov models, theoretical values of probabilities $P_{ij}^{(n)}$ can be determined.

Based on the data of the observed trace of states of a real system, the frequencies of transition of the system from the $i$-th to the $j$-th state in $k = \overline{1, N}$ steps are calculated, represented by the matrix $V_i = \{v_{ij}^{(k)}\}$.

In this case, the following condition is satisfied:

$$\sum_{(j)} v_{ij}^{(k)} = n_i^{(k)} . \quad (19)$$

Provided that,

$$n_i^{(k)} \ge 50$$

according to the Pearson agreement criterion [31], the value can be taken as statistics characterizing the deviation of experimental frequencies from the corresponding theoretical values

$$\chi_n^2(V_i) = \sum_{(j)} \frac{[v_{ij}^{(k)} - n_i^{(k)} \cdot P_{ij}^{(k)}]^2}{n_i^{(k)} \cdot P_{ij}^{(k)}} = = \sum_{(j)} \frac{[v_{ij}^{(k)}]^2}{n_i^{(k)} \cdot P_{ij}^{(k)}} - n_i^{(k)} . \quad (20)$$

Given the significance level α, the Markov hypothesis about the behaviour of a system with the number of states $L$ is rejected when the value $\chi_n^2(V_i)$ is exceeded. This value is distributed according to the chi-square law with $L$–1 degrees of freedom corresponding to the table value.

The Pearson goodness-of-fit test gives reliable results only if the $V_j$ matrix row elements are approximately equal [31]. It can only be applied when theoretical $P_{ij}^{(k)} \ne 0$ for all $k$.

To check the correspondence of experimental frequencies to theoretical one's at large values of $L$, the information criterion can be used

$$J_C = \frac{\hat{H}_i^{(k)} - M(H_i^{(k)})}{\sqrt{D(H_i^{(k)})}} , \quad (21)$$

where $\hat{H}_i^{(k)} = -\sum_{j=1}^{L} \frac{v_{ij}^{(k)}}{n_i^{(k)}} \cdot \ln\left(\frac{v_{ij}^{(k)}}{n_i^{(k)}}\right)$ –

statistical estimation of the entropy of an empirical distribution;

$$M(H_i^{(k)}) = h_i^{(k)} - (L-1)/n_i^{(k)} -$$

expected value;

$$D(H_i^{(k)}) = = \frac{1}{n_i^{(k)}} \left( \sum_{j=1}^{L} P_{ij}^{(k)} \cdot \ln^2 (P_{ij}^{(k)}) - [h_i^{(k)}]^2 \right) -$$

entropy dispersion of theoretical distribution;

$$h_i^{(k)} = -\sum_{j=1}^{L} P_{ij}^{(k)} \cdot \ln(P_{ij}^{(k)}) .$$

If the empirical frequency distribution coincides with the expected one, the JC statistic is normally distributed with zero mathematical expectation and unit variance. In this case, for a given significance level α and a quantile of the normalized Gaussian distribution $u_{1-\alpha}$, the equality

$$|J_C| \le u_{1-\alpha} . \quad (22)$$

In addition, it should be taken into account that

$$D(H_i^{(k)}) = 0$$

indicates the absence of stochasticity in the behavior of the system. This condition may be fully consistent with the chosen model.

The information criterion is almost as powerful as the Pearson criterion. However, the probability of rejecting a correct hypothesis for some types of distributions is much lower.

**Model evaluation example**

Let us consider the state trace of a simple system (Fig. 1), consisting of 48 observations. Let the trace be described by transitions in three states:

$$s_0,\ s_2,\ s_0,\ s_2,\ s_0,\ s_1,\ s_0,\ s_1,\ s_0,\ s_2,\ s_0,\ s_1,$$
$$s_0,\ s_2,\ s_0,\ s_2,\ s_0,\ s_1,\ s_0,\ s_1,\ s_0,\ s_2,\ s_0,\ s_1,$$
$$s_0,\ s_2,\ s_0,\ s_2,\ s_0,\ s_1,\ s_0,\ s_1,\ s_0,\ s_2,\ s_0,\ s_1,$$
$$s_0,\ s_2,\ s_0,\ s_2,\ s_0,\ s_1,\ s_0,\ s_1,\ s_0,\ s_2,\ s_0,\ s_1.$$

Let us accept the assumption that a model of system behavior can be built without taking into account the history of transitions. Then it can be described by a Markov chain with a matrix of transition probabilities

$$Q = \begin{bmatrix} 0 & q & 1-q \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

where $q$ for a given route will be equal to 0.5.

The theoretical values of the probability of a system transition from the *i*-th to the *j*-th state in n steps are determined by formula (9).

Non-zero eigenvalues $r_\lambda$ of the matrix $Q$ are found from the equation

$$\det(Q - rI) = r(1-r)(1+r) = 0 ,$$

then, $r_1 = 1$ и $r_2 = -1$.

Then the systems of equations necessary to determine the coefficients $g_{ij}^{(\lambda)}$

$$\sum_{k=1}^{\rho} q_{ik} x_k - r \cdot x_i = 0; \quad i = \overline{1, \rho} ,$$

$$\sum_{k=1}^{\rho} y_k q_{kj} - r \cdot y_j = 0; \quad j = \overline{1, \rho} ,$$

look like:

$$\begin{cases} qx_2 + (1-q)x_3 - rx_1 = 0, \\ x_1 - rx_2 = 0, \\ x_1 - rx_3 = 0; \end{cases}$$

$$\begin{cases} y_2 + y_3 - ry_1 = 0, \\ y_1 q - ry_2 = 0, \\ y_1(1-q) - ry_3 = 0. \end{cases}$$

Let $a_1, a_2, b_1, b_2$ is be some nonzero constants. Then for $r_1$ a non-zero solution of the indicated systems of equations of the form can be chosen

$$x_1^{(1)} = x_2^{(1)} = x_3^{(1)} = a_1 ,$$

$$y_1^{(1)} = b_1,\ y_2^{(1)} = b_1 q ,\ y_3^{(1)} = b_1(1-q) ;$$

and for $r_2$ the non-zero solution is defined as

$$x_1^{(2)} = a_2 ,\ x_2^{(2)} = -a_2 ,\ x_3^{(2)} = -a_2 ,\ y_1^{(2)} = b_2 ,$$

$$y_2^{(2)} = -b_2 q ,\ y_3^{(2)} = -b_2(1-q) .$$

Since the coefficients $g_{ij}^{(\lambda)}$ satisfy the equality

$$g_{ij}^{(\lambda)} = C^{(\lambda)} x_i^{(\lambda)} y_j^{(\lambda)} ,$$

where $$C^{(\lambda)} \cdot \sum_{k=1}^{\rho} x_k^{(\lambda)} y_k^{(\lambda)} = 1 ,$$

then the constant factors is equal:

$$C^{(1)} = 1/(2a_1 b_1) ,$$

$$C^{(2)} = 1/(2a_2 b_2) .$$

Finally, the expressions for the theoretical probabilities of the transition of the system model to the zero, first and second states have the form:

$$P_{00}^{(n)} = \frac{1}{2} + \frac{(-1)^n}{2} ,$$

$$P_{01}^{(n)} = \frac{q}{2} - \frac{q}{2} \cdot (-1)^n ,$$

$$P_{02}^{(n)} = \frac{1-q}{2} - \frac{1-q}{2} \cdot (-1)^n ;$$

$$P_{10}^{(n)} = \frac{1}{2} - \frac{(-1)^n}{2} ,$$

$$P_{11}^{(n)} = \frac{q}{2} + \frac{q}{2} \cdot (-1)^n ,$$

$$P_{12}^{(n)} = \frac{1-q}{2} + \frac{1-q}{2} \cdot (-1)^n ;$$

$$P_{20}^{(n)}=\frac{1}{2}-\frac{(-1)^n}{2},$$

$$P_{21}^{(n)}=\frac{q}{2}+\frac{q}{2}\cdot(-1)^n,$$

$$P_{22}^{(n)}=\frac{1-q}{2}+\frac{1-q}{2}\cdot(-1)^n.$$

Let us set the maximum number $N$ of analyzed state changes equal to 5.

Then the experimental frequencies of the system's transition to the zero, first and second states are determined, based on the analysis of the trace, by the matrices $V_i=\{v_{ij}^{(k)}\}$, where $k=\overline{1,N}$, with the following values:

$$V_0=\begin{vmatrix}0 & 12 & 12\\ 23 & 0 & 0\\ 0 & 12 & 11\\ 22 & 0 & 0\\ 0 & 12 & 10\end{vmatrix},\quad V_1=\begin{vmatrix}11 & 0 & 0\\ 0 & 4 & 7\\ 11 & 0 & 0\\ 0 & 4 & 7\\ 10 & 0 & 0\end{vmatrix},$$

$$V_2=\begin{vmatrix}12 & 0 & 0\\ 0 & 8 & 4\\ 11 & 0 & 0\\ 0 & 8 & 3\\ 11 & 0 & 0\end{vmatrix}.$$

The number of states of the system $L = 3$ is small and $q = 0.5$ assumes a uniform distribution law. Therefore, as a statistic characterizing the deviation of experimental frequencies from theoretical probability values, we apply the Pearson criterion

$$\chi_n^2(V_i)=\sum_{(j)}\frac{[v_{ij}^{(k)}]^2}{n_i^{(k)}\cdot P_{ij}^{(k)}}-n_i^{(k)},$$

$$n_i^{(k)}=\sum_{(j)}v_{ij}^{(k)}.$$

We do not take into account zero probabilities, for the significance level α = 0.1 and value $\chi_{0.9,\,2}^2$ = 0.211.

Then the hypothesis about the Markovian transition process of the system is rejected when analyzing the even rows of the matrix $V_2$ ($\chi_n^2(V_2^{(2)})$ = 0.333).

Consequently, the accepted assumption about the type of model is incorrect.

Accordingly, to adequately describe the dynamics of the observed system, it is necessary to take into account the prehistory of its transitions across selected states.

The disadvantage of the considered method is a certain complexity in obtaining expressions for theoretical probabilities $P_{ij}^{(n)}$, especially if the number of system states is quite large. In this case, as an alternative to the analytical approach, the values of theoretical probabilities can be obtained by the Monte Carlo method, i.e. when simulating the behavior of the corresponding Markov chain.

In this case, the problem of testing the hypothesis about the correspondence of experimental and model probability distributions can be solved on the basis of a criterion of the form

$$\chi_n^2(V_i)=$$
$$=n_i^{(k)}m_i^{(k)}\sum_{(j)}\frac{[v_{ij}^{(k)}/n_i^{(k)}-\omega_{ij}^{(k)}/m_i^{(k)}]^2}{v_{ij}^{(k)}+\omega_{ij}^{(k)}},$$

where $\omega_{ij}^{(k)}$ – frequency of transition of the model from the i-th to the j-th state for $k=\overline{1,N}$ steps,

$$\sum_{(j)}\omega_{ij}^{(k)}=m_i^{(k)}.$$

To test the proposed method, a study of the algorithm for managing the transport system of a metropolis was carried out.

The algorithm was run 1000 times with different initial data. The execution time of this algorithm was considered as the parameter under study. The results of the analysis are shown in Fig. 2.

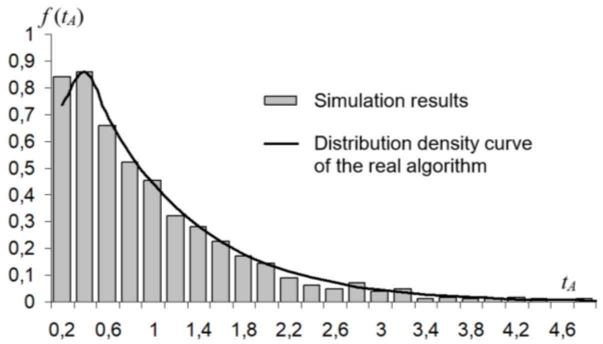


**Fig. 2.** Density distribution function of the execution time of the metropolis transport system control algorithm

As can be seen from Fig. 2, the simulation results practically coincided with the real results.

## Conclusions

Thus, the developed method, based on the analysis of processes described by models based on Petri nets, allows us to assess their adequacy up to the introduced assumptions. In addition, the method can identify cause-and-effect relationships between system states, take into account the background of ongoing processes and justify the choice of its length.

The use of various criteria when testing hypotheses about the correspondence of theoretical to experimental probability distributions improves the quality of the decision made. At the same time, the possibility of irrationally increasing the dimension of the synthesized model is reduced.

The proposed method is applicable to assess the adequacy of a wide class of models, which are based on a description of the process of changing the states of a system by a Markov chain with absorbing states.

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**Розробка методу оцінки адекватності моделі комп'ютерної системи на основі мереж Петрі**

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**Анотація. Актуальність.** Метою моделювання будь-якої системи за допомогою мережі Петрі є дослідження поведінки змодельованої системи на основі аналізу визначених властивостей мережі Петрі. Тому необхідно розробити метод оцінки адекватності моделі, заснований на оцінці ступеня відповідності її поведінці системи. **Об'єктом** дослідження є поведінка моделі системи, побудованої за допомогою мережі Петрі. **Предметом** дослідження є значення відхилення змодельованих процесів від реальних значень. **Мета дослідження**: розробка методу оцінки адекватності опису динаміки досліджуваного процесу в моделі комп'ютерної системи, яка базується на мережі Петрі. Отримані **результати**. Побудована математична модель, яка визначається кількістю станів системи. В моделі розглянуті варіанти аналізу траси станів системи. Аналіз адекватності синтезованої моделі та її уточнення проводяться за розробленим ітеративним алгоритмом. Детально розглянутий варіант перевірки гіпотези щодо марківського характеру процесів зміни станів системи. Для цього запропоновані відповідні статистичні критерії. Розглянутий приклад оцінки заданої траси станів. Для перевірки запропонованого методу було проведено дослідження алгоритму управління транспортною системою мегаполісу. Результати моделювання практично співпали з реальними результатами. **Висновки**. Розроблений метод дозволяє оцінити адекватність моделі на основі мереж Петрі з точністю до введених допущених. Метод дозволяє вчасно передісторію динамічних процесів і обґрунтувати вибір її довжини. Метод також дозволяє знизити можливість нераціонального збільшення розміру синтезованої моделі.

**Ключові слова:** комп'ютерна система; мережа Петрі; модель; адекватність; марківський процес; стан системи; статистичний критерій; керування транспортною системою.