Adaptive control methods

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DEVELOPMENT OF CONTROL LAWS OF UNMANNED AERIAL VEHICLES FOR PERFORMING GROUP FLIGHT AT THE STRAIGHT-LINE HORIZONTAL FLIGHT STAGE

Abstract. The article proposes an improved approach to controlling groups of unmanned aerial vehicles (UAVs) aimed at increasing the overall efficiency and flexibility of the control process. The use of a heterogeneous external field, which varies both in magnitude and direction, allows achieving greater adaptability and accuracy in controlling a group of UAVs. A vector field for unmanned aerial vehicles determines the direction and intensity of the vehicles' movement in space. Such vector fields can be used to develop UAV control laws, including determining optimal flight paths, controlling speed, avoiding obstacles, and ensuring coordination of a group of UAVs. The **subject of the study is the** methods of controlling groups of autonomous UAVs, where each vehicle may have different speeds and flight directions. To solve this problem, various methods of using a heterogeneous field have been developed and proposed. Instead of using a homogeneous field that provides a constant flight speed, a vector field is used that adapts to different conditions and characteristics of the vehicles in the group. This method allows for effective group management, ensuring the necessary coordination and interaction between the vehicles. An analysis of recent research and publications in the field of autonomous system control indicates the feasibility of using machine learning, vector fields, and a large amount of data to successfully coordinate the movement of autonomous systems. These approaches make it possible to create efficient and reliable control systems. The **aim of the study** is to develop laws for controlling the movement of a group of autonomous unmanned aerial vehicles at the stage of straight-line horizontal flight based on natural analogues to improve the efficiency and reliability of their coordinated movement in different conditions. The **main conclusions of the research** are that the proposed method of controlling groups of UAVs based on a heterogeneous field can be implemented. It takes into account a variety of vehicle characteristics and environmental conditions that are typical for real-world use scenarios. This work opens up prospects for further improving the management of UAV groups and their use in various fields of activity. The article emphasises the relevance of technology development for autonomous unmanned systems, especially in the context of autonomous transport systems.

Keywords: unmanned aerial vehicles; group flight; control of autonomous systems; Lyapunov vector fields; neural networks;autopilot technologies.

Introduction

Unmanned aerial vehicles (UAVs) can be used in single and group flights to perform military and economic tasks.

Managing a group of UAVs may involve deploying a different number of aircraft with different characteristics, such as speed and direction of flight. Group flight is carried out to achieve common goals. Distances between aircraft and other group parameters can be measured by sensors and calculated in real time. Control laws take these distances and other parameters into account to make decisions about movement, trajectory control, and mission execution for the group of UAVs.

The management of autonomous transport systems is evolving rapidly, opening up new opportunities to improve traffic flow and reduce accidents. One of the ways to achieve these goals is group driving in a straight line. This innovative technology promises to ensure efficient and safe operation of autonomous vehicles on the road, but for its successful implementation, appropriate laws and regulations need to be developed. In this article, we will look at important aspects of the development of governance laws for group driving in a straight line and their implications for collaborative use.

The relevance of this article lies in the fact that the development of technologies for autonomous transport systems is becoming increasingly important in the modern world. Swarming unmanned aerial vehicles capable of following a straight line in a group promise great potential for improving road safety and reducing traffic congestion. However, in order to ensure their safe and efficient operation, it is necessary to develop appropriate laws and regulations that take into account all the nuances of this innovative technology. Therefore, this article will look at the importance of establishing an appropriate legal framework for cooperative tracking and discuss the possible consequences and benefits of this process.

Analysis of the latest research and publications

A literature review and analysis of articles and studies by other authors in the field helps to provide more information and context on the issue. Below are examples of the results of the literature review. Control methods in the field of autonomous driving. Many studies, including those by Waymo engineers, consider the use of machine learning to solve autopilot control tasks. For example, they use neural networks to analyse the traffic situation and make driving decisions [1–4].

Vector fields in control. Articles by researchers from Stanford University consider the use of Lyapunov vector fields for autonomous control. They use these fields to create optimal paths for a car to reach a given position [5–7].

Availability of a large amount of data. Studies show that one of the key requirements for the successful management of an autonomous system is the availability of a large amount of data for training and analysis. For example, Tesla collects large amounts of data on traffic situations to improve the efficiency of its autopilot [8–13].

Successful implementations. Some articles provide examples of successful implementations of driverless autonomous vehicles. For example, Google Waymo has a pilot project with self-driving taxis in selected cities [14–16].

The ability to process large amounts of data in real time. As a result of research conducted by Ukrainian scientists, new methods for numerical solution of integro-differential equations have been proposed. The main advantage is the reduction of computational costs in solving dynamic models for remote control of unmanned aerial vehicles expressed through integro-differential equations with K-positive defined K-symmetric operators based on the improvement and application of the theory of variational gradient methods to these equations [17, 18].

The literature review helps to understand the trends and advances in the development of control laws for autonomous vehicles. The use of examples helps to concretise these approaches and technologies used by researchers and engineers to achieve the goal of autonomous driving and coordinated movement of multiple vehicles in a limited space.

The aim of the study is to develop laws for controlling the movement of a group of autonomous unmanned aerial vehicles at the stage of straight-line horizontal flight based on natural analogues to improve the efficiency and reliability of their coordinated movement in different conditions. It is assumed that the implementation of the developed control laws will improve the accuracy of controlling a group of unmanned aerial vehicles and ensure the adaptability of the process to changing external conditions.

Performing research

1. Development of control laws for the flight of the group at the gathering stage in accordance with the general heading angle

The vector field for UAVs includes vectors that define paths or control vectors for the vehicles at different points in space. This field can represent parameters such as speed, direction, location, and other important values that affect the movement of the UAV. UAV control can be autonomous, when the vehicles make decisions on their own based on algorithms and sensors, or remote, when the operator or pilot controls them manually. It is important to determine which control method is most suitable for a particular task or scenario. Managing a UAV group involves coordination and collaboration between a different number of aircraft. Groups can be of different shapes and arrangements, and it is important to determine how each vehicle interacts with the others and how they achieve common goals. The shape of the group depends on the task it is performing. For example, a group of UAVs conducting a reconnaissance mission may form a row or a circle to ensure maximum coverage.

A group of UAVs performing an attack mission can form a wedge or a diamond to ensure maximum strike power (Fig. 1).

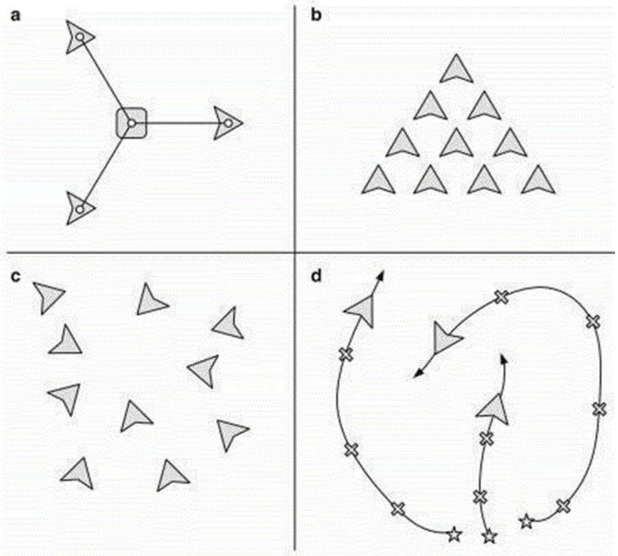

Fig. 1. Possible representation of different classes of UAV groups: a – direct physical interaction; b – formations; c – swarms of vehicles; d – cooperation

Group parameters include distances between vehicles, intervals between vehicles, distances between targets, and possible overruns. These parameters are determined based on the needs and objectives of the team, and their selection is important for the successful completion of the mission. Distances between vehicles can be measured using sensors such as GPS, radar, or embedded laser distance systems. This data can be transmitted wirelessly or over a data network for further analysis and use in the control system. Control laws are the rules, algorithms and strategies that define how UAVs should respond to different situations and perform tasks. They can include calculating optimal paths, speed control, obstacle avoidance, and coordination between vehicles. Properly designed governance laws ensure that UAVs operate efficiently and safely.

There are two main strategies for synthesising standard UAV autopilots [1].

1) The method of successive loop closure.

In this approach, the control loops are arranged in such a way that the reference signal for the inner loop is generated by the outer control loop (Fig. 2).

The most important advantage of this method is the simplicity of implementing input constraints for flight parameters (e.g., roll and pitch angles) and actuators, since the control inputs can be limited before they enter the inner loop. The basic rule of thumb is to ensure "fast" dynamics of the internal loops and "slower than before" dynamics for each new loop that is added. The main difficulty here is the lack of formalisation of the concepts of "fast" and "slow", as well as the difficulty in determining whether the interaction between the outer and inner loop is too strong (for example, closing the outer loop may degrade the efficiency of the inner loop, requiring the synthesis to be performed again). The bandwidth of the outer loop is usually chosen to be 5-10 times smaller than that of the inner loop [2].

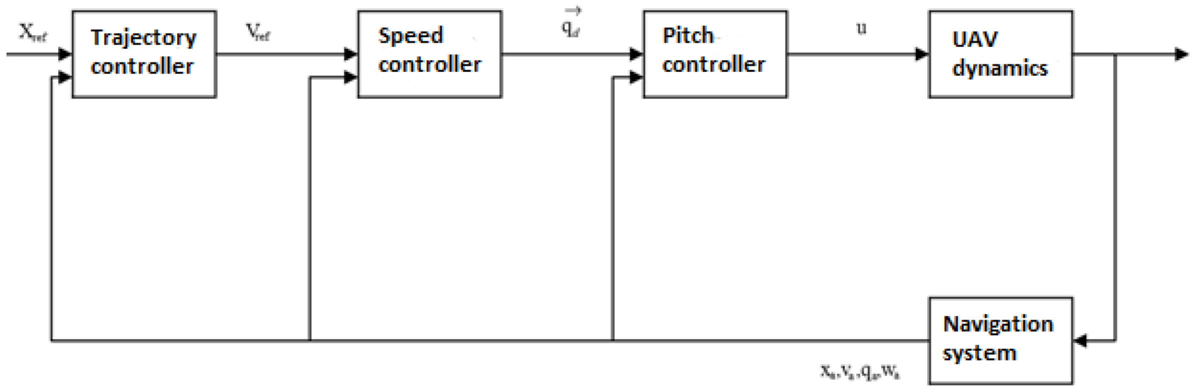

Fig. 2. An example of a series circuit closure

2) Autopilot synthesis for full dynamics - linear or nonlinear.

The advantage of this approach is the ability to use state-space methods for the full coupled dynamics of the UAV. However, it is difficult to take into account the saturation of actuators in this approach and it is difficult to include state constraints. In addition, such autopilots, especially for UAV flight with high flight dynamics, are very sensitive to errors and simplifications in mathematical models [3].

By modifying the interaction architecture, a heterogeneous method can be described that is proposed both in terms of the magnitude and direction of the path vector field.

To control single UAVs, the paper proposes the method of the path following vector field. A similar method is called the Lyapunov vector field method. It is assumed that a single UAV maintains a constant flight speed, so the field is homogeneous in size.

Various methods of a vector field that is heterogeneous in both magnitude and direction have been developed to control a group of UAVs.

Let's choose the control law for the UAV speeds as follows:

$$V^c = (v_i^c)_{i=\overline{1,N}} =$$

$$= \left(\sqrt{ \frac{ v_f^n \frac{2}{\pi} arctg(k_v^n \tilde{e}_i^n))^2 + }{ +(v + v_f^\tau \frac{2}{\pi} arctg(k_v^\tau \tilde{e}_i^\tau))^2 } } \right)_{i=\overline{1,N}} \in \mathbb{R}^{(N X1)}, \quad (1)$$

where $k_v^{(n,\tau)}$ are positive constants that determine the smoothness of the vehicles' reaching the given relative positions in the n and τ directions, respectively, $v_f^{(n,\tau)}$ are the maximum values of the norm of the additional velocity vectors v_f^n and v_f^τ in the *n-direction* and τ direction, v is the swarm *cruising* speed, optimal in terms of the UAV's aerodynamic characteristics.

Let's choose the control law for the UAV's heading angles as follows:

$$\chi^c =$$

$$= \left(\begin{matrix} \chi^q + \\ + arcsin\left(v_f^n \frac{2}{\pi} arctg(k_v^n \tilde{e}_i^n) \big/ v_i^c \right) \end{matrix} \right)_{i=\overline{1,N}} \in \mathbb{R}^{(N X1)}, \quad (2)$$

where χ^q is the final heading angle of the UAV formation.

When developing and implementing control laws, it is advisable to take into account the range of heights and speeds of straight-line horizontal flight of UAVs. This range usually represents the area in which a UAV is able to perform a straight-line horizontal flight at a constant speed without external influences (Fig. 3).

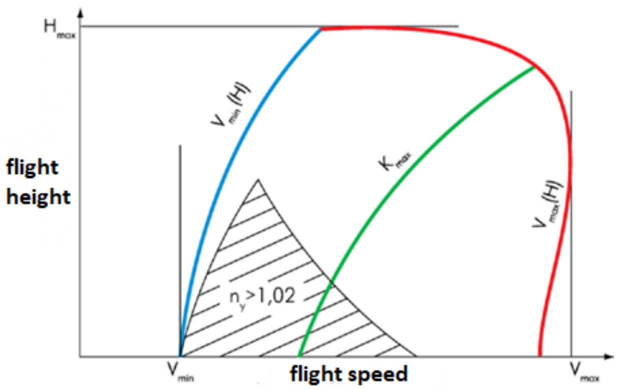

Fig. 3. Example of UAV altitude and speed range

As shown in Fig. 3, the range of altitudes and speeds is limited by the lines of minimum Vmin(H) and maximum Vmax(H) permissible flight speeds. Inside the range is the flight line with the maximum aerodynamic quality Kmax, which corresponds to the maximum flight duration. The operational range of altitude and flight speed (shaded area) includes the area limited by the minimum permissible normal overload n_y = 1.02 (minimum acceleration in g units that ensures the execution of a turn).

Proposition 1. If the control laws, v^c χ^c are given in accordance with equations (1) and (2), then in the dynamic system of a UAV swarm, the equilibrium point is asymptotically stable in general (given without proof).

Thus, the control laws (1) and (2) define a heterogeneous vector field of the path $F:\mathbb{R}^2 \to \mathbb{R}$ of each UAV in the two-dimensional flight space of a group of UAVs. The norm of the field vector at a particular point in space is a speed command for the vehicle located at that point, and the direction is a heading command [5].

2. Development of control laws for the flight of the group at the stage of the group's entry into a straight path

Let's choose the control law for the UAV speeds as follows:

$$v^{0l} = (v_i^0)_{i=\overline{1,N}} = \sqrt{\left(\begin{matrix} vsin(\psi_i) - v_f^{line}\dfrac{2}{\pi}\times \\ \times arctg(k_v^{line}\tilde{e}_i^{line}) \end{matrix}\right)^2 + \left(\begin{matrix} vcos(\psi_i) - v_f^{\tau}\dfrac{2}{\pi}\times \\ \times arctg(k_v^{\tau}\tilde{e}_i^{\tau}) \end{matrix}\right)^2}_{\,i=\overline{1,N}} \in \mathbb{R}^{(NX1)}. \qquad (3)$$

The control law for the heading angle will be chosen as follows:

$$\chi^{0l} = \chi^q + \\ +arcsin\left(\left(\begin{matrix} vsin(\psi_i) - v_f^{n}\dfrac{2}{\pi}\times \\ \times arctg(k_v^{line}\tilde{e}_i^{line}) \end{matrix}\right) \Big/ v_i^c\right)_{i=1,N} \in \mathbb{R}^{(NX1)}. \qquad (4)$$

Fig. 4 shows an example of a group of four UAVs moving along a single straight path.

Let us consider the following vector:

$$A^l = \\ = \aleph\left(\begin{matrix} A_{line}\otimes[1 \quad 0]^T + \\ +[A_\tau \quad 0]^T \otimes [1 \quad 0]^T \end{matrix}\right) \in R^{(2N-1)\times 1}, \qquad (5)$$

where $A_{line} = \left(arctg(\tilde{e}_i^{line})\right)_{i=\overline{1,N}} \in \mathbb{R}^{(N\times 1)}$

and $A_\tau = \left(arctg(\tilde{e}_{i-1,i}^{\tau})\right)_{i=\overline{2,N}} \in R^{(N-)\times 1}$.

For each *i-th* device, we can choose the following Lyapunov function:

$$V_s = \frac{1}{2}(\tilde{e}_i^{line})^2. \qquad (6)$$

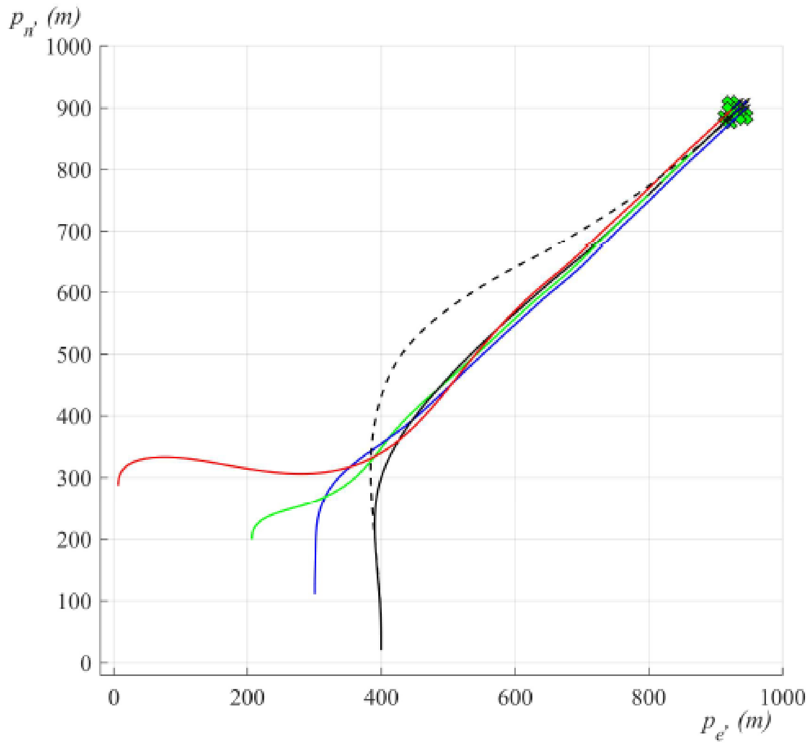

Fig. 4. An example of forming a group of four UAVs during formation building

The derivative of the function along the trajectories of the UAV swarm system using the control laws (3) and (4):

$$\dot{V}_s(\tilde{e}_i^{line}) = \tilde{e}_i^{line}\left[\begin{matrix} -vsin(arctg(k_\chi \tilde{e}_i^{line})) - \\ -v_f^{line}\dfrac{2}{\pi}arctg(k_v^{line}\tilde{e}_i^{line}) \end{matrix}\right].$$

With the help of the Gronwall-Bellman Lemma, it is possible to obtain:

$$V_s(\tilde{e}_i^{line}) \le exp(-\sigma\tau)V_s(\tilde{e}_i^{line}(0)). \qquad (7)$$

It follows from (6) and (7) that

$$\left|\tilde{e}_i^{line}(t)\right| \le exp(-\sigma\tau/2)\left|\tilde{e}_i^{line}(0)\right| \le$$

$$\le exp(-\sigma\tau/2)e_{max}{}^{line}, \quad \forall t \in [0,+\infty).$$

Thus, we can consider $\tilde{e}_i^{line}(t)$ for each *i-th* UAV as a function that disappears over time [6].

Assertion 2. If the control laws v^{0l} and χ^{0l} are given by equations (3) and (4), respectively, then in the dynamic system of UAV operation, the equilibrium point is asymptotically stable in general [7].

Proof. Let us choose the following positive quadratic form as the Lyapunov function:

$$v_0^l(\tilde{e}^l) = (\tilde{e}^l)^T A^l - \\ -\sum_{(i=2)}^{N} ln\left((\tilde{e}_{i-1,i}^{\tau})^2+1\right) - \sum_{(i=2)}^{N} ln\left((\tilde{e}_i^{line})^2+1\right). \quad (8)$$

The derivative of this function can be transformed as follows:

$$\dot{V}_0^l(\tilde{e}^l,t) = (A^l)^T \tilde{e}^l .$$

Under the assumption of no wind, the derivative of the Lyapunov function (8) along the trajectories of the UAV group system can be expressed as follows [8]:

$$\dot{V}_0^l(\tilde{e}^l,t) = \\ = \sum_{(i=2)}^{N}\begin{pmatrix} arctg\left(\tilde{e}_i^{line}\right)v_i^g(t)\times \\ \times sin(\chi_i-\chi^q)\end{pmatrix}\cdot \dot{V}_1^l(\tilde{e}^{line},t)+ \\ +\sum_{(i=2)}^{N} arctg\left(\left(\tilde{e}_{i-1,i}^{\tau}\right)\begin{pmatrix} -v_i^g(t)\times \\ \times cos\left(\chi_{i-1}-\chi^q\right)+ \\ +v_i^g(t)cos\left(\chi_i-\chi^q\right)\end{pmatrix}\right)\times \quad (9) \\ \times \dot{V}_2^l(\tilde{e}^{\tau},t).$$

Taking into account (3) and (4), when using the dynamics in the form of components $\dot{V}_1^l(\tilde{e}^{line},t)$ and $\dot{V}_2^l(\tilde{e}^{\tau},t)$ in equation (9), they take the form:

$$\dot{V}_1^l(\tilde{e}^{line}) = \\ = arctg\left(\dot{V}_1^l(\tilde{e}_1^{line}\right)\begin{bmatrix} -vsin\left(arctg\left(k_\chi \tilde{e}_1^{line}\right)-\right. \\ \left.-v_f^{line}\frac{2}{\pi}arctg\left(k_v^{line}\tilde{e}_1^{line}\right)\right)\end{bmatrix}+ \\ +\sum_{(k=2)}^{(N-2)} arctg\left(\tilde{e}_k^{line}\right)\begin{bmatrix} -vsin\left(arctg\left(k_\chi \tilde{e}_k^{line}\right)-\right. \\ \left.-v_f^{line}\frac{2}{\pi}arctg\left(k_v^{line}\tilde{e}_k^{line}\right)\right)\end{bmatrix}+ \\ +arctg\left(\tilde{e}_N^{line}\right)\begin{bmatrix} -vsin\left(arctg\left(k_v \tilde{e}_N^{line}\right)-\right. \\ \left.-v_f^{line}\frac{2}{\pi}arctg\left(k_v^{line}\tilde{e}_N^{line}\right)\right)\end{bmatrix}$$

and

$$\dot{V}_2^l\left(\tilde{e}^{\tau},t\right) = \\ = arctg(\tilde{e}^{\tau}{}_{12})\begin{bmatrix} -vcos\left(arctg\left(k_\chi \tilde{e}_1^{line}(t)\right)\right)- \\ -v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\tilde{e}^{\tau}{}_{12}\right)+ \\ +vcos(arctg\left(k_\chi \tilde{e}_2^{line}(t)\right))- \\ v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\left(-\tilde{e}^{\tau}{}_{12}+\tilde{e}^{\tau}{}_{23}\right)\right)\end{bmatrix}+ \\ +\sum_{k=2}^{N-2}\begin{pmatrix} arctg\left(\tilde{e}_{k,k+1}^{\tau}\right)\times \\ \times\begin{bmatrix} -vcos\left(arctg\left(k_\chi \tilde{e}_1^{line}(t)\right)\right)- \\ -v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\begin{pmatrix} -ek_{k-1,k}^{\tau}+ \\ +\tilde{e}_{k,k+1}^{\tau}\end{pmatrix}\right)+ \\ +vcos\left(arctg\left(k_\chi \tilde{e}_{k+1}^{line}(t)\right)\right)+ \\ +v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\begin{pmatrix} -e_{k,k+1}^{\tau}+ \\ +\tilde{e}_{k+1,k+2}^{\tau}\end{pmatrix}\right)\end{bmatrix}\end{pmatrix}+ \\ +arctg\left(\tilde{e}_{N-1,N}^{\tau}\right)\begin{bmatrix} -vcos\left(arctg\left(k_\chi e_{(N-1)}^{line}(t)\right)\right)- \\ -v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\left(-e_{N-2,N-1}^{\tau}+\right.\right. \\ \left.\left.+\tilde{e}_{N-1,N}^{\tau}\right)\right)- \\ +vcos\left(arctg\left(k_\chi \tilde{e}_N^{line}(t)\right)\right)+ \\ +v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\left(-e_{N-1,N}^{\tau}\right)\right)\end{bmatrix}.$$

Next, $\dot{V}_2^l\left(\tilde{e}^{\tau},t\right)$ can be converted in this way:

$$\dot{V}_2^l\left(\tilde{e}^{\tau},t\right) = \\ = arctg\left(\tilde{e}_N^{\tau}\right)\begin{bmatrix} -vcos\left(arctg\left(k_\chi \tilde{e}_1^{line}(t)\right)\right)- \\ -v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\tilde{e}_{12}{}^{\tau}\right)\end{bmatrix}+ \\ +\sum_{k=2}^{N-1}\begin{pmatrix}\left(arctg\left(\tilde{e}_{k-1,k}^{\tau}\right)+arctg\left(\tilde{e}_{k,k+1}^{\tau}\right)\right)\times \\ \times\begin{bmatrix} -vcos\left(arctg\left(k_\chi \tilde{e}_1^{line}(t)\right)\right)- \\ -v_f^{\tau}\frac{2}{\pi}arctg\left(k_v^{\tau}\begin{pmatrix}\tilde{e}_{k-1,k}^{\tau}+ \\ +\tilde{e}_{k,k+1}^{\tau}\end{pmatrix}\right)\end{bmatrix}\end{pmatrix}+$$

$$+arctg\left(\tilde{e}^{\tau}_{N-1,N}\right)\begin{bmatrix} vcos\left(arctg\left(k_{\chi}\tilde{e}^{line}_{N}(t)\right)\right)- \\ -v^{\tau}_{f}\frac{2}{\pi}arctg\left(k^{\tau}_{v}\tilde{e}^{\tau}_{N-1,N}\right)\end{bmatrix}.$$

Further, the derivative of the Lyapunov function can be represented as [9]

$$\dot{V}^{l}_{0}\left(\tilde{e}^{\tau},t\right)=\dot{V}^{l}_{1}\left(\tilde{e}^{line}\right)+\dot{V}^{l}_{2}\left(\tilde{e}^{\tau},t\right)=$$

$$=W_1\left(\tilde{e}^{l}\right)+W_2\left(\tilde{e}^{\tau},t\right),$$

where.

$$W_1\left(\tilde{e}^{l}\right)=\dot{V}^{l}_{1}\left(\tilde{e}^{line}\right)-$$

$$-arctg\left(\tilde{e}_{12}^{\tau}\right)v^{\tau}_{f}\frac{2}{\pi}arctg\left(k^{\tau}_{v}\tilde{e}_{12}^{\tau}\right)+$$

$$+\sum_{k=2}^{N-1}\begin{pmatrix}\left(arctg\left(\tilde{e}^{\tau}_{k-1,k}\right)+arctg\left(\tilde{e}^{\tau}_{k,k+1}\right)\right)\times \\ \times\left[v^{\tau}_{f}\frac{2}{\pi}arctg\left(k^{\tau}_{v}\begin{pmatrix}\tilde{e}^{\tau}_{k-1,k}+ \\ +\tilde{e}^{\tau}_{k,k+1}\end{pmatrix}\right)\right]\end{pmatrix}+$$

$$+arctg\left(\tilde{e}^{\tau}_{N-1,N}\right)\left[-v^{\tau}_{f}\frac{2}{\pi}arctg\left(k^{\tau}_{v}\tilde{e}^{\tau}_{N-1,N}\right)\right]$$

and

$$W_2\left(\tilde{e}^{l},t\right)=arctg\left(\tilde{e}_{12}^{\tau}\right)\times$$

$$\times\left[-vcos\left(arctg\left(k_{\chi}\tilde{e}^{line}_{1}(t)\right)\right)\right]+$$

$$+\sum_{k=2}^{N-1}\begin{pmatrix}\left(arctg\left(\tilde{e}^{\tau}_{k-1,k}\right)+arctg\left(\tilde{e}^{\tau}_{k,k+1}\right)\right)\times \\ \times\left[-vcos\left(arctg\left(k_{\chi}\tilde{e}^{line}_{k}(t)\right)\right)\right]\end{pmatrix}+$$

$$+arctg\left(\tilde{e}^{\tau}_{N-1,N}\right)\left[vcos\left(arctg\left(k_{\chi}\tilde{e}^{line}_{N}(t)\right)\right)\right]=$$

$$=\sum_{k=2}^{N-1}\left(arctg\left(\tilde{e}^{\tau}_{k-1,k}\right)\begin{bmatrix}-vcos\left(arctg\left(k_{\chi}\tilde{e}^{line}_{k-1}\right)\right)+ \\ +vcos\left(arctg\left(k_{\chi}\tilde{e}^{line}_{k}\right)\right)\end{bmatrix}\right).$$

According to the Lagrange's theorem of the mean, since the function arctg is continuous and differentiable over its entire domain, then

$$-arctg\left(\tilde{e}^{\tau}_{i-1,i}\right)+arctg\left(\tilde{e}^{\tau}_{i,i+1}\right)=$$

$$=\frac{-\tilde{e}^{\tau}_{i-1,i}+\tilde{e}^{\tau}_{i,i+1}}{1+z^2},$$

where $z\in\left(\tilde{e}^{\tau}_{i-1,i};\tilde{e}^{\tau}_{i,i+1}\right)$.

In addition, the arctg function is odd over its entire domain, so

$$\begin{pmatrix}-\tilde{e}^{\tau}_{i-1,i} \\ +\tilde{e}^{\tau}_{i,i+1}\end{pmatrix}arctg\left(k^{\tau}_{v}\begin{pmatrix}-\tilde{e}^{\tau}_{i-1,i} \\ +\tilde{e}^{\tau}_{i,i+1}\end{pmatrix}\right)\geq 0.$$

From the above, we conclude that

$$\begin{pmatrix}-arctg\left(\tilde{e}^{\tau}_{i-1,i}\right)+ \\ +arctg\left(\tilde{e}^{\tau}_{i,i+1}\right)\end{pmatrix}arctg\left(k^{\tau}_{v}\begin{pmatrix}-\tilde{e}^{\tau}_{i-1,i}+ \\ +e^{\tau}_{i,i+1}\end{pmatrix}\right)\geq 0\ .$$

Thus, we can conclude that in our case, $W_1(\tilde{e}^{l})$ is a negative function because

$$W_1(\tilde{e}^{l})<0,\forall(\tilde{e}^{l})\neq 0\ .$$

However, $W_2(\tilde{e}^{l},t)$ can acquire positive values [13].

According to the Modified Invariance Principle, the state vector on all constrained solution trajectories

$$\tilde{e}^{l}=\left\{\tilde{e}^{line}_{1},\tilde{e}_{12}^{\tau},...,\tilde{e}^{line}_{k},\tilde{e}^{\tau}_{k,k+1},...,e^{\tau}_{N-1,N},\tilde{e}^{line}_{N}\right\}$$

ends inside the region, is defined as

$$\Omega_e\triangleq\left\{\begin{matrix}\left\{\tilde{e}^{line}_{1},\tilde{e}_{12}^{\tau},...,\tilde{e}^{line}_{k},\tilde{e}^{\tau}_{k,k+1},...,\tilde{e}^{\tau}_{N-1,N},\tilde{e}^{line}_{N}\right\}: \\ W_1(\tilde{e}^{l})=0\end{matrix}\right\},$$

which involves

$$\tilde{e}^{line}_{1}\equiv 0,\tilde{e}^{\tau}_{12}\equiv 0...,\tilde{e}^{line}_{k}\equiv 0,$$
$$(\tilde{e}^{\tau}_{k-1,k}\tilde{e}^{\tau}_{k,k+1})\equiv 0..,\tilde{e}^{\tau}_{N-1,N}\equiv 0..,\tilde{e}^{line}_{N}\equiv 0.$$

The possibly nonnegative term $W_2(\tilde{e}^{l},t)$ is bounded along all bounded trajectories and tends to zero with time, since it has been shown that $\tilde{e}^{line}_{1}(t)$ for each *i-th* UAV is a vanishing function [14].

At the same time, the application $W_2(\tilde{e}^{l},t)$ does not allow us to say which trajectories are constrained and whether constrained trajectories exist in our case at all.

If $\left\|\tilde{e}^{l}\right\|$ is growing, then

$$\dot{V}^{l}_{0}(\tilde{e}^{l})\leq W_2=$$

$$=\sum_{k=2}^{N-1}\left(arctg(\tilde{e}^{\tau}_{k-1,k})\begin{bmatrix}-vcos\left(arctg\left(k_{\chi}\tilde{e}^{line}_{k-1}\right)\right)+ \\ +vcos\left(arctg\left(k_{\chi}e^{line}_{k}\right)\right)\end{bmatrix}\right)\leq$$

$$\leq\left(-\frac{v}{\sqrt{1+(k_{\chi}exp(-\sigma t/2)\tilde{e}^{line}_{\max})^2}}+v\right)V_0(\tilde{e}^{l})\leq$$

$$\leq\left(-\frac{v}{1+(k_{\chi}exp(-\sigma t/2)\tilde{e}^{line}_{\max})^2}+v\right)V_0(\tilde{e}^{l})=$$

$$= \left(-\frac{\left(k_\chi exp(-\sigma t/2)\tilde{e}_{max}^{line}\right)^2}{1+\left(k_\chi exp(-\sigma t/2)\tilde{e}_{\max}^{line}\right)^2} \right) V_0(\tilde{e}^l).$$

Let us introduce the notation [16]:

$$I \triangleq \frac{\left(k_\chi exp(-\sigma t/2)\tilde{e}_{\max}^{line}\right)^2}{1+\left(k_\chi exp(-\sigma t/2)e_{\max}^{line}\right)^2}.$$

Note that the integral of $\int_0^{+\infty} Idt$ is convergent because

$$\lim_{t\to\infty} (1/(exp(-\sigma t/2))^2 I =$$

$$= \left(k_\chi \tilde{e}_{\max}^{line}\right)^2 > 0$$

and integral $\int_0^{+\infty} (exp(-\sigma t/2))^2 dt$ is the same.

Accordingly, it can be argued that $\int_0^{+\infty} Idt =: \gamma \in \mathbb{R}$, where γ is a bounded constant whose value depends on the initial positions of the devices, the parameters of the final path, the tuning coefficients and the characteristics of the devices themselves [17].

Then, applying the Gronwall-Bellman Lemma, we can obtain:

$$V_0^l\left(\tilde{e}^l\right) \le V_0^l\left(\tilde{e}^l(0)\right) exp\left[\int_0^{+\infty} Idt\right] =$$

$$= V_0^l\left(\tilde{e}^l(0)\right) exp(\gamma).$$

Since the chosen Lyapunov function is positively defined and bounded, all trajectories are bounded.

The scope of Ω_e implies that $W_1(\tilde{e}^l) \equiv 0$ is equivalent to

$$\tilde{e}_1^{line} \equiv 0, \tilde{e}_{12}^{\tau} \equiv 0..., \tilde{e}_k^{line} \equiv 0,$$

$$\left(-\tilde{e}_{k-1,k}^{\tau}\tilde{e}_{k,k+1}^{\tau}\right) \equiv 0.., \tilde{e}_{N-1,N}^{\tau} \equiv 0, \tilde{e}_N^{line} \equiv 0.$$

Thus, all trajectories end in

$$\tilde{e}_1^{line} \equiv 0 \wedge \tilde{e}_{12}^{\tau} \equiv 0 \wedge ... \wedge \tilde{e}_{N-1,N}^{\tau} \equiv 0 \wedge \tilde{e}_N^{line} \equiv 0$$

and the system is asymptotically stable as a whole [18].

Conclusions

This paper investigates and proposes a method for controlling groups of unmanned aerial vehicles (UAVs) based on a heterogeneous path vector field. This work extends the existing approaches aimed at improving the accuracy and coordination of UAV group actions and takes into account the diversity of their characteristics, such as speed and direction of flight.

The main conclusion is that the proposed method of a heterogeneous vector field proved to be effective and efficient for controlling groups of UAVs. It provides greater flexibility and adaptability compared to traditional methods that use homogeneous vector fields.

The studies conducted have shown the importance of further research in the field of UAV group management and the development of more complex strategies for interaction between vehicles. In the future, these methods can be applied in various fields, including military, commercial and scientific use, to improve the coordination and efficiency of group tasks, thereby ensuring the further development of unmanned aerial systems.

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Розробка законів управління безпілотними літальними апаратами для виконання групового польоту на етапі прямолінійного горизонтального польоту

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Анотація. В статті пропонується удосконалений підхід до управління групами безпілотних літальних апаратів (БПЛА), що спрямовано на підвищення загальної ефективності та гнучкості процесу управління. Використання неоднорідного зовнішнього поля, яке змінюється як за величиною, так і за напрямом дозволяє досягти більшої адаптивності та точності в управлінні групою БПЛА. Векторне поле для безпілотних літальних апаратів визначає напрямок та інтенсивність руху апаратів у просторі. Такі векторні поля можуть бути використані для розробки законів управління БПЛА, включаючи визначення оптимальних шляхів руху, керування швидкістю, уникнення перешкод та забезпечення координації групи БПЛА. **Предметом дослідження** є методи управління групами автономних БПЛА, де кожен апарат може мати різну швидкість та напрямки польоту. Для вирішення цієї проблеми розроблено та запропоновано різні методи використання неоднорідного поля. Замість використання однорідного поля, яке забезпечує постійну швидкість польоту, використовують векторне, яке адаптується до різних умов і характеристик апаратів у групі. Цей метод дозволяє ефективно керувати групою, забезпечуючи необхідну координацію та взаємодію між апаратами. Аналіз останніх досліджень та публікацій у галузі управління автономними системами вказує на доцільність використання машинного навчання, векторних полів та великої кількості даних для успішної координації руху автономних систем. Дані підходи дозволяють створити ефективні та надійні системи управління. **Метою дослідження** є розробка законів управління руху групи автономних безпілотних літальних апаратів на етапі прямолінійного горизонтального польоту на основі природних аналогів для підвищення ефективності та надійності їх координованого руху в різних умовах. **Основними висновками досліджень** є те, що запропонований метод управління групами БПЛА на основі неоднорідного поля може бути реалізованим. Він враховує різноманітність характеристик апаратів та умов довкілля, що є типовими для реальних сценаріїв використання. Ця робота відкриває перспективи для подальшого вдосконалення управління групами БПЛА та їх використання в різних сферах діяльності. В статті підкреслюється актуальність розвитку технологій для автономних безпілотних систем, особливо в контексті автономних транспортних систем.

Ключові слова: безпілотні літальні апарати; груповий політ; управління автономними системами; векторні поля Ляпунова; нейронні мережі; технології автопілоту.