

# Methods of information systems synthesis

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doi: <https://doi.org/10.20998/2522-9052.2024.2.04>Volodymyr Bezkorovainyi<sup>1</sup>, Liudmyla Kolesnyk<sup>1</sup>, Viktors Gopejenko<sup>2,3</sup>, Viktor Kosenko<sup>1,4</sup><sup>1</sup> Kharkiv National University of Radio Electronics, Kharkiv, Ukraine<sup>2</sup> ISMA University of Applied Sciences, Riga, Latvia<sup>3</sup> Ventspils International Radio Astronomy Centre, Ventspils, Latvia<sup>4</sup> National University "Yuri Kondratyuk Poltava Polytechnic", Poltava, Ukraine

## THE METHOD OF RANKING EFFECTIVE PROJECT SOLUTIONS IN CONDITIONS OF INCOMPLETE CERTAINTY

**Abstract.** The **subject** of research in the article is the process of ranking options in support systems for project decision-making under conditions of incomplete certainty. The **goal** of the work is to increase the efficiency of technologies for automated design of complex systems due to the development of a combined method of ranking effective options for building objects in conditions of incomplete certainty of input data. The following **tasks** are solved in the article: analysis of the current state of the problem of ranking options in support systems for project decision-making; decomposition of problems of system optimization of complex design objects and support of project decision-making; development of a variant ranking method that combines the procedures of lexicographic optimization and cardinal ordering in conditions of incomplete certainty of input data. The following **methods** are used: systems theory, utility theory, optimization, operations research, interval and fuzzy mathematics. **Results.** According to the results of the analysis of the problem of supporting project decision-making, the existence of the problem of correctly reducing subsets of effective options for ranking, taking into account factors that are difficult to formalize and the experience of the decision-maker (DM), was established. Decomposition of the problems of system optimization of complex design objects and support for project decision-making was carried out. For the case of ordinalistic presentation of preferences between local criteria, an estimate of the size of the rational reduction of subsets of optimal and suboptimal options for each of the indicators is proposed. Its use allows for one approach to obtain a subset of effective variants of a given capacity for analysis and final selection of the DM. A method of transforming the ordinalistic presentation of preferences between local criteria to their quantitative presentation in the form of weighting coefficients is proposed. **Conclusions.** The developed methods expand the methodological foundations of the automation of processes supporting the adoption of multi-criteria project decisions. They make it possible to correctly reduce the set of effective alternatives in conditions of incomplete certainty of the input data for the final choice, taking into account factors that are difficult to formalize, knowledge and experience of ODA. The practical use of the obtained results will allow to reduce the time and capacity complexity of the procedures for supporting project decision-making, and due to the use of the technology of selection of subsets of effective options with intervally specified characteristics - to guarantee the quality of project decisions and to provide a more complete assessment of them.

**Keywords:** design automation; multi-criteria evaluation; effective options; interval analysis; support for making project decisions; utility theory.

### Introduction

The anthropogenic objects that are designed, created, and used in all spheres of human activity are subject to increasingly high requirements for their efficiency, productivity, reliability, and survivability, etc. As a result, they are becoming increasingly complex in terms of their structural organization, and the costs of their creation and operation are growing [2]. The growing complexity of objects and increasing requirements for their functional characteristics leads to a corresponding complication of technologies and means of their design [3, 4]. To obtain effective design solutions for facilities, it is considered expedient to jointly solve the problems of their structural, parametric, and technological optimization [5, 6]. At the same time, most optimization problems of designing complex objects are solved by a set of functional and cost indicators and are quite complex in terms of computing resources. Typical examples of complex design objects are geographically or spatially distributed logistics, telecommunications, and monitoring systems, the cost and functional characteristics of which are significantly dependent on

their topology (territorial or spatial organization) [7]. Synthesis tasks in the design, development planning, or reengineering of distributed objects involve topology optimization, which leads to a significant increase in the power of the sets of possible options for their construction [8]. It is known that the vast majority of options for building such objects generated using automatic procedures are inefficient according to Pareto [9]. In this regard, there is a need to preliminarily identify the Pareto front among them and reduce the set of effective options, taking into account the established preferences between quality indicators. The final choice of the best option is made by a decision maker (DMP) who is able to analyze only a few alternatives. Due to the complexity of justifying a single scalar criterion for assessing the effectiveness of facility construction options, the DMP makes a choice based on the analysis of a certain set of conflicting functional and cost indicators. To evaluate the effectiveness of acceptable options, utility theory and modern methods of individual or collective expert evaluation are used, in particular: bipolar fuzzy methods [10]; methods based on the model of the ratio of possibilities using fuzzy numbers [11];

fuzzy soft choice methods [12]; lexicographic optimization [13]. At the same time, the evaluation of design solutions according to local functional and cost criteria is based on modeling results with a certain error [14]. As a result, decisions on the best option are made in conditions of incomplete certainty of the option assessments and the relationship between potential effects and the costs of achieving them. The use of fuzzy or interval mathematics methods for ranking options under conditions of this kind of uncertainty requires formalizing operations for comparing estimates according to local and general performance criteria.

### Analysis of the current state of the problem

Complex object design technologies are based on the methodology of systemic and aggregative-decomposition approaches. A systematic approach to solving the problem of ranking effective design solutions in conditions of incomplete certainty involves preliminary determining its place in the process of systemic optimization of the object being designed [15]. Within the framework of the aggregative-decomposition approach, the problem of system optimization of an object is divided into sets of tasks of different levels or aspects of design, followed by combining their solutions at the appropriate level of detail [16].

Formally, the problem of system optimization is considered as a certain MetaTask that covers all the main stages of its life cycle [17]:

$$\text{MetaTask} = \{Task^l\}, \quad Task^l = \{Task_i^l\}, \quad (1)$$

$$l = \overline{1, n_l}, i = \overline{1, i_l},$$

where:  $n_l$  – number of levels of problem description;  $i_l$  – number of tasks at the level  $l$ .

Each of the local tasks of the problem represents a certain data converter:

$$Task_i^l : In_i^l \rightarrow Out_i^l, \quad l = \overline{1, n_l}, i = \overline{1, i_l}, \quad (2)$$

where  $In_i^l, Out_i^l$  – input and output data of the  $i$ -th task of the  $l$ -st level.

At the macro level ( $l = 1$ ) the tasks of determining the requirements for the object and its system optimization arise  $Task^1 = \{Task_i^1\}, i = \overline{1, 5}$ , which differ in restrictions that take into account the specifics of the life cycle stages:  $Task_1^1$  – formulation of requirements for the object being created and development of technical specifications for its optimization;  $Task_2^1$  – system design of the object;  $Task_3^1$  – planning of the object development;  $Task_4^1$  – adaptation of the object;  $Task_5^1$  – reengineering of the object.

The set of microlevel tasks ( $l = 2$ ) covers the entire range of tasks of system optimization of the facility that arise at the stages of its pre-design research, design, construction and operation  $Task^2 = \{Task_i^2\}, i = \overline{1, 6}$

[18]:  $Task_1^2$  – choosing the principles of object construction;  $Task_2^2$  – selection of the object's structure;  $Task_3^2$  – determination of the topology of elements and connections of the object;  $Task_4^2$  – selection of technology for the object's operation;  $Task_5^2$  – defining the parameters of the elements and relationships of the object;  $Task_6^2$  – performance evaluation and selection of design solutions.

Solution ranking tasks are solved within each of the system optimization tasks discussed above. This task is most difficult when solving the problem of evaluating the effectiveness of options and selecting final design solutions  $Task_6^2$ . Due to the incomplete data, it is considered as a problem represented by the logical statement "It is necessary  $s^o$ " or  $\langle -, s^o \rangle$  (where  $s^o \in S$  – optimal design solution;  $S$  – set of acceptable solutions) [19]. At the same time, the decision-making situation  $d$  (formally  $\langle d, - \rangle$ ) is usually not clearly defined due to the inaccuracy of assessments of construction options and the incomplete certainty of the relationship between the potential effects of using the facility and the costs of achieving them. To move to a decision-making problem of the classical type "Given, Required  $\gg = \langle d, s^o \rangle$  the problem is decomposed into a set of problems of the form:

$$\langle \langle d, - \rangle, \langle d, s^o \rangle \rangle; \langle \langle -, s^o \rangle, \langle d, s^o \rangle \rangle. \quad (3)$$

The task of evaluating the effectiveness and selection of design solutions  $Task_6^2$  is a special case of the multi-criteria decision-making task. In the most general case, at the third level, it is decomposed into a set of tasks [19].:

$$Task_6^2 = \langle Tasks, ReIs \rangle, \quad Tasks = \{Task_i^3\}, \quad i = \overline{1, 6}, \quad (4)$$

where  $Tasks$  – is the set of tasks obtained as a result of decomposition;  $ReIs$  – a set of relations between tasks that define the scheme of their connections by input and output data (solution order);  $Task_1^3$  – formalization of the purpose of the object creation;  $Task_2^3$  – establishing a universal set of design solutions  $S^U$ ;  $Task_3^3$  – selection of a set of permissible solutions  $S \subseteq S^U$ ;  $Task_4^3$  – selection of a subset of effective solutions (Pareto front)  $S^E \subset S \subset S^U$ ;  $Task_5^3$  – ranking of solutions  $s \in S^E$ ;  $Task_6^3$  – selection of the best design solution  $s^o \in S^E$ .

The task of formalizing the purpose of creating an object  $Task_1^3$  is to determine the set and importance of performance indicators (local criteria)  $k_j(s), j = \overline{1, m}$ , which adequately characterize the options for its construction [19]. It determines the relationship between

functional  $k_j(s) \in Q(s)$  and cost  $k_l(s) \in C(s)$  indicators of the quality of design solutions.

The definition of a universal set of design solutions  $S^U$   $Task_2^3$  is carried out in the process of solving tasks  $Task_1^2 - Task_5^2$ . The task  $Task_2^3$  is combinatorial in nature and can have a time complexity of up  $O[2^n]$  to  $O[n!]$ . To solve it, branches and boundaries or directed search technologies are widely used, which can significantly reduce the set of alternative solutions that are generated and analyzed during the design process [5, 6].

The task of determining the set of permissible options for constructing an object  $S = S^U \setminus \bar{S}$  ( $Task_3^3$ ) is to exclude from the universal set  $S^U$  a subset of options  $\bar{S}$ , that do not satisfy the established functional or cost constraints [19]:

$$\begin{aligned} k_j(s) &\geq k_j^* \quad \forall k_j(s) \in Q(s); \\ k_l(s) &\leq k_l^* \quad \forall k_l(s) \in C(s). \end{aligned} \quad (5)$$

The task of selecting a subset of effective options for constructing an object  $S^E \subset S$  ( $Task_4^3$ ) is to remove from the set of permissible options  $S \subset S^U$  a subset of ineffective options  $\bar{S}^E \subset S$ . In this case, the option of constructing an object  $s^E \in S^E$  is called effective if there is no option  $s \in S$  on the set of permissible options  $S$  for which the inequalities are true [10]:

$$k_j(s) \geq k_j(s^E), \text{ if } k_j(s) \rightarrow \max, \quad (6)$$

$$k_j(s) \leq k_j(s^E), \text{ if } k_j(s) \rightarrow \min \quad (7)$$

and at least one of them was strict.

To solve the task  $Task_4^3$  we use the methods of discrete choice, weighting, pairwise comparisons, based on Karlin's lemma, based on the Germeier theorem, and evolutionary search [9].

The ranking of options (task  $Task_5^3$ ) and the selection of the best among them  $s^o \in S^E$  (task  $Task_6^3$ ) is carried out on the Pareto front [20] by maximizing their utility [21] using the relative priority scale of the hierarchy analysis method [22], the ELECTRE family of methods [23], the PROMETHEE-GAIA methodology [24], TOPSIS, VIKOR, SIR [25].

When using the ordinalistic approach, the ordering of non-powerful sets of effective options  $s \in S^E$  is carried out by DMP. When using the cardinalistic approach, a general efficiency criterion is formed, which is used to perform a scalar evaluation and select the best option for building an object:

$$s^o = \arg \max_{s \in S^E} P(s). \quad (8)$$

The value of the general efficiency criterion  $P(s)$  determines the ordering of the options for building an

object by value [19]:

$$\begin{aligned} \forall s, v \in S: s \sim v &\leftrightarrow P(s) = P(v); \\ s \succ v &\leftrightarrow P(s) > P(v); \quad s \succeq v \leftrightarrow P(s) \geq P(v). \end{aligned} \quad (9)$$

When formalizing the overall performance criterion, utility functions are used to determine the value of the established values of local criteria  $k_j(s)$ ,  $j = \overline{1, m}$ . Essentially, they are membership functions of the fuzzy set "Best value of the local criterion" and allow to formalize the degree of uncertainty of this concept.

The fuzzy set "The best value of the local criterion"  $G$  on a certain set  $K = \{k_j(s)\}$  is defined by the membership function  $\xi_G: K \rightarrow [0, 1]$ , which corresponds to each element of the base set  $k_j(s) \in K$  to a real number  $\xi_G$  from the interval  $[0, 1]$ :  $G = \{ \langle k_j(s), \xi_G[k_j(s)] \rangle \}$ .

When using an additive convolution for pointwise specified values of local criteria, the certainty of the weighting coefficients is taken into account with an error in the model [26, 27]. As functions of the overall utility of the options for constructing an object  $P(s)$  weighted by parameters  $\lambda_j$  additive, multiplicative, or mixed convolutions of the utility functions of local criteria  $\xi_j(s)$ ,  $j = \overline{1, m}$ , are used. When using additive convolution for point values of local criteria, the determination of weighting coefficients with an error  $\delta$  is taken into account in the model [27]:

$$s^o = \arg \max_{s \in S} \left\{ P(s) = \left\{ \frac{1}{m} \sum_{j=1}^m [\lambda_j \xi_j(s)]^\beta \right\}^{1/\beta} \right\}, \quad (10)$$

where  $\beta = -\log m / \log(1 + \delta)$  – a parameter that depends on the error in determining the weighting coefficients  $\delta$ .

If the quantitative values of the weighting estimates are unknown, and the local criteria are ordered by their importance  $k_1(s) \succ k_2(s) \succ \dots \succ k_m(s)$ , then the lexicographic optimization method is used to select the best option. Option  $s \in S$  is considered lexicographically better than the variant  $v \in S$ , if one of the following conditions is met for their local criteria utility functions [19]:

$$\begin{aligned} \xi_1(x) &> \xi_1(y); \quad \xi_1(s) = \xi_1(v); \quad \xi_2(s) > \xi_2(v); \\ \xi_j(s) &= \xi_j(v); \quad j = \overline{1, m-1}; \quad \xi_m(s) > \xi_m(v). \end{aligned} \quad (11)$$

The choice of the rational size of concessions (deviation from the optimal value of local criteria) remains an unresolved problem in the lexicographic optimization method.

The general utility function built on the basis of the Kolmogorov-Gabor polynomial is considered universal [19]. Existing methods of forming a function of total utility of options for building an object  $P(s)$  in the tasks  $Task_5^3$  i  $Task_6^3$  are designed to be used on a variety of

efficient low-power options. There is a need to develop a method for ranking effective options for constructing design objects under conditions of incomplete certainty of input data, taking into account factors that are difficult to formalize, knowledge and experience of DMP.

The aim of the study is to increase the efficiency of computer-aided design technologies for complex systems by developing a combined method for ranking sets of effective options for constructing large-capacity facilities under conditions of incomplete certainty of input data.

### Task statement and selection of basic methods for its solution

Based on the results of the review of the current state of the problem of ranking effective options for constructing facilities under conditions of incomplete certainty of input data, it was found that

- most problems of system design of complex objects are multicriteria and combinatorial in nature;
- the process of solving them involves the automatic generation and analysis of powerful sets of design solutions, and the vast majority of solutions generated in the design process are inefficient according to Pareto;
- evaluation of design solutions is carried out by means of mathematical modeling with some errors;
- the process of selecting the best option for the construction of the facility is carried out using expert evaluation methods, during which only a small number of design solutions can be analyzed;
- fuzzy and interval mathematics methods are used to take into account the incomplete certainty of input data. Comparing fuzzy sets or performing operations on them is possible only when they are defined on the same universes. In the case of interval representation of input data, the comparison of object characteristics is problematic.

To solve tasks in which the values of the weighting coefficients  $\lambda_j$ ,  $j = \overline{1, m}$  and local criteria  $k_j(s)$ ,  $j = \overline{1, m}$  values set within certain limits, it is proposed to use the apparatus of interval mathematics [27]. For some values of the object's characteristics  $a \in [a^-; a^+]$  and  $b \in [b^-; b^+]$  the rules for performing operations of classical interval arithmetic are as follows:

$$[a] + [b] = [a^- + b^-; a^+ + b^+]; \quad (12)$$

$$[a] - [b] = [a^- - b^+; a^+ - b^-]; \quad (13)$$

$$[a] \cdot [b] = \left[ \min \{ a^- \cdot b^-, a^- \cdot b^+, a^+ \cdot b^-, a^+ \cdot b^+ \}; \max \{ a^- \cdot b^-, a^- \cdot b^+, a^+ \cdot b^-, a^+ \cdot b^+ \} \right]; \quad (14)$$

$$[a]/[b] = [1/b^+; 1/b^-], \quad 0 \notin [b]. \quad (15)$$

Solving problems of systematic optimization of object construction options  $Task^2 = \{Task_i^2\}$ ,  $i = \overline{1, 6}$  on the selected set of problems (1) will be considered according to the scheme represented as a tuple [17]:

$$SysOpt = \langle Task^2, InDat, Res, DesDec, ProcDec \rangle, \quad (16)$$

where  $Task^2$  – ordered set of tasks  $\langle Task_i^2 \rangle$ ,  $i = \overline{1, 6}$ ;  $InDat^2 = \{InDat_i^2\}$ ,  $i = \overline{1, 6}$  – a set of input data of tasks;  $Res^2 = \{Res_i^2\}$ ,  $i = \overline{1, 6}$  – a set of constraints of tasks;  $DesDec^2 = \{DesDec_i^2\}$ ,  $i = \overline{1, 6}$  – a set of design options for constructing an object;  $ProcDec^2 = \{ProcDec_i^2\}$ ,  $i = \overline{1, 6}$  – a solving procedure that assigns each pair "Input data - Constraints"  $\langle InDat_i^2, Res_i^2 \rangle$  a non-empty subset of options for constructing an object  $\{DesDec_i^2\}$ ,  $i = \overline{1, 6}$ .

The general task of ranking effective project solutions  $Task_5^2$  and choosing the best among them  $Task_6^2$  under conditions of incomplete certainty is formulated as follows.

Given:

- the set of possible options for constructing an object  $S$ ;
- a set of local criteria for assessing the effectiveness of facility construction options  $k_j(s)$ ,  $j = \overline{1, m}$ ;
- evaluation of each of the permissible options for the construction of the object  $s \in S$  by local criteria determined with an error  $\varepsilon_j$ ,  $[k_j(s) - \varepsilon_j; k_j(s) + \varepsilon_j]$ ,  $j = \overline{1, m}$ ;
- values of weighting coefficients of local criteria  $\lambda_j$ ,  $j = \overline{1, m}$  unknown or determined with a significant error  $\varepsilon_j$ ,  $j = \overline{1, m}$  so that local criteria can only be ordered by importance  $k_1(s) \succ k_2(s) \succ \dots \succ k_m(s)$ .

Required:

- determine a subset of Pareto-efficient options for building an object  $S^E \subset S$ ;
- to rank effective design solutions (facility construction options) for situations of incomplete certainty of local criteria for the effectiveness of options  $k_j(s)$ ,  $j = \overline{1, m}$  and their weight coefficients  $\lambda_j$ ,  $j = \overline{1, m}$ ;
- choose the best option from a subset of effective ones  $s^o \in S^E$ .

To solve the problem, we will use the idea of a combined expert-machine method, which involves the sequential implementation of stages [19]:

- selection on the set of permissible  $S = \{s\}$  subsets of effective options for building an object  $S^E \subseteq S$ , for which in the general case  $Card(S^E) \ll Card(S)$ ;
- determining the preferences of experts on the importance of certain properties of options on a subset of effective options  $s \in S^E$  by partial criteria  $k_j(s)$ ,  $j = \overline{1, m}$ ;
- parametric synthesis of the total utility function of options  $[P(s)]$ ;

- ranking of options using the synthesized function of total utility of options  $[P(s)]: [P(s)] > [P(v)] \leftrightarrow s \succ v \forall s, v \in S^E$ ;

- selection on a subset  $S^E$  of several most effective options  $S' \subseteq S^E$ ,  $Card(S') \ll Card(S^E)$ ;

- determination of the rank value  $P(s)$  of a subset of the most effective options;

- selecting the best option from a subset of the most effective ones  $s^o \in S^E$ .

### Selecting a subset of effective options

The selection of a subset of effective options (Pareto front) involves comparing pairs of permissible options  $S = \{s\}$  for each of the local criteria  $k_j(s)$ , which are presented in interval form  $k_j(s) = [k_j^-(s); k_j^+(s)]$ ,  $j = \overline{1, m}$ . Comparison of interval estimates that do not overlap will be performed by comparing their centers (mean values). If the intervals do overlap, the choice of the best one will depend on their relative position [28, 29]. To compare the overlapping intervals, we will use the Hukuhara generalized difference score  $A \overset{-}{gH} B$  and the comparison index  $\gamma_{A,B}$ , which allows us to compare hypothetical gains and losses based on the results of the choice [30, 31].

Let us represent the interval values of the  $j$ -th characteristic of the options  $s_i, s_l \in S$  as intervals  $A = [k_j^-(s_i); k_j^+(s_i)]$  and  $B = [k_j^-(s_l); k_j^+(s_l)]$  in the following form  $A = [\hat{a}; \bar{a}]$  and  $B = [\hat{b}; \bar{b}]$  where  $\hat{a}, \hat{b}, \bar{a}, \bar{b}$  are the centers and radii of the intervals  $A$  and  $B$ :

$$\begin{aligned} \hat{a} &= [a^- + a^+]/2, & \bar{a} &= [a^+ - a^-]/2, \\ \hat{b} &= [b^- + b^+]/2, & \bar{b} &= [b^+ - b^-]/2. \end{aligned} \quad (17)$$

The generalized Hukuhara difference  $A \overset{-}{gH} B$  and the comparison index  $\gamma_{A,B}$  built on its basis for intervals  $A = [\hat{a}; \bar{a}]$  and  $B = [\hat{b}; \bar{b}]$  are determined by the ratio [32, 33]:

$$A \overset{-}{gH} B = \left[ \begin{array}{l} \min \{a^- - b^-; a^+ - b^+\}; \\ \max \{a^- - b^-; a^+ - b^+\} \end{array} \right] = (\hat{a} - \hat{b}; |\bar{a} - \bar{b}|). \quad (18)$$

$$\gamma_{A,B} = (\bar{a} - \bar{b}) / (\hat{a} - \hat{b}). \quad (19)$$

When comparing options by the indicators that are maximized  $k_j(s) \rightarrow \max_{s \in S}$ , subject to a positive average gain  $\hat{a} > \hat{b}$  the following situations of intervals crossing are possible [33].

Situation 1.1:  $a^- < b^-$ . In this situation, some values of the first interval  $a \in A$  are worse than all values of the second interval  $b \in B$ . The possible loss of quality when choosing  $A$  instead  $B$  in the worst case is  $\bar{a} - \bar{b} < 0$ . The ratio of the worst-case loss to the average gain is:

$$I_{1,1}(A, B) = (a^- - b^-) / (\hat{a} - \hat{b}) = 1 - \gamma_{A,B} < 0. \quad (20)$$

Situation 2.1:  $a^- \geq b^-$ . In this situation, some values of the second interval  $b \in B$  are worse than all values of the first interval  $a \in A$ . No losses in the worst case scenario:

$$I_{2,1}(A, B) = (a^- - b^-) / (\hat{a} - \hat{b}) = 1 - \gamma_{A,B} > 0. \quad (21)$$

Situation 3.1:  $a^+ < b^+$ . In this case, all values of the first interval are worse than some values of the second interval  $b \in B$ . Negative value of the difference between the upper limits of the intervals  $a^+ - b^+ < 0$  reflects possible losses in the worst case scenario. Ratio of losses to average gain in the worst case scenario:

$$I_{3,1}(A, B) = (a^+ - b^+) / (\hat{a} - \hat{b}) = 1 + \gamma_{A,B} < 0. \quad (22)$$

Situation 4.1:  $a^+ \geq b^+$ . In this case, some values of the first interval  $a \in A$  are better than all values of the second interval  $b \in B$  and when choosing  $A$  instead of  $B$  no losses in the worst case scenario:

$$I_{4,1}(A, B) = (a^+ - b^+) / (\hat{a} - \hat{b}) = 1 + \gamma_{A,B} > 0. \quad (23)$$

When comparing options by the indicators that are maximized  $k_j(s) \rightarrow \max_{s \in S}$ , the interval  $A$  is preselected comparing with  $B$ , if  $\hat{a} < \hat{b}$ . Using the value of the comparison index  $\gamma_{A,B}$  (19) the risk measure for this choice is set. A significant difference  $\hat{b} - \hat{a} > 0$  indicates the correct choice. In this case, you should take into account the type of intersection of the intervals  $A$  and  $B$  [33].

Situation 1.2:  $a^- < b^-$ . In this situation, for each value of the second interval  $b \in B$  there are such values  $a \in A$ , that  $a < b$ :

$$I_{1,2}(A, B) = (a^- - b^-) / (\hat{a} - \hat{b}) = 1 - \gamma_{A,B} > 0. \quad (24)$$

Situation 2.2:  $a^- \geq b^-$ . In this situation, some values of the second interval  $b \in B$  are smaller (better) than all values of the first interval  $a \in A$ . A positive difference  $a^- - b^- > 0$  indicates the possible losses in the worst case.

The ratio of the worst-case loss to the average gain in this case is given by:

$$I_{2,2}(A, B) = \frac{a^- - b^-}{\hat{a} - \hat{b}} = 1 - \gamma_{A,B} < 0. \quad (25)$$

Situation 3.2:  $a^+ < b^+$ . In this situation, some values of the second interval  $b \in B$  are larger (worse) than all values of the first interval  $a \in A$ :

$$I_{3,2}(A, B) = (a^+ - b^+) / (\hat{a} - \hat{b}) = 1 + \gamma_{A,B} > 0. \quad (26)$$

Situation 4.2:  $a^+ \geq b^+$ . In this situation, some values of the interval  $a \in A$  are larger (worse) than all values of the interval  $b \in B$ . Positive difference of upper limits of intervals  $a^+ - b^+ > 0$  indicates possible losses in the worst case.

The ratio of losses in the worst case to the average profit in this case will be:

$$I_{4,2}(A, B) = (a^+ - b^+) / (\hat{a} - \hat{b}) = 1 + \gamma_{A,B} < 0. \quad (27)$$

### Methods for ranking options

To solve the problem of ranking effective design solutions and choosing the best among them, we propose the following variant of the lexicographic optimization method for conditions of incomplete certainty of DMP preferences.

Let us assume the following ordering of local criteria by importance  $k_1(s) \succ k_2(s) \succ \dots \succ k_m(s)$ . The idea of the method development is to use indices (20) - (27) to compare options from a subset of effective  $s \in S^E$  for each of the local criteria  $[k_j(s)]$ ,  $j = \overline{1, m}$ .

On the set of effective  $S^E$  it is necessary to find a subset of options  $S_1^o \subseteq S^E$ , the best according to local criteria  $k_1(s)$ , then a subset of options  $S_2^o \subseteq S_1^o$  on it, the best according to local criteria  $k_2(s)$  and so on. At the last stage, from the subset of options  $S_{m-1}^o \subseteq S_{m-2}^o$  by criteria  $k_m(s)$  the best option  $s^o \in S_{m-1}^o$  is chosen. If in the process of choosing according to the criteria  $k_j(s)$ ,  $j = \overline{1, m-1}$  only one option will be obtained, the corresponding set  $S_j^o$ ,  $j = \overline{1, m-1}$  should be expanded to include quasi-optimal options.

When implementing the method, the problem is to choose a rational amount of concession  $\Delta \xi_j(s)$  to determine the composition of subsets  $S_j^o$ ,  $j = \overline{1, m-1}$ . Too small a concession size (deviation from the best value) does not allow taking into account the values of all local criteria of options  $s \in S^E$ , and too large a size does not allow to take into account the given ordering of the criteria.

Traditionally, the size of the concession is determined by selecting by repeatedly solving the optimization problem.

Determining the optimal size of a concession  $\Delta \xi_j^o(s)$ ,  $j = \overline{1, m-1}$  is proposed to be carried out based on the required number of options  $n = |S_{m-1}^o|$  for DMP to choose the best solution at the last stage. Let us denote by  $N = |S^E|$  the power of the set of effective design solutions. If the choice is made on the entire set of permissible options, then  $N = |S|$ . In the absence of additional information about the benefits of DMP on the set of local criteria, the values of all concessions can be assumed to be equal  $\Delta \xi_j^o(s) = const \quad \forall j = \overline{1, m}$ . Then the choice of the optimal size of the concession can be replaced by the search for the optimal size of the reduction of the set of suboptimal solutions  $\Delta_N = (N - n) / (m - 1)$ .

Its value will be the solution to the equation describing the uniform reduction of subsets at each stage  $S \supseteq S_1^o \supseteq S_2^o \supseteq \dots \supseteq S_m^o$ :

$$N_1 = N - \Delta N, \quad N_2 = N_1 - \Delta N = N - 2\Delta N, \dots, \quad (28)$$

$$N_{m-1} = N_{m-2} - \Delta N = N - (m-1)\Delta N.$$

To perform automatic ranking and selection of a single option at the last stage, it is proposed to reduce the original problem to the problem of maximizing the total utility function built on the basis of the Kolmogorov-Gabor polynomial [34]:

$$[P(s)] = \sum_{i=1}^m \lambda_i [\xi_i(s)] + \sum_{i=1}^m \sum_{j=i}^m \lambda_{ij} [\xi_i(s)] [\xi_j(s)] + \dots \rightarrow \max_{s \in S^E} \quad (29)$$

$$+ \sum_{i=1}^m \sum_{j=i}^m \sum_{l=j}^m \lambda_{ijl} [\xi_i(s)] [\xi_j(s)] [\xi_l(s)] + \dots$$

where  $[P(s)] = [P^-(s); P^+(s)]$  – is the interval value of the total utility function for the option  $s$ ;  $[\xi_i(s)] = [\xi_i^-(s); \xi_i^+(s)]$  – is the interval value of the option utility function  $s$  by the  $i$ -th local criterion.

To approximate the estimates of the value of partial criteria values  $[\xi_j(s)]$ ,  $j = \overline{1, m}$  it is proposed to use the universal gluing function [34]:

$$\xi_j(s) = \begin{cases} \bar{a}_j (b_{j1} + 1) \left( 1 - \left( b_{j1} / \left( b_{j1} + \frac{\bar{k}_j(s)}{\bar{k}_{ja}} \right) \right) \right), & 0 \leq \bar{k}_j(s) \leq \bar{k}_{ja}; \\ \bar{a}_j + (1 - \bar{a}_j)(b_{j2} + 1) \times & \\ \times \left( 1 - \left( b_{j2} / \left( b_{j2} + \frac{\bar{k}_j(s) - \bar{k}_{ja}}{1 - \bar{k}_{ja}} \right) \right) \right), & \bar{k}_{ja} < \bar{k}_j(s) \leq 1, \end{cases} \quad (30)$$

where  $\bar{k}_{ja}, \bar{a}_j$  – normalized values of the coordinates of the function gluing points,  $0 \leq \bar{k}_{ja} \leq 1, 0 \leq \bar{a}_j \leq 1$ ;  $\bar{k}_j(s) = [k_j(s) - k_j^-] / (k_j^+ - k_j^-)$  is the normalized value of the  $j$ -th criterion for the option  $s$ ;  $b_{j1}, b_{j2}$  – parameters that determine the type of dependencies on the initial and final segments of the function. If the ordering by importance is specified  $k_1(s) \succ k_2(s) \succ \dots \succ k_m(s)$ , weighting coefficients of the total utility function  $P(s)$  (29) are proposed to be chosen to ensure equal advantage  $\Delta\lambda = const$  between neighboring local criteria when the condition is met

$$\sum_{j=1}^m \lambda_j = 1.$$

In particular, for the additive total utility function, as a special case of (29), the relations for calculating the weighting coefficients are given as follows:

$$\lambda_j = (m - j + 1)\Delta\lambda, \quad \Delta\lambda = \left( \frac{2}{m^2 + m} \right), \quad j = \overline{1, m}. \quad (31)$$

The basic method of lexicographic optimization involves the sequential ordering of options according to each of the local criteria.

The proposed transition from the scheme of sequential optimization by local criteria to the selection of the best options by the values of their total utility (29) involves only one ordering of the set of permissible  $S$  or effective options  $S^E$ .

### Experimental results

Let's consider an example of solving the problem of ranking effective options when designing a corporate

computer network in conditions of incomplete certainty of the input data in this formulation.

The characteristics of the set of options for permissible design solutions (network construction options) are given  $S = \{s_i\}, i = \overline{1, 8}$ , which are evaluated by three local criteria: efficiency (time of access to network resources)  $k_1(s) \rightarrow \min$ , reliability  $k_2(s) \rightarrow \max$  and the estimated costs of creating and operating the network  $k_3(s) \rightarrow \min$ . Local criteria are organized by their importance  $k_1(s) \succ k_2(s) \succ k_3(s)$ . The characteristics of the system design options are determined with an error  $\varepsilon_j \approx 0,05, j = \overline{1, 3}$ :

$$k_1(s) \in [3,601; 5,359];$$

$$k_2(s) \in [0,910; 0,993];$$

$$k_3(s) \in [9,683; 13,927] \quad [34].$$

It is necessary to determine a subset of effective network construction options on the set of permissible ones, to rank them by their overall efficiency  $s_i \succ s_j \succ \dots \succ s_k$  and determine the best among them  $s^o \in S^E$ .

In order to reduce the number of checks (20) - (27), depending on the direction of the desired change in local criteria, we represent the characteristics of the options by the values of their utility (value) functions  $\xi_j^-(s_i), \xi_j^+(s_i), j = \overline{1, 3}, s \in S$  (30).

For the given characteristics of the options, we calculate the values of their centers and radii  $\hat{\xi}_j(s_i), \bar{\xi}_j(s_i), j = \overline{1, 3}$  (Tabl. 1).

Table 1 – Values of utility functions of local criteria for network construction options

| Option | $\xi_1^-(s)$ | $\xi_1^+(s)$ | $\hat{\xi}_1(s)$ | $\bar{\xi}_1(s)$ | $\xi_2^-(s)$ | $\xi_2^+(s)$ | $\hat{\xi}_2(s)$ | $\bar{\xi}_2(s)$ | $\xi_3^-(s)$ | $\xi_3^+(s)$ | $\hat{\xi}_3(s)$ | $\bar{\xi}_3(s)$ |
|--------|--------------|--------------|------------------|------------------|--------------|--------------|------------------|------------------|--------------|--------------|------------------|------------------|
| $s_1$  | 0,001        | 0,248        | 0,124            | 0,124            | 0,036        | 0,614        | 0,325            | 0,289            | 0,867        | 0,999        | 0,934            | 0,066            |
| $s_2$  | 0,836        | 0,999        | 0,918            | 0,082            | 0,139        | 0,723        | 0,431            | 0,292            | 0,001        | 0,176        | 0,088            | 0,088            |
| $s_3$  | 0,228        | 0,453        | 0,341            | 0,113            | 0,264        | 0,855        | 0,560            | 0,295            | 0,835        | 0,970        | 0,902            | 0,067            |
| $s_4$  | 0,260        | 0,539        | 0,399            | 0,139            | 0,310        | 0,904        | 0,607            | 0,297            | 0,504        | 0,655        | 0,579            | 0,076            |
| $s_5$  | 0,265        | 0,543        | 0,404            | 0,139            | 0,299        | 0,892        | 0,595            | 0,296            | 0,127        | 0,297        | 0,212            | 0,085            |
| $s_6$  | 0,342        | 0,556        | 0,449            | 0,107            | 0,402        | 1,000        | 0,701            | 0,299            | 0,761        | 0,899        | 0,830            | 0,069            |
| $s_7$  | 0,220        | 0,502        | 0,361            | 0,141            | 0,161        | 0,747        | 0,454            | 0,293            | 0,163        | 0,331        | 0,247            | 0,084            |
| $s_8$  | 0,625        | 0,810        | 0,717            | 0,093            | 0,001        | 0,578        | 0,290            | 0,289            | 0,258        | 0,421        | 0,339            | 0,082            |

To determine the composition of a subset of effective options, we use the method of pairwise comparisons [9]. All utility functions of local criteria are maximized  $\xi_j(s_i) \rightarrow \max, j = \overline{1, 3}$ .

Using formulas (19) - (23), we calculate the values of the generalized Hukuhara difference and comparison indices for the utility functions of all local criteria.

Based on the results of analyzing the values of the

comparison indices, the ratio of strict preference is established  $R_j(S)$  for each of the local criteria  $k_j(s), j = \overline{1, 3}$ :

$$R_1(S) = \left\{ \begin{array}{l} \langle s_2, s_3 \rangle, \langle s_2, s_4 \rangle, \langle s_2, s_5 \rangle, \\ \langle s_2, s_6 \rangle, \langle s_2, s_7 \rangle, \langle s_2, s_8 \rangle, \\ \langle s_4, s_7 \rangle, \langle s_5, s_7 \rangle, \langle s_6, s_7 \rangle \end{array} \right\}, \quad (32)$$

$$R_2(S) = \left\{ \begin{array}{l} \langle s_1, s_8 \rangle, \langle s_2, s_8 \rangle, \langle s_3, s_7 \rangle, \\ \langle s_3, s_8 \rangle, \langle s_4, s_5 \rangle, \langle s_4, s_7 \rangle, \\ \langle s_4, s_8 \rangle, \langle s_5, s_7 \rangle, \langle s_5, s_8 \rangle, \\ \langle s_6, s_7 \rangle, \langle s_6, s_8 \rangle, \langle s_7, s_8 \rangle \end{array} \right\}; \quad (33)$$

$$R_3(S) = \left\{ \begin{array}{l} \langle s_1, s_2 \rangle, \langle s_1, s_3 \rangle, \langle s_1, s_4 \rangle, \\ \langle s_1, s_5 \rangle, \langle s_1, s_7 \rangle, \langle s_1, s_8 \rangle, \\ \langle s_3, s_4 \rangle, \langle s_3, s_5 \rangle, \langle s_3, s_6 \rangle, \\ \langle s_3, s_7 \rangle, \langle s_3, s_8 \rangle, \langle s_4, s_5 \rangle, \\ \langle s_4, s_7 \rangle, \langle s_4, s_8 \rangle, \\ \langle s_6, s_7 \rangle, \langle s_6, s_8 \rangle \end{array} \right\}. \quad (34)$$

From the strict advantage relations (32)-(34), we establish the composition of subsets of effective  $S^E$  and ineffective options  $\bar{S}^E$ :

$$\begin{aligned} S^E &= \{s_1, s_2, s_3, s_6, s_8\}; \\ \bar{S}^E &= \{s_4, s_5, s_7\}. \end{aligned} \quad (35)$$

For the sets of permissible options for building a corporate computer network, it has been experimentally established that subsets of effective options have much lower relative capacities (Tabl. 2, Fig. 1):

$$\delta S = \text{Card}(S^E) / \text{Card}(S)$$

Table 2 – Relative power of subsets of effective options  $\delta S$

| $m$ | The power of the set of permissible options $\text{Card}(S)$ |        |        |        |        |        |        |        |        |        |
|-----|--|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|     | 10000  | 20000  | 30000  | 40000  | 50000  | 60000  | 70000  | 80000  | 90000  | 100000 |
| 2   | 0,0015   | 0,0008 | 0,0004 | 0,0003 | 0,0002 | 0,0002 | 0,0001 | 0,0001 | 0,0001 | 0,0001 |
| 3   | 0,0062   | 0,0025 | 0,0017 | 0,0015 | 0,0014 | 0,0011 | 0,0009 | 0,0008 | 0,0008 | 0,0007 |
| 4   | 0,0152   | 0,0108 | 0,0063 | 0,0061 | 0,0058 | 0,005  | 0,0042 | 0,0038 | 0,0037 | 0,0034 |
| 5   | 0,0405   | 0,0251 | 0,0148 | 0,0148 | 0,0145 | 0,0132 | 0,0129 | 0,0123 | 0,0108 | 0,0103 |
| 6   | 0,0823   | 0,0655 | 0,0472 | 0,0397 | 0,0371 | 0,0353 | 0,032  | 0,0311 | 0,0257 | 0,0245 |
| 7   | 0,1763   | 0,1286 | 0,0827 | 0,0821 | 0,0721 | 0,0679 | 0,0677 | 0,0618 | 0,0582 | 0,0539 |

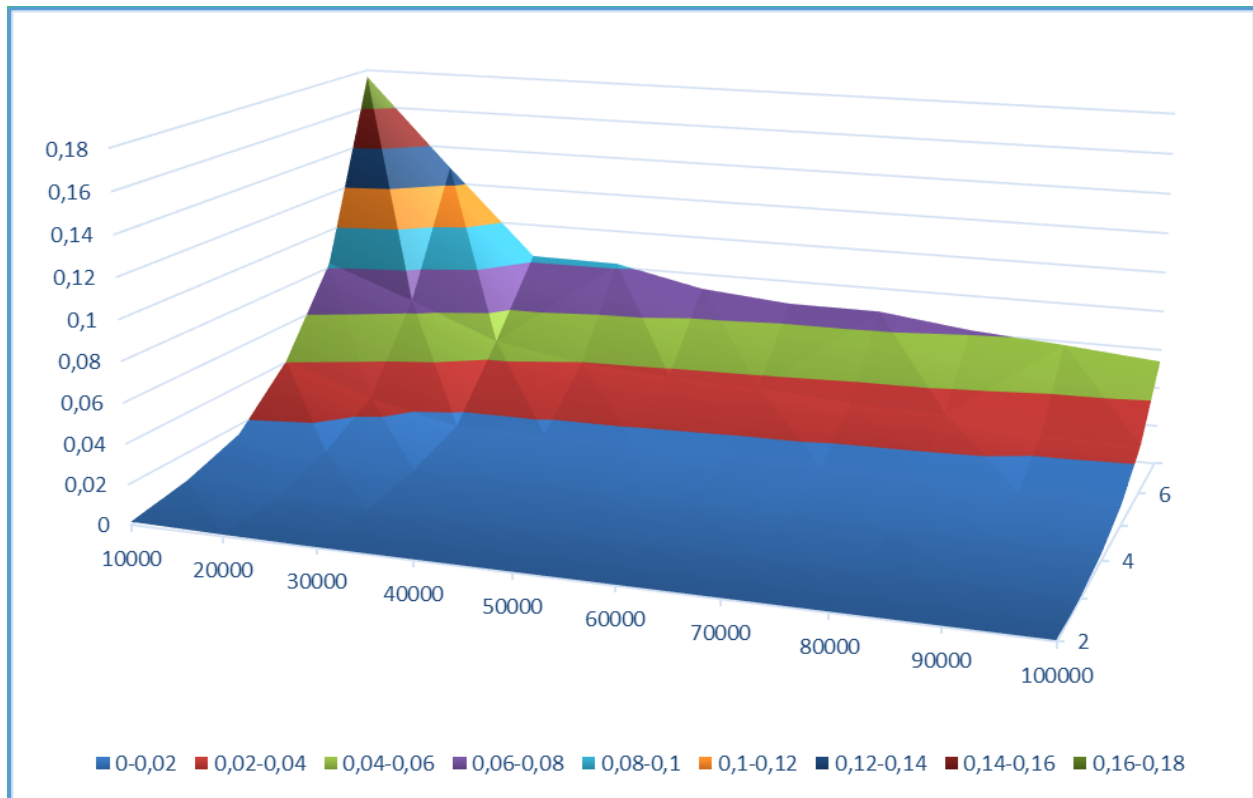


Fig. 1. Dependence of the relative powers of subsets of effective options  $\delta S$  on the power of permissible options  $\text{Card}(S)$  and the number of local criteria  $m$

Based on this, it is recommended that in the process of generating options for constructing a facility, they should be evaluated and those that are dominant should

be discarded. This will significantly reduce the required memory for storing information, the time for ranking and selecting the best design solution.



When implemented on the set of permissible options  $S = \{s_i\}$ ,  $i = \overline{1,8}$  classical scheme of the lexicographic optimization method with concessions  $\Delta\xi_j(s) = 0,1$  and  $\Delta\xi_j(s) = 0,2$ ,  $j = \overline{1,2}$  already at the first stage we will get subsets of suboptimal options  $S_1^o = \{s_2\}$  by power

$$N_1 = Card(S_1^o) = 1.$$

This requires an increase in the size of the concession.

For the size of the concession  $\Delta\xi_j(s) = 0,3$ ,  $j = \overline{1,2}$  at both the first and second stages, we get a subset of suboptimal options  $S_1^o = S_2^o = \{s_2, s_8\}$  by power

$$N_1 = N_2 = Card(S_1^o) = 2.$$

At the third stage, the best option is  $s^o = s_8$ . The option  $s_8$  will be the best for the size of the concession

$$\Delta\xi_j(s) = 0,4, \quad j = \overline{1,2}.$$

This option has an average gain

$$\hat{\xi}_3(s_8) - \hat{\xi}_3(s_2) = 0,251$$

compared to the option  $s_2$  only by the most unimportant indicator  $k_3(s)$ , however, it loses by the most important indicator

$$\hat{\xi}_1(s_2) - \hat{\xi}_1(s_8) = 0,201$$

and by the second most important indicator –

$$\hat{\xi}_2(s_2) - \hat{\xi}_2(s_8) = 0,141.$$

For the amount of concession  $\Delta\xi_j(s) = 0,5$ ,  $j = \overline{1,2}$  at the first and second stages, we obtain subsets of suboptimal options

$$S_1^o = S_2^o = \{s_2, s_6, s_8\},$$

and the third best option will be  $s^o = s_6$ .

The option  $s_6$  has average gains

$$\hat{\xi}_3(s_6) - \hat{\xi}_3(s_2) = 0,742$$

compared to the option  $s_2$  by the most unimportant indicator  $k_3(s)$  and

$$\hat{\xi}_2(s_6) - \hat{\xi}_2(s_2) = 0,270$$

on the second indicator, but loses on the most important indicator

$$\hat{\xi}_2(s_2) - \hat{\xi}_2(s_6) = 0,469.$$

To reduce the time for solving the task, we implement it on the set of permissible options

$$N = Card(S) = 8$$

the proposed lexicographic optimization scheme in order to obtain the two best options at the last stage  $n = 2$ . The number of options that are rejected after optimization at each stage using the proposed method:

$$\Delta_N = \lfloor (N - n) / (m - 1) \rfloor = \lfloor (8 - 2) / (3 - 1) \rfloor = 3. \quad (36)$$

The number of options that are included in the subset of suboptimal ones according to the local criterion  $k_1(s)$ :

$$N_1 = N - \Delta N = 8 - 3 = 5.$$

Composition of a subset of suboptimal ones by the local criterion  $k_1(s)$

$$S_1^o = \{s_2, s_4, s_5, s_6, s_8\}.$$

The number of options that are included in the subset of suboptimal ones according to the local criterion  $k_2(s)$ :

$$N_2 = N_1 - \Delta N = 5 - 3 = 2.$$

Composition of a subset of suboptimal solutions by local criterion  $k_2(s)$ :

$$S_2^o = \{s_2, s_8\}.$$

The final choice can be made by the DMP or by using a total utility assessment [ $P(s)$ ] (29).

The proposed estimation of the size of the rational reduction of subsets of optimal and suboptimal options for each of the indicators allows us to obtain a subset of effective options for a given capacity in one approach for the analysis and final selection of DMP.

Using a cardinalistic approach to ordering the options from the set of permissible ones  $S$  for  $m = 3$  using the relations (31), we calculate the value of the distance between the importance estimates of neighboring local criteria  $\Delta\lambda = 1/6 \approx 0,167$ . Taking this into account, the weight coefficients of the local criteria for the additive function of total utility will be:

$$\lambda_1 = 3/6 = 0,5; \quad \lambda_2 = 2/6 \approx 0,333; \quad \lambda_3 = 1/6 \approx 0,167.$$

Interval values of the additive function of total utility calculated for the established values of the weighting coefficients [ $P(s)$ ] are given in Table 3.

Table 3 – Interval values of the total utility function for acceptable network design options

| Option | $P^-(s)$ | $P^+(s)$ | Option | $P^-(s)$ | $P^+(s)$ | Option | $P^-(s)$ | $P^+(s)$ | Option | $P^-(s)$ | $P^+(s)$ |
|--------|----------|----------|--------|----------|----------|--------|----------|----------|--------|----------|----------|
| $s_1$  | 0,1795   | 0,4955   | $s_3$  | 0,3639   | 0,6732   | $s_5$  | 0,2817   | 0,6181   | $s_7$  | 0,2189   | 0,5550   |
| $s_2$  | 0,4937   | 0,7702   | $s_4$  | 0,3426   | 0,6799   | $s_6$  | 0,4550   | 0,7611   | $s_8$  | 0,3831   | 0,6678   |

Using the values of the comparison index  $\gamma_{A,B}$  (19) and the fulfillment of inequalities (20) - (23), the following order of options is established by interval values of the additive function of total utility  $[P(s)]$ :

$$s_2 \succ s_6 \succ s_8 \succ s_3 \succ s_4 \succ s_5 \succ s_7 \succ s_1. \quad (37)$$

The option  $s_2 = s^o$  is the best option for building a network according to three ordered local criteria. It belongs to the subset of efficient  $s_2 \in S^E$ , to subsets of the best  $S_1^o$  and  $S_2^o$ , obtained for the amount of concession  $\Delta\xi_j(s) = 0,1 \div 0,5$ ,  $j = \overline{1,2}$ , and obtained at the penultimate stage by lexicographic optimization with the proposed method of selecting the size of the concession.

The proposed method of transforming the ordinalistic representation of preferences between local criteria to their quantitative representation in the form of weighting coefficients allows us to move from solving the lexicographic optimization problem to the optimization problem based on an interval function of the total utility of facility construction options. This greatly simplifies the process of selecting the most effective design solutions for facility construction options.

### Conclusions

Based on the results of the analysis of the problem of ranking design decisions, it was found that the process of solving partial problems involves the automatic generation and analysis of powerful sets of alternative options, which are evaluated by means of mathematical modeling with some errors. This leads to the need to correctly reduce subsets of design decisions, to rank them in conditions of incomplete certainty of multicriteria evaluations of options and preferences of the decision maker. Based on the results of the decomposition of the problems of system optimization of complex objects and support for design decision-making under conditions of

incomplete certainty, the problem of ranking and selecting the best among effective options is formulated.

Based on the results of the study, a solution to the scientific and practical problem of increasing the efficiency of computer-aided design technologies for complex objects by developing a combined method for ranking effective options for constructing objects under conditions of incomplete certainty of input data using the interval analysis apparatus is proposed. For the case of an ordinalistic representation of preferences between local criteria, an estimate of the size of the rational reduction of subsets of optimal and suboptimal options for each of the indicators is proposed. Its use makes it possible to obtain a subset of effective options of a given capacity in one approach for analysis and final selection by the decision maker. A method of transforming the ordinalistic representation of preferences between local criteria to their quantitative representation in the form of weighting coefficients is proposed. This makes it possible to move from solving the problem of lexicographic optimization to the problem of optimizing an interval function of the total utility of the options for constructing an object. The efficiency and effectiveness of the proposed methods have been confirmed by experimental studies with different sets of permissible options.

The developed methods expand the methodological foundations for automating the processes of supporting the adoption of multi-criteria design decisions. They allow for the correct reduction of the set of effective alternatives in conditions of incomplete certainty of input data for the final choice, taking into account factors that are difficult to formalize, knowledge and experience of DMP. The practical use of the results obtained will reduce the time and capacity complexity of design decision support procedures, and by using the technology of selecting subsets of effective options with interval-specific characteristics, it will guarantee the quality of design decisions and provide a more complete assessment.

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## ВІДОМОСТІ ПРО АВТОРІВ/ ABOUT THE AUTHORS

- Безкоровайний Володимир Валентинович** – доктор технічних наук, професор, професор кафедри системотехніки, Харківський національний університет радіоелектроніки, Харків, Україна;  
**Volodymyr Bezkorovainyi** – Doctor of Sciences (Engineering), Professor, Professor of the Department of System Engineering, Kharkiv National University of Radio Electronics, Kharkiv, Ukraine;  
e-mail: [vladimir.beskorovainyi@nure.ua](mailto:vladimir.beskorovainyi@nure.ua); ORCID ID: <https://orcid.org/0000-0001-7930-3984>;  
Scopus ID: <https://www.scopus.com/authid/detail.uri?authorId=57190427154>.
- Колесник Людмила Володимирівна** – кандидат технічних наук, доцент, професор кафедри системотехніки, Харківський національний університет радіоелектроніки, Харків, Україна;  
**Liudmyla Kolesnyk** – Candidate of Technical Sciences, Associate Professor, Professor of the Department of System Engineering, Kharkiv National University of Radio Electronics, Kharkiv, Ukraine;  
e-mail: [liudmyla.kolesnyk@nure.ua](mailto:liudmyla.kolesnyk@nure.ua); ORCID ID: <https://orcid.org/0000-0003-4417-7759>;  
Scopus ID: <https://www.scopus.com/authid/detail.uri?origin=resultslist&authorId=57192820253&zone=>.
- Вікторс Гопеєнко** – доктор інженерних наук, професор, проректор з наукової роботи, директор магістерської програми «Комп'ютерні системи», ISMA університет прикладних наук, Рига, Латвія; провідний науковий співробітник, Вентспілський університет прикладних наук, Вентспілський міжнародний радіоастрономічний центр Вентспілс, Латвія.  
**Viktors Gopejenko** – Doctor of Sciences (Engineering), Professor, Vice-Rector for Research, Director of Study Programme Computer Systems (MSc) ISMA University of Applied Sciences Department of Natural Science and Computer Technologies, Riga, Latvia; Leading Researcher Ventspils University of Applied Sciences, Ventspils International Radio Astronomy Centre, Ventspils, Latvia;  
e-mail: [viktors.gopejenko@isma.lv](mailto:viktors.gopejenko@isma.lv); ORCID ID: <https://orcid.org/0000-0002-7783-4519>;  
Scopus ID: <https://www.scopus.com/authid/detail.uri?authorId=55038229400>.
- Косенко Віктор Васильович** – доктор технічних наук, професор, професор кафедри автоматики, електроніки та телекомунікацій, Національний університет «Полтавська політехніка імені Юрія Кондратюка», Полтава, Україна; професор кафедри комп'ютерно-інтегрованих технологій, автоматизації та робототехніки, Харківський національний університет радіоелектроніки, Харків, Україна;  
**Viktor Kosenko** – Doctor of Sciences (Engineering), Professor, Department of Automation, Electronic and Telecommunication Department of National University "Yuri Kondratyuk" Poltava Polytechnic, Poltava, Ukraine; Professor of the Department of Computer-Integrated Technologies, Automation and Robotics, Kharkiv National University of Radio Electronics, Kharkiv, Ukraine;  
e-mail: [kosvict@gmail.com](mailto:kosvict@gmail.com); ORCID ID: <https://orcid.org/0000-0002-4905-8508>;  
Scopus ID: <https://www.scopus.com/authid/detail.uri?authorId=57190443921>.

## Метод ранжування ефективних проектних рішень в умовах неповної визначеності

В. В. Безкоровайний, Л. В. Колесник, В. Гопеєнко, В. В. Косенко

**Анотація.** Предметом дослідження в статті є процес ранжування варіантів у системах підтримки прийняття проектних рішень в умовах неповної визначеності. **Мета роботи** – є підвищення ефективності технологій автоматизованого проектування складних систем за рахунок розроблення комбінованого методу ранжування ефективних варіантів побудови об'єктів в умовах неповної визначеності вхідних даних. У статті вирішуються такі **завдання**: аналіз сучасного стану проблеми ранжування варіантів у системах підтримки прийняття проектних рішень; декомпозиція проблем системної оптимізації складних об'єктів проектування та підтримки прийняття проектних рішень; розроблення методу ранжування варіантів, який об'єднує процедури лексикографічної оптимізації та кардиналістичного впорядкування в умовах неповної визначеності вхідних даних. Використовуються такі **методи**: теорії систем, теорії корисності, оптимізації, дослідження операцій, інтервальної та нечіткої математики. **Результати.** За результатами аналізу проблеми підтримки прийняття проектних рішень встановлено існування проблеми коректного скорочення підмножин ефективних варіантів для ранжування з урахуванням факторів, що важко піддаються формалізації, та досвіду особи, що приймає рішення (ОПР). Виконана декомпозиція проблем системної оптимізації складних об'єктів проектування та підтримки прийняття проектних рішень. Для випадку ординалістичного подання переваг між локальними критеріями запропоновано оцінку розміру раціонального скорочення підмножин оптимального та субоптимальних варіантів за кожним з показників. Її використання дозволяє за один підхід отримувати підмножину ефективних варіантів заданої потужності для аналізу й остаточного вибору ОПР. Запропоновано метод трансформації ординалістичного подання переваг між локальними критеріями до їх кількісного подання у вигляді вагових коефіцієнтів. **Висновки.** Розроблені методи розширюють методологічні засади автоматизації процесів підтримки прийняття багатокритеріальних проектних рішень. Вони дозволяють здійснювати коректне скорочення множини ефективних альтернатив в умовах неповної визначеності вхідних даних для остаточного вибору з урахуванням факторів, що важко піддаються формалізації, знань і досвіду ОПР. Практичне використання отриманих результатів дозволить скорочувати часову й смісну складність процедур підтримки прийняття проектних рішень, а за рахунок використання технології виділення підмножин ефективних варіантів з інтервально заданими характеристиками – гарантувати якість проектних рішень та надавати їх більш повну оцінку.

**Ключові слова:** автоматизація проектування; багатокритеріальне оцінювання; ефективні варіанти; інтервальний аналіз; підтримка прийняття проектних рішень; теорія корисності.