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## ASSESSING AND FORECASTING THE STATE OF DETERIORATING SYSTEMS WITH THE USE OF MODIFIED REGRESSION POLYNOMIALS ON THE BASIS OF FUNCTIONAL APPROXIMATION OF THEIR COEFFICIENTS

**Abstract:** **Object of research** is technical state of deteriorating systems whose operating conditions depend on a large number of interacting factors. The caused inhomogeneity of the sample of initial data on the technical state leads to impossibility of correct use of traditional methods of assessing the state of a system (meaning methods using mathematical tools of regression analysis). **Subject of research** is developing a method for constructing a regression polynomial based on the results of processing a set of controlled system parameters. Non-linearity of the polynomial describing the evolution of the technical state of real systems leads to an increase in the number of regression polynomial coefficients subject to estimation. The problem is further complicated by the growing number of factors affecting the technical state of the system. In these circumstances, the so-called <small sample effect> occurs. **Goal the research** consists in developing a method for constructing an approximation polynomial that describes evolution of the system state in a situation where the volume of the initial data sample is insufficient for correct estimating coefficients of this polynomial. **The results obtained.** The paper proposes a method for solving the given problem, based on implementation of a two-stage procedure. At the first stage a functional description of the approximation polynomial coefficients is performed; and this radically reduces the number of regression polynomial parameters to be estimated. This polynomial is used for preliminary estimation of its coefficients with the aim of filtering out insignificant factors and their interactions. At the second stage, parameters of the truncated polynomial are estimated by means of using standard technologies of mathematical statistics. Two approaches to constructing a modified polynomial have been studied: the additive one and the multiplicative one. It has been shown that the additive approach is, on average, an order of magnitude more effective than the multiplicative one.

**Keywords:** assessment of technical state; deteriorating systems; functional representation of regression equation coefficients.

### Introduction

Assessing and forecasting the technical state of systems taking into account conditions of their operation is carried out with the use of appropriate mathematical models. Suppose  $F = \{F_1, F_2, \dots, F_m\}$  is a set of factors determining the mode of operation of the system and presumably influencing its state, and  $R(F)$  is an indicator of the technical state of the system. The mathematical model traditionally used for describing  $R(F)$ , is represented as a multifactorial regression Kolmogorov-Gabor Polynomial [1–3]:

$$R(F) = a_0 + \sum_{i=1}^m a_i F_i + \sum_{i_1=1}^m \sum_{i_2 > i_1}^m a_{i_1 i_2} F_{i_1} F_{i_2} + \dots + \sum_{i_1=1}^m \sum_{i_2 > i_1}^m \dots \sum_{i_d > i_{d-1}}^m a_{i_1 i_2 \dots i_d} F_{i_1} F_{i_2} \dots F_{i_d}. \quad (1)$$

Technology and procedure for calculating multiple values of coefficients of the polynomial (1) is determined by the nature of the problem set. If, in particular, the problem of assessing reliability of the system is considered, then it shall be solved in the following way [4–7]. During the procedure of system operation technical state of the system shall be controlled at specific instants in time  $T_1, T_2, \dots, T_K$ . Therewith, values of factors  $F_i(T_k)$ ,  $i = 1, 2, \dots, m$ ,  $k = 1, 2, \dots, K$ , and results of control displayed by a set of indicators  $h_k$  are registered;  $h_k = 1$ , if a system failure is determined at the moment  $T_k$ ,  $h_k = 0$ , if at the moment  $T_k$  the system is in good order.

As a state index  $R(F)$  we choose the failure rate defined in the following way. If we accept the

hypothesis that the failure flow is the simplest, then the probability of failure-free operation of the system on the interval  $T_K$  is equal to

$$P(T_K) = e^{-\lambda(F(T_K))T_K}. \quad (2)$$

To obtain the vector  $A = (a_0, \dots, a_m, \dots, a_{i_1 i_2, \dots, i_d})$  the maximum likelihood method [8, 9] is used. In this case calculated is the probability of the fact that in the result of  $K$  checks the set  $\{h_1, h_2, \dots, h_k\}$  will be obtained. Then

$$P(A) = \prod_{k=1}^K \left( \left[ \exp \int_0^{T_k} \lambda(F(t)) dt \right]^{h_k} \times \left[ 1 - \exp(-\lambda(F(t))T_k) \right]^{1-h_k} \right).$$

Hence

$$L = \ln P(A) = \sum_{k=1}^K (1 - h_k) \ln \left( 1 - \exp \left\{ - \int_{T_k}^{T_{k+1}} \lambda(F(t)) dt \right\} \right) - \sum_{k=1}^K h_k \int_{T_k}^{T_{k+1}} \lambda(F(t)) dt. \quad (3)$$

Next, the logarithmic likelihood function (3) is differentiated by the components of vector  $A$  and the result of differentiation is set to zero.

Let the non-stationary law of variation of failure flow intensity has the form [10, 11]:

$$\lambda(F(t)) = \sum_{i=0}^d a_i(\varepsilon) t^i. \quad (4)$$

Then the resulting system of equations will have the following form:

$$\frac{\partial L}{\partial a_i} = \sum_{k \in E_1} \frac{e^{-\sum_{i=0}^d \frac{a_i(\varepsilon)}{i+1} T_k^{i+1}}}{1 - e^{-\sum_{i=0}^d \frac{a_i(\varepsilon)}{i+1} T_k^{i+1}}} - \sum_{k \in E_0} \frac{T_k^{i+1}}{i+1} = 0, \quad (5)$$

$i = 0, 1, 2, \dots, d,$

where  $E_0 = \{k: k \in E, h_k = 0\}$ ,  $E_1 = \{k, h_k = 1\}$ .

The system of nonlinear equations (5) can be solved by any known numerical method (for example, Newton's method [12, 13]) whence we get (4). Let us consider the simplest particular case, when the failure rate  $\lambda(\varepsilon)$  is estimated at the stage of normal operation of the system under fixed conditions. Herewith, the system of equations (5) is reduced to one equation:

$$\sum_{k \in E_1} \frac{T_k e^{-\lambda_\varepsilon T_k}}{1 - e^{-\lambda_\varepsilon T_k}} - \sum_{k \in E_0} T_k = 0. \quad (6)$$

Let's solve this equation by expanding the denominator in its left-hand side to the Maclaurin series, restricting ourselves to the linear term. Herewith we receive the following

$$\sum_{k \in E_1} \frac{e^{-\lambda_\varepsilon T_k}}{\lambda_\varepsilon} = \sum_{k \in E_0} T_k,$$

whence 
$$\lambda_\varepsilon^{(V)} = \frac{\sum_{k \in E_1} e^{-\lambda(V-1) T_k}}{\sum_{k \in E_0} T_k}. \quad (7)$$

We obtain an initial approximation by limiting ourselves to the linear terms of the Maclaurin series:

$$\lambda_\varepsilon^{(O)} = \frac{\sum_{k=1}^m h_k}{\sum_{k=1}^m T_k}. \quad (8)$$

Let us point out obvious shortcomings of the described approach. The adequacy of the estimates obtained when using it depends on the size of the sample and the degree of homogeneity of the statistical material used. The arising need to split the sample generates unpredictable differences in the obtained results for different subsamples. At the same time, the desire to make each of the subsamples as homogeneous as possible leads to a decrease in its volume and to decrease in accuracy of the obtained estimates.

In accordance with this, a rational approach to solving problems of assessing and forecasting system reliability consists in joint processing of results of monitoring the state of systems operated in different conditions.

Herewith, for a set of possible operating modes  $S = 1, 2, \dots, \delta$  we introduce a set of operational factors  $F = (F_1, F_2, \dots, F_m)$ ,  $F_S = (f_{1S}, f_{2S}, \dots, f_{mS})$  and the laws of change in the failure rate corresponding to these operational factors.

$$\lambda_S(t) = \sum_{i=0}^d a_{iS} t^i, \quad S = 1, 2, \dots, \delta, \quad (9)$$

$$a_{iS} = \varphi_{iS}(f_{1S}, f_{2S}, \dots, f_{mS}) = b_{i0} + \sum_{j=1}^m b_{ij} f_{jS} + \sum_{j_1=1}^m \sum_{j_2>j_1}^m b_{ij_1 j_2} f_{j_1 S} f_{j_2 S} + \dots +$$

$$+ \sum_{j_1=1}^m \sum_{j_2>j_1}^m \dots \sum_{j_{d_0}>j_{d_0-1}}^m b_{ij_1 j_2 \dots j_{d_0}} f_{j_1 S} f_{j_2 S} \dots f_{j_{d_0} S}. \quad (10)$$

The above relation (10) corresponds to the situation when the failure rate in each of  $\delta$  the operation options is affected by all factors, as well as their interactions of the order not higher than  $d_0$ .

The obtained relations (9) - (10) make it possible to carry out joint statistical processing of the results of operation of multiple objects of the same type, functioning in different conditions. In accordance with this, the entire set of objects is divided into  $\delta$  subsets, and in each of these subsets the operating conditions are characterized by their individual set of values of influencing factors.  $F_S = (f_{1S}, f_{2S}, \dots, f_{mS})$ ,  $S = 1, 2, \dots, \delta$ . At the same time, for the selected set of  $T_1, T_2, \dots, T_K$  moments used for monitoring operability of the objects, their results are recorded. Next, with the use of relations (9), (10) probabilities of fault-free operation of the objects are determined and the probability function is formed. Logarithmic representation of this function is differentiated and the results are set to zero. Solution of the resulting system of equations determines coefficients of the analytical description of the multifactor polynomial model of the law of change in failure rate (9). The principal difficulty of practical implementation of the described approach consists in high dimension of the resulting problem. If the number of factors taken into account is equal to  $m$ , the maximum order of factor interactions taken into account is equal to  $d_0$  and the law of change in the failure rate is described by means of a polynomial of degree  $d$  then the number of unknown coefficients to be estimated shall be determined by the following relation

$$J = (d + 1) C_{m+d_0}^{d_0}.$$

Herewith, if  $m = 10$ ,  $d_0 = 4$ ,  $d = 3$ , which is quite real, then  $J \cong 4000$ . It is clear that assessment of such a number of parameters is not possible.

To solve the problem, the following two-step approach is offered. At the first step, we introduce calculation relations for analytical description of the relation coefficients (10). In this connection, the following two approaches to formation of approximating values turn out to be the most easily implemented  $f_{ij}$ : additive one and multiplicative one.

$$f_{i_1 i_2 \dots i_K} = \sum_{j=1}^K b_{kj}, \quad (11)$$

$$f_{i_1 i_2 \dots i_K} = \prod_{j=1}^K b_{kj}, \quad i_1, i_2, \dots, i_K \in \{1, 2, \dots, m\}. \quad (12)$$

Let us consider the approximation procedure for the simplest problem, when  $d = 2$ .

Let us introduce the matrix  $A$ :

$$A = \begin{pmatrix} a_0 & a_{01} & \dots & a_{0m} \\ a_{10} & a_{11} & \dots & a_{1m} \\ - & - & - & - \\ a_{m0} & a_{m1} & \dots & a_{mm} \end{pmatrix}.$$

Next, to describe the additive approximation polynomial, we introduce the following

$$B_{\Sigma} = \begin{bmatrix} b_0 & b_{01} & \dots & b_{0m} \\ b_{10} & b_{01} + b_{10} & \dots & b_{0m} + b_{10} \\ - & - & - & - \\ b_{m0} & b_{01} + b_{m0} & \dots & b_{0m} + b_{m0} \end{bmatrix},$$

$$B_{\Pi} = \begin{bmatrix} b_0 & b_{01} & \dots & b_{0m} \\ b_{10} & b_{01}b_{10} & \dots & b_{0m}b_{10} \\ - & - & - & - \\ b_{m0} & b_{01}b_{m0} & \dots & b_{0m}b_{m0} \end{bmatrix}.$$

Thus, in the proposed method for constructing modified regression equations coefficients  $b_{ij}, i \neq 0, j \neq 0$ , are to be calculated according to the formula  $b_{ij} = b_{i0} + b_{0j}$  (for functional (11)) and according to the formula  $b_{ij} = b_{i0} b_{0j}$  (for functional (12)). In this connection the number of unknown coefficients is equal to  $\frac{(m+1)(m+2)}{2}$ .

Let us turn to the problem of finding the optimal set  $B = (b_0, b_{11}, \dots, b_{1m}, b_{21}, \dots, b_{2m}, \dots)$ , determining the approximating polynomial in the class (11). Let us write the relation for this polynomial of order  $d$ :

$$L_A(x) = b_0 + \sum_{i_1=1}^m b_{1i_1}x_{i_1} + \sum_{i_1=1}^m \sum_{i_2=1}^m (b_{2i_1} + b_{2i_2})x_{i_1}x_{i_2} + \dots + \sum_{i_1=1}^m \dots \sum_{i_d=1}^m (b_{di_1} + \dots + b_{di_d})x_{i_1} \dots x_{i_d}. \quad (13)$$

The obtained relation (13) is linear with respect to the required variables. Therefore, the method of least squares can be used for their estimation. For this purpose, we transform (13):

$$L(x) = b_0 + \sum_{i_1=1}^m b_{1i_1}x_{i_1} + \sum_{i_1=1}^m \sum_{i_2=1}^m (b_{2i_1} + b_{2i_2})x_{i_1}x_{i_2} + \dots + \sum_{i_1=1}^m \dots \sum_{i_d=1}^m (b_{di_1} + \dots + b_{di_d})x_{i_1} \dots x_{i_d} =$$

$$= b_0 + \sum_{i_1=1}^m b_{1i_1}x_{i_1} +$$

$$+ \left( \sum_{i_1=1}^m b_{2i_1}x_{i_1} \sum_{i_2=1}^m x_{i_2} + \sum_{i_2=1}^m b_{2i_2}x_{i_2} \sum_{i_1=1}^m x_{i_1} \right) +$$

$$+ \dots + \left( \sum_{i_1=1}^m b_{di_1}x_{i_1} \sum_{i_2=1}^m \dots \sum_{i_{d-1}=1}^m x_{i_2}x_{i_3} \dots x_{i_d} +$$

$$+ \sum_{i_2=1}^m b_{di_2}x_{i_2} \sum_{i_1=1}^m \sum_{i_3=1}^m \dots \sum_{i_d=1}^m x_{i_1}x_{i_3} \dots x_{i_d} + \dots +$$

$$+ \sum_{i_d=1}^m b_{di_d}x_{i_d} \sum_{i_1=1}^m \dots \sum_{i_{d-1}=1}^m x_{i_1}x_{i_2} \dots x_{i_{d-1}} \right) =$$

$$= b_0 + \sum_{i_1=1}^m b_{1i_1}x_{i_1} + 2 \sum_{i_1=1}^m b_{2i_2}x_{i_1} \sum_{i_2=1}^m x_{i_2} + \dots +$$

$$+ d \sum_{i_1=1}^m b_{di_1} \sum_{i_2=1}^m \dots \sum_{i_d=1}^m x_{i_2}x_{i_3} \dots x_{i_d}.$$

Let us consider that a set of function values at  $N$  points is used to estimate the coefficients. Let us

introduce the following:  $x_{ij}$  – value of factors  $x_j$  at the point  $i, i = 1, 2, \dots, N, j = 1, 2, \dots, m; y_i$  – value of function at the point  $i, i = 1, 2, \dots, N;$

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ - & - & - & - \\ x_{N1} & x_{N2} & \dots & x_{Nm} \end{pmatrix},$$

$$H_1 = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ - & - & - & - \\ 0 & 0 & \dots & 1 \end{pmatrix};$$

$$d_{im}x = Nx_m;$$

$$H_2 = 2 \begin{pmatrix} \sum_{i_1=1}^m x_{1i_1} & 0 & \dots & 0 \\ 0 & \sum_{i_1=1}^m x_{2i_1} & \dots & 0 \\ - & - & - & - \\ 0 & 0 & \dots & \sum_{i_1=1}^m X_{Ni_1} \end{pmatrix};$$

$$H_d = d \begin{pmatrix} \sum_{i_2=1}^m \dots \sum_{i_d=1}^m x_{1i_2} & \dots & x_{i_d} & \dots & 0 \\ - & - & - & - & - \\ 0 & \dots & \sum_{i_1=1}^m & \dots & \sum_{i_{d-1}=1}^m x_{1i_1} & \dots & x_{1i_{d-1}} \end{pmatrix},$$

$$B_d = (b_0, b_{11}, \dots, b_{1m}, b_{21}, \dots, b_{2m}, \dots, b_{d1}, \dots, b_{dm})^T.$$

$$H_{\Sigma} = (I_N : H_1 X : H_2 X : H_d X).$$

The desired vector  $B$  shall be calculated according to the formula

$$B_d = (H_{\Sigma}^T H_{\Sigma})^{-1} H_{\Sigma}^T Y. \quad (14)$$

Let us now construct the approximating functional that arises when using the multiplicative approach:

$$L_m(x) = b_0 + \sum_{i_1=1}^m l_{1i_1}x_{i_1} +$$

$$+ \sum_{i_1=1}^m \sum_{i_2=1}^m (b_{2i_1}b_{2i_2})x_{i_1}x_{i_2} + \dots + \sum_{i_1=1}^m \dots$$

$$\dots \sum_{i_d=1}^m (b_{di_1}b_{di_2} \dots b_{di_d})b_{di_1} \dots b_{di_d}x_{i_1} \dots x_{i_d}. \quad (15)$$

The functional (15) is nonlinear with respect to the required variables. Therefore, direct use of the method of least squares in the form (14) is impossible. However, relation (15) specifies the value of the approximating polynomial for any sets of values of factors and polynomial coefficients. Therefore, the optimal set of parameters  $B$  can be calculated by any numerical method of zero-order optimization. Analysis of numerical values of elements in this set makes it possible to filter out insignificant factors and their interactions. In this regard, reduced dimension of problem concerning estimating parameters of the truncated polynomial allows us to use standard methods of mathematical statistics at the second stage of the proposed procedure. Comparing effectiveness of

applying the proposed approaches to the construction of truncated approximation polynomials is of practical importance. A simulation experiment was conducted for this purpose. Let us compare two approaches: additive one and multiplicative one.

The simulation experiment was organized as follows. Preliminarily, for the given values of  $d$  (degree of polynomial),  $m$  (number of factors), a test regression polynomial was introduced, its coefficients were randomly selected from the interval  $[0,1]$ . Next, for a random set of factor values, the corresponding value of the test polynomial was calculated a given number of times  $N$ . These values were used to estimate the coefficients of approximation polynomials (additive one and multiplicative one). The described procedure was repeated  $R$  number of times, and after that a criterion was calculated for comparing effectiveness of the approaches

$$\eta = \frac{\zeta_M}{\zeta_A}, \quad \zeta_A = \frac{1}{R} \sum_{r=1}^R \sqrt{\frac{1}{N} \sum_{j=1}^N (y_{jr} - \hat{y}_{jr}^{(A)})^2},$$

$$\zeta_M = \frac{1}{R} \sqrt{\frac{1}{N} \sum_{j=1}^N (y_{jr} - \hat{y}_{jr}^{(M)})^2}$$

where  $R$  – quantity of conducted series of experiments;  $r$  – consecutive number of the respective experiment set,  $r = 1, 2, \dots, R$ ;  $y_{jr}$  – value of the test polynomial in the  $j$  – th experiment of the  $r$  – th set,  $j = 1, 2, \dots, N$ ;  $\hat{y}_{jr}^{(A)}$  – value of the approximating polynomial in the  $j$  - th experiment of the  $r$  - th set,  $j = 1, 2, \dots, N$ , calculated using the additive approach;  $\hat{y}_{jr}^{(M)}$  – value of the approximating polynomial in the  $j$  - th experiment of the  $r$  - th series,  $j = 1, 2, \dots, N$ , calculated using the multiplicative approach. Results of the simulation experiment are summarized in Table 1.

Table 1 – Results of the simulation experiment

	$d = 2$			$d = 3$		
$m$	5	10	15	5	10	15
$\eta$	3.4	5.3	8.4	7.2	11.2	14,7

Analysis of the presented results of the simulation experiment allows us to formulate the following recommendations. First, the additive approach is more effective than the multiplicative one in all cases. Second, advantages of the additive approach compared to the multiplicative approach grow with the increase in the degree of the approximating polynomial and the number of factors taken into account.

In conclusion, we should note that the proposed method of constructing truncated polynomials describing relationship between a certain resulting variable and a set of influencing variables can be effectively used in solving many other problems. For example, it can be used in the problems of evaluating effectiveness of mass service systems, in problems of managing a multi-nomenclature stock [14–16], etc. Direction of further research consists in development of the proposed method of constructing truncated regression equations for the case when the initial data are not specified clearly. In this regard the technology considered in [17] can be used.

### Conclusions

1. We have considered the actual problem of assessing the technical state of deteriorating systems, whose operating conditions depend on a large number of interacting factors.

2. It has been demonstrated that the problem of processing a heterogeneous sample of initial data on the state of a system cannot be solved correctly by traditional methods.

3. To solve the set problem concerning multifactor assessing and forecasting the technical state of deteriorating systems in conditions of a small sample, proposed we propose a two-step method of constructing modernized approximating polynomials based on a constructed functional description of coefficients of the required regression equation.

4. It has been demonstrated that the additive method of forming coefficients of the approximating polynomial is, on average, an order of magnitude more effective than the multiplicative method.

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Received (Надійшла) 14.07.2023

Accepted for publication (Прийнята до друку) 02.11.2023

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### Оцінка та прогнозування стану старіючих систем з використанням модифікованих регресійних поліномів на основі функціональної апроксимації їх коефіцієнтів

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**Анотація.** Об'єкт дослідження – технічний стан старіючих систем, умови експлуатації яких залежить від значної частини взаємодіючих чинників. Неоднорідність вибірки вихідних даних про технічний стан, що виникає у зв'язку з цим, призводить до неможливості коректного використання традиційних методик оцінки стану системи, що використовують математичний інструментарій регресійного аналізу. **Предмет дослідження** – розробка методу побудови регресійного полінома за результатами обробки набору контрольованих параметрів системи. Нелінійність полінома, що описує еволюцію технічного стану реальних систем, призводить до збільшення числа коефіцієнтів регресійного полінома, що підлягають оцінюванню. Проблема додатково ускладнюється із зростанням числа чинників, які впливають на технічний стан системи. У цих обставин виникає так званий «ефект малої вибірки». **Мета дослідження** - розробка методу побудови апроксимаційного полінома, що описує еволюцію стану системи у ситуації, коли обсяг вибірки вихідних даних недостатній для коректного оцінювання коефіцієнтів цього полінома. **Отримані результати.** У роботі запропоновано метод вирішення поставленого завдання, що базується на реалізації двоетапної процедури. На першому етапі виконується функціональний опис коефіцієнтів апроксимаційного полінома, що радикально знижує число параметрів регресійного полінома, що підлягають оцінюванню. Цей поліном використовується для попередньої оцінки його коефіцієнтів з метою відсіву малозначимих факторів та їх взаємодій. На другому етапі проводиться оцінка параметрів зрізаного полінома з використанням стандартних технологій математичної статистики. Досліджено два підходи до побудови модифікованого полінома: адитивний та мультиплікативний. Показано, що адитивний підхід у середньому на порядок ефективніший за мультиплікативний.

**Ключові слова:** оцінка технічного стану, старіючі системи, функціональне уявлення коефіцієнтів рівняння регресії.