# Information systems modeling

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## SELECTION OF THE SET OF ALLOWABLE VALUES OF THE VARIABLE PARAMETERS OF THE STABILIZER OF A COMPLEX DYNAMIC OBJECT

**Abstract. Topicality.** One of the most important tasks in the application of computational methods for the parametric synthesis of controllers of complex dynamic objects is the task of determining the set of permissible values of the variable parameters of the controller, where the target function is calculated based on the solutions of the mathematical model of the disturbed motion of the dynamic object with its subsequent minimization. **The purpose of the work** is to construct the set of permissible values of variable parameters of the stabilizer of a complex dynamic object when applying the algorithmic combined method of parametric synthesis of stabilizers of complex dynamic objects, the essence of which is the direct calculation of the integral quadratic functional on the solutions of a closed dynamic system with subsequent finding of its global minimum through a sequential combination of two algorithms – the Sobol grid algorithm and the Nelder-Mead algorithm. **Results.** With the help of the Sobol grid construction algorithm, the starting point of the computational process is brought to the node of the Sobol grid, which is located in the small vicinity of the point of the global minimum. At the second stage of optimization, the found Sobol grid node is selected as the starting point for applying the Nelder-Mead method, which is implemented by the Optimization Toolbox software product of the MATLAB package or the Minimize software product of the MATHCAD package and leads the computational process to the point of the global minimum. **Conclusion.** The paper proves a theorem according to which the stability region of a closed system of the first approximation can be taken as such a set, and also gives an example of a solution to the problem of parametric synthesis of the stabilizer of the car's course stability system during its emergency braking.

**Keywords:** complex dynamic object; mathematical model of the disturbed motion of a closed dynamic system; parametric synthesis of the stabilizer; variable parameters.

## Introduction

**Literary review and problem statement.** In the 1960s, American [1,2] and Soviet [3-5] scientists created the foundations of modern control theory, in particular, the theory of analytical design of optimal regulators (ADOR theory). The ADOR theory essentially represents a method of structural-parametric synthesis of stabilizers of dynamic systems. In its original form, this theory considered linear dynamic objects. Later, the ADOR theory was extended to nonlinear objects [6]. But the practical application of this theory did not spread widely for the following reasons:

- firstly, the structure of the optimal regulator, obtained using the ADOR theory, involves the use of information about all, without exception, the components of the state vector of the stabilized object, obtaining which is associated with significant difficulties, or is completely impossible;
- secondly, the methods of the ADOR theory are not designed for the parametric synthesis of stabilizers of objects containing non-analytical nonlinearities;
- thirdly, the ADOR theory gives only general recommendations regarding the selection of weighting coefficients of integral additive quadratic functionals of quality;
- fourthly, parametric synthesis of digital stabilizers for complex nonlinear dynamic objects using the ADOR theory is almost impossible because mathematical models of closed control systems contain both ordinary differential equations and difference relations.

The listed features of the ADOR theory restrained its practical application. Until the beginning of the 90s of the last century, publications about the use of the ADOR theory in engineering practice rarely appeared. Examples of such publications are monographs [7, 8] and articles [6,9]. In these works, the authors proved that the value of the integral quadratic functional, which is calculated based on the solutions of the mathematical model of the disturbed motion of the closed dynamic system, is equal to the value of the Lyapunov function of the closed system at the final moment of the time of control $t=\tau$. At the same time, the optimal values of the variable parameters of the stabilizer should be chosen under the condition of reaching the minimum of the Lyapunov function of the closed system at these values. Thus, the problem of parametric synthesis of a dynamic system regulator was reduced to a problem of mathematical programming, where the Lyapunov function of a closed dynamic system was used as the objective function. But this approach did not lead to a fundamental solution the problem of parametric synthesis of stabilizers of dynamic systems. The application of the Lyapunov function does not take into account the influence of non-analytical nonlinearities of the executive body of the dynamic system on the processes being stabilized, as well as the influence on them of the "code-to-analog" and "analog-to-code" converters of the digital stabilizer. In addition, the methods of optimization of the Hook-Jives and Nelder-Mead functions [10], which are used in the implementation of the search for the minimum of the Lyapunov function, are unable to find the point of the global minimum of the Lyapunov function in the space of varied parameters of the stabilizer, but only find the point of the local minimum closest to the starting point of the computing process.

At the beginning of the 20th century, the authors of the article [11] proposed the method of the main coordinates of complex dynamic objects, which are understood as those components of the state vector of the object that characterize its dynamic properties to the greatest extent and are used to form a stabilization algorithm. On the basis of this method, an algorithmic combined method of parametric synthesis of stabilizers of complex dynamic objects was developed, the essence of which is the direct calculation of the additive integral quadratic functional on the solutions of a closed dynamic system, followed by finding its global minimum in the space of varied stabilizer parameters through a sequential combination of two algorithms – the Sobol grid algorithm [13] and the Nelder-Mead algorithm [10]. With the help of the Sobol grid method, the starting point of the computational process is brought to the node of the Sobol grid, which is located in the small vicinity of the point of the global minimum. At the second stage of optimization, the found Sobol grid node is selected as the starting point for applying the Nelder-Mead method, which is implemented by the Optimization Toolbox software product of the MATLAB package or the Minimize software product of the MATHCAD package and leads the computational process to the point of the global minimum.

With the help of a combined algorithmic method, the authors solved the problems of parametric synthesis of a digital stabilizer of a resilient tank gun [14], a digital stabilizer of the space stage of a carrier rocket with a liquid jet engine on the active part of the flight path [15], as well as a digital stabilizer of the course stability system of a refueling car [16].

One of the most difficult problems of solving any optimization problem is the determination of the set of permissible values of the variable parameters of the dynamic system. The proposed work considers the method of choosing such a set while minimizing the functional, which is calculated on the solutions of a complex nonlinear dynamic system.

## Main material

**Selection of the set of permissible values of variable parameters of the stabilizer of a complex dynamic object.** We will assume that the disturbed motion of a dynamic object is described by a vector-matrix differential equation

$$\dot{X}(t) = \Phi[X(t)] + B \cdot U(t), \tag{1}$$

where $X(t)$ is the $n$-dimensional vector of the state of the object; $U(t)$ $m$-dimensional control vector; $B$ – control matrix $n \times m$; $\Phi[X(t)]$– $n$-dimensional nonlinear analytic vector function.

The automatic regulator implements a control vector $U(t)$ in the form

$$U(t) = \Psi[G(t)], \tag{2}$$

where $\Psi[G(t)]$ is a $m$-dimensional non-analytic vector function; $G(t)$ – $m$-dimensional control vector function, which is formed by an electronic stabilizer in the form

$$G(t) = K \cdot X(t), \tag{3}$$

where $K$ is the matrix of variable parameters of the stabilizer with the size $m \times n$.

The set of vector-matrix differential equation (1) and vector-matrix ratios (2) and (3) creates a mathematical model of the disturbed motion of a closed dynamic system.

In accordance with works [11, 12], the components of the state vector $X(t)$, which to the greatest extent characterize the behavior of the control object, will be called the main coordinates of the stabilization object. Usually, the measurement of the main coordinates does not cause difficulties, therefore, when forming the vector function (3), only the main coordinates of the object of stabilization are taken into account. At the same time, the columns of the matrix $K$ corresponding to the main coordinates of the object are non-zero, and all other columns are zero. The task of the parametric synthesis of the stabilizer of a dynamic object is to determine the non-zero elements of the matrix $K$ such that the integral quadratic functional reaches a minimum on the solutions of the mathematical model (1)–(3)

$$I(K) = \int_0^{\tau} \langle X(t), QX(t) \rangle dt, \tag{4}$$

where $Q$ is the diagonal Sylvester matrix of size $n \times n$

$$Q = \begin{bmatrix} \beta_1^2 & 0 & ... & 0 \\ 0 & \beta_1^2 & ... & 0 \\ . & . & . & . \\ 0 & 0 & ... & \beta_1^2 \end{bmatrix}. \tag{5}$$

Some of the diagonal elements of the matrix (5) are equal to zero, and its non-zero diagonal elements correspond to the main components of the state vector $X(t)$.

In accordance with the combined algorithmic method of parametric synthesis of the stabilizer of a closed dynamic system, the value of the functional (4) is directly calculated on the solutions of the closed dynamic system (1)–(3) with subsequent finding of its minimum on the set of permissible values $G_K$, which is built in the space of variable parameters of the stabilizer

$$I(K^*) = \min_{K \in G_K} \int_0^{\tau} \langle X(t), QX(t) \rangle dt, \tag{6}$$

where $K^* \in G_K$ is the matrix of optimal values of variable parameters.

To build a computational process of search of the matrix $K^*$, it is necessary to define a set $G_K$ in the space of variable parameters of the stabilizer. To do this, let's move from the nonlinear system (1)–(3) to the system of the first approximation [17]. Instead of the nonlinear differential equation of the perturbed motion of the stabilization object (1), let's go to the linear differential equation

$$\dot{X}(t) = A \cdot X(t) + B \cdot U(t), \qquad (7)$$

where the elements of the matrix $A$ are calculated by the formula

$$a_{ij} = \left( \frac{\partial \varphi_i [x_1(t), x_2(t), \ldots, x_n(t)]}{\partial x_j(t)} \right)_0, \quad \left(i, j = \overline{1,n}\right). \qquad (8)$$

Through $\varphi_i [x_1(t), x_2(t), \ldots, x_n(t)]$ in formula (8), the elements of the vector function $\Phi[X(t)]$ are marked, and the derivatives are calculated at the point of stable equilibrium of the object, which is determined by the algebraic equation

$$\Phi[X(t)] + B \cdot U(t) = 0. \qquad (9)$$

The non-analytical vector function (2) takes into account non-analytical nonlinearities of the executive bodies of the stabilizer - saturation, insensitivity zone, etc. If these properties of the executive body are neglected, formula (2) can be presented in a linear form

$$U(t) = G(t), \qquad (10)$$

and the mathematical model of the closed system of the first approximation can be written in the form of a linear-matrix differential equation

$$\dot{X}(t) = A \cdot X(t) + B \cdot K \cdot X(t) = (A + B \cdot K) X(t). \qquad (11)$$

The following theorem applies: as a set of allowable values $G_K$ of the variable parameters of the stabilizer of the closed nonlinear dynamic system (1)–(3), the region of stability of the corresponding system of the first approximation (11) in the space of variable parameters can be chosen.

The proof of the formulated theorem will be carried out from the opposite. Let's choose a point $K$ in the set of variable parameters that belongs to the region of stability of the system of the first approximation (11). In this point, in accordance with Lyapunov's theorem on stability to the first approximation [18], the nonlinear system (1)–(3) is also stable. Therefore, it is quite likely that the minimum point of the functional (4) is in the middle of the region of stability of the system of the first approximation (11).

Now suppose that the minimum point of the functional (4), calculated on the solutions of the nonlinear system (1)–(3), is outside the region of stability of the system of the first approximation (11), that is, the system of the first approximation is unstable. But then the corresponding nonlinear system (1)–(3) is unstable and any point chosen outside the region of stability cannot be the minimum point of the functional (4) calculated on the solutions of the nonlinear system (1)–(3). Therefore, the minimum point of the functional (4) is in the region of stability of the system of the first approximation (11), which can be taken as a set $G_K$ of allowable values of the variable parameters of the stabilizer of the nonlinear system (1)–(3). The theorem is proved.

To construct the region of stability of the system of the first approximation (11), we write down its characteristic equation

$$\det[A + B \cdot K - E \cdot s] = 0, \qquad (12)$$

where $s$ is the complex variable of the Laplace transform. Using the $D$-distribution method [19] in the set of variable parameters of the stabilizer, we will select a set $G_K$ and when organizing the computational process of searching for elements of the matrix of variable parameters $K$, we will use the condition

$$K \in G_K. \qquad (13)$$

**Example.** As an example, consider the construction of the region of stability $G_K$ of the closed system of course stability of the car [16]. Mathematical model of the disturbed movement of the car is written in the form:

$$\begin{aligned} \dot{v}_x(t) &= -\frac{1}{M}\left[2k_b p_0(t) + Gf_0\right]; \\ \ddot{\psi}(t) &= -\frac{Bk_b}{2I}\Delta p(t) - \frac{2H_m M}{I} f_0 v_x(t)\dot{\psi}(t); \\ \Delta\ddot{p}(t) &= -\frac{f_r}{I_r}\Delta\dot{p}(t) - \frac{c_r}{I_r}\Delta p(t) + \frac{k_u}{I_r}u(t); \\ \dot{y}(t) &= -v_x(t)\psi(t), \end{aligned} \qquad (14)$$

where $v_x(t)$ – the current speed of the center of mass of the car; $\psi(t)$ – angle of deviation of the main longitudinal central axis of inertia of the car relative to the given direction of movement; $y(t)$ – lateral shift of the center of mass of the car body relative to the given direction of movement (Fig. 1); $p_0(t)$ – brake fluid pressure in the main brake cylinder; $\Delta p(t)$ – brake fluid pressure difference in the brake lines of the right and left sides of the car; $M$ – mass of the car; $I$ – the moment of inertia of the car body relative to its own central vertical axis of inertia; $f_0$ – the car's movement resistance coefficient; $G$ – weight of the car; $I_r$ – the moment of inertia of the rocker arm of the electromagnet of the executive body; $f_r$ – coefficient of liquid friction in the axis of rotation of the rocker arm; $c_r$ – stiffness coefficient of the fixing spring of the rocker arm; $H_m$ – the distance from surface of movement of the car to its center of mass; $k_u$ – gain coefficient of the electro-hydraulic amplifier; $B$ – track width of the car; $k_b$ – coefficient of proportionality.

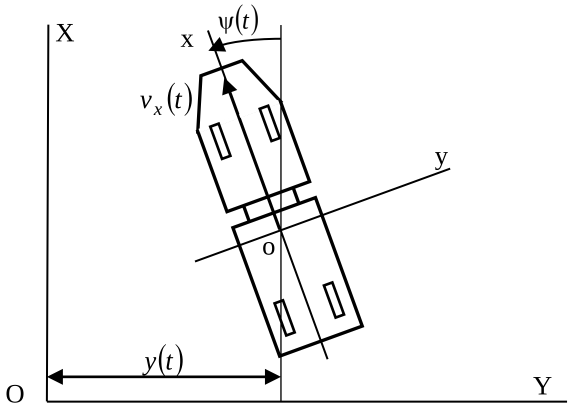


**Fig. 1.** For determination the disturbed car movement

System (14) is essentially non-linear, since it contains the multiplication of system state variables in the right parts of the second and fourth differential equations. At the same time, in the process of emergency braking $p_0(t)=p_{0\max}$, the right part of the first equation of system (14) represents a constant value, and the current speed of the car changes according to the formula

$$v_x(t)=v_0-a\cdot t\,, \tag{15}$$

where $v_0$ is the initial speed at the beginning of the emergency braking mode; $a$ – acceleration during braking.

Substitute ratio (15) into system (14). As a result, we get a mathematical model of disturbed car movement in the form of a linear non-stationary system

$$\begin{aligned}
&\ddot{\psi}(t)=-\frac{Bk_b}{2I}\Delta p(t)-\frac{2H_mM}{I}f_0(v_0-a\cdot t)\dot{\psi}(t);\\
&\Delta\ddot{p}(t)=-\frac{f_r}{I_r}\Delta\dot{p}(t)-\frac{c_r}{I_r}\Delta p(t)+\frac{k_u}{I_r}u(t);\\
&\dot{y}(t)=-(v_0-a\cdot t)\psi(t).
\end{aligned} \tag{16}$$

The main coordinates of the system under consideration are the angular deviation $\psi(t)$, the angular speed of rotation of the car body $\omega_\psi=\dot{\psi}(t)$ and the lateral shift of the center of mass of the body $y(t)$. It is these coordinates that determine the disturbed movement of the car body, are measured by the corresponding sensors and are used in the formation of the control function

$$\sigma(t)=k_\psi\psi(t)+k_{\dot{\psi}}\omega_\psi(t)+k_y y(t), \tag{17}$$

The static characteristic of the executive body of the car's course stability system is presented in Fig. 2 and can be written in the form of a non-analytical function

$$u(t)=\begin{cases} u^*\operatorname{sign}\sigma(t) \text{ при } |\sigma(t)|<u^*;\\ \sigma(t) \text{ при } u^*\le|\sigma(t)|<u^{**};\\ u^{**}\operatorname{sign}\sigma(t) \text{ при } |\sigma(t)|\ge u^{**}.\end{cases} \tag{18}$$

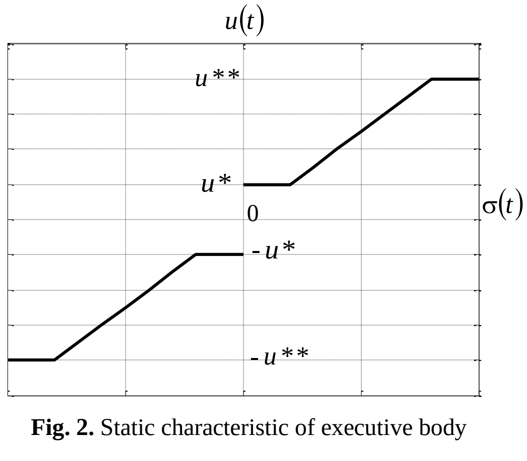


**Fig. 2.** Static characteristic of executive body

The system of differential equations (14) and ratios (17) and (18) will be called the mathematical model of the disturbed motion of the closed system of course stability of the car.

If the non-linearities of the static characteristic of the executive body of the system are neglected due to the fact that the value of the insensitivity zone $u^*$ is small and the value of saturation $u^{**}$ is large enough, then the non-analytical function (18) can be replaced by a linear function

$$u(t)=\sigma(t),$$

or taking into account formula (17)

$$u(t)=k_\psi\psi(t)+k_{\dot{\psi}}\omega_\psi(t)+k_y y(t). \tag{19}$$

The system of linear non-stationary differential equations (16) and relation (19) will be called the mathematical model of the closed system of the first approximation.

Construction of the region of stability of a linear non-stationary system on the set of its variable parameters is a rather complex problem. In engineering practice, the method of "frozen coefficients" [20,21] has become widespread, according to which several moments of time $t_1,t_2,...,t_s,...,t_q$ are selected in the interval $[0,\tau]$ and for each of the moments $t_s,(s=\overline{1,q})$ the time-varying coefficients are "frozen", that is, their values are assumed to be constant, and for each of the moments $t_s,(s=\overline{1,q})$ a mathematical model of disturbed motion with time-constant coefficients is built. In other words, instead of a non-stationary system of the first approximation, $q$ stationary systems are considered

$$\begin{aligned}
&\ddot{\psi}(t)=-\frac{Bk_b}{2I}\Delta p(t)-\frac{2H_mM}{I}f_0v_s\dot{\psi}(t);\\
&\Delta\ddot{p}(t)=-\frac{f_r}{I_r}\Delta\dot{p}(t)-\frac{c_r}{I_r}\Delta p(t)+\\
&+\frac{k_u}{I_r}\left[k_\psi\psi(t)+k_{\dot{\psi}}\dot{\psi}(t)+k_y y(t)\right];\\
&\dot{y}(t)=-v_s\psi(t);\qquad (s=\overline{1,q}).
\end{aligned} \tag{20}$$

where $v_s=v_0-a\cdot t_s,\quad (s=\overline{1,q})$.

Let's introduce the notation

$$a'_{\psi\psi}=\frac{2H_mM}{I}f_0;\ \ a_{\psi p}=\frac{Bk_b}{2I};\ \ a_{pp}=\frac{c_r}{I_r};$$

$$a'_{pp}=\frac{f_r}{I_r};\ \ \bar{k}_u=\frac{k_u}{I_r}.$$

Then linear stationary systems for each moment $t_s,(s=\overline{1,q})$ are written in the form

$$\begin{aligned}
&\ddot{\psi}(t)=-a_{\psi p}\Delta p(t)-a'_{\psi\psi}v_s\dot{\psi}(t);\\
&\Delta\ddot{p}(t)=-a'_{pp}\Delta\dot{p}(t)-a_{pp}\Delta p(t)+\\
&+\bar{k}_u\left[k_\psi\psi(t)+k_{\dot{\psi}}\dot{\psi}(t)+k_y y(t)\right];\\
&\dot{y}(t)=-v_s\psi(t);\qquad (s=\overline{1,q}).
\end{aligned} \tag{21}$$

We write each system of differential equations (21) in the normal Cauchy form, for which we introduce the state vector of the closed system

$$X(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix}=\begin{bmatrix} \psi(t) \\ \dot{\psi}(t) \\ \Delta p(t) \\ \Delta\dot{p}(t) \\ y(t) \end{bmatrix}.$$

In the new variables, each of the systems (21) takes the form:

$$\begin{aligned} &\dot{x}_1(t)=x_2(t); \\ &\dot{x}_2(t)=-a'_{\psi\psi}v_s x_2(t)-a_{\psi p}x_3(t); \\ &\dot{x}_3(t)=x_4(t); \\ &\dot{x}_4(t)=\bar{k}_u k_{\psi}x_1(t)+\bar{k}_u k_{\dot{\psi}}x_2(t)- \\ &-a_{pp}x_3(t)-a'_{pp}x_4(t)+\bar{k}_u k_y x_5(t); \\ &\dot{x}_5(t)=-v_s x_1(t); \qquad \left(s=\overline{1,q}\right). \end{aligned} \tag{22}$$

Each of the systems (22) will be written in vector-matrix form

$$\dot{X}(t)=A(v_s)\cdot X(t); \qquad \left(s=\overline{1,q}\right), \tag{23}$$

where the matrix $A(v_s)$ is written in the form

$$A(v_s)=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -a'_{\psi\psi}v_s & -a_{\psi p} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \bar{k}_u k_{\psi} & \bar{k}_u k_{\dot{\psi}} & -a_{pp} & -a'_{pp} & \bar{k}_u k_y \\ -v_s & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{24}$$

The characteristic equations of each of the systems (22) have the form

$$\det\left[A(v_s)-E\cdot s\right]=0; \qquad \left(s=\overline{1,q}\right), \tag{25}$$

where $s$ is the complex variable of the Laplace transform.

Substitution of matrices (24) into characteristic equations (25) allows to write the latter in the canonical form:

$$\begin{aligned} &s^5+\left(a'_{\psi\psi}v_s+a'_{pp}\right)s^4+ \\ &+\left(a'_{\psi\psi}a'_{pp}v_s+a_{pp}\right)s^3+ \\ &+\left(a_{\psi p}\bar{k}_u k_{\dot{\psi}}+a'_{\psi\psi}a_{pp}v_s\right)s^2+ \\ &+a_{\psi p}\bar{k}_u k_{\psi}s-a_{\psi p}v_s\bar{k}_u k_y=0; \qquad \left(s=\overline{1,q}\right). \end{aligned} \tag{26}$$

In Fig. 3 shows the structural and functional scheme of the closed system of the car's course stability, where the notations are adopted:

C – car;
BS – braking system;
B – car body;
SU – sensor unit;
EU – electronic unit.

Analysis of Fig. 3 allows us to conclude that the electronic unit implements two control circuits, namely, the internal control circuit for the angle of deviation $\psi(t)$ and angular speed $\omega_{\psi}(t)=\dot{\psi}(t)$, as well as the external control circuit for the shift of the car's center of mass $y(t)$.

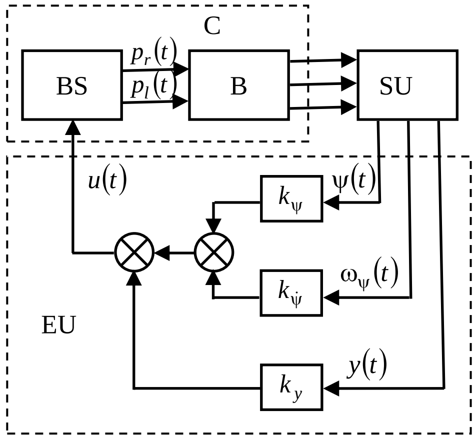


**Fig. 3.** Closed system of car course stability

Let's break the outer contour by putting $k_y=0$ in the characteristic equations (26). As a result, the characteristic equations (26) take the form $\left(s=\overline{1,q}\right)$:

$$\begin{aligned} &s^4+\left(a'_{\psi\psi}v_s+a'_{pp}\right)s^3+\left(a'_{\psi\psi}a'_{pp}v_s+a_{pp}\right)s^2+ \\ &+\left(a_{\psi p}\bar{k}_u k_{\dot{\psi}}+a'_{\psi\psi}a_{pp}v_s\right)s+a_{\psi p}\bar{k}_u k_{\psi}=0. \end{aligned} \tag{27}$$

In the characteristic equations (27), we will make a replacement $s=j\omega$, separate the real and imaginary parts and set them equal to zero. As a result, to construct the stability region $G_K^{\psi}(v_s)$ for the internal control loop, we obtain the ratio

$$k_{\psi}=\frac{\omega^2}{a_{\psi p}\bar{k}_u}\left\{a'_{\psi\psi}a'_{pp}v_s+a_{pp}-\omega^2\right\}; \tag{28}$$

$$k_{\dot{\psi}}=\frac{1}{a_{\psi p}\bar{k}_u}\left\{\left(a'_{\psi\psi}v_s+a'_{pp}\right)\omega^2-a'_{\psi\psi}a_{pp}v_s\right\}. \tag{29}$$

Changing $\omega$ from zero to infinity, in the area of the variable parameters $\left(k_{\psi},k_{\dot{\psi}}\right)$ for different values $v_s$, we construct regions of stability $G_K^{\psi}(v_s),\left(s=\overline{1,q}\right)$, which are presented in Fig. 4.

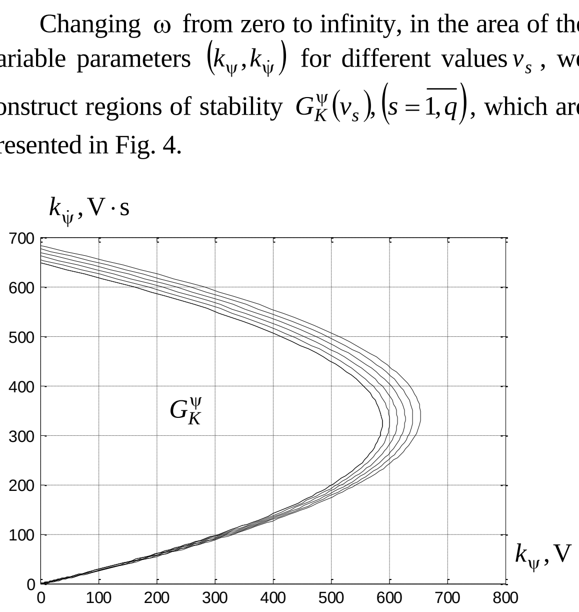


**Fig. 4.** Stability regions $G_K^{\psi}(v_s)$

At the same time, the region $G_K^{\psi}$ represents the intersection of regions $G_K^{\psi}(v_s), \left(s=\overline{1,q}\right)$

$$G_K^{\psi}=G_K^{\psi}(v_1)\cap G_K^{\psi}(v_2)\cap\ldots\cap G_K^{\psi}(v_q) \qquad (30)$$

and is the region of allowable values of variable parameters $k_{\psi}$ and $k_{\dot\psi}$ in the plane $\left(k_{\psi},k_{\dot\psi}\right)$. The values of the coefficients of the characteristic polynomial (27) in the example under consideration are equal to:

$$a'_{\psi\psi}=0,8\cdot 10^{-2}\,m^{-1};\quad a_{pp}=200\ s^{-2};$$

$$a'_{pp}=55\ s^{-1};\quad \bar{k}_u=0,5\cdot 10^{9}V^{-1}\cdot Pa\cdot s^{-1};$$

$$a_{\psi p}=0,34\cdot 10^{-7}\,Pa^{-1}\cdot s^{-2}.$$

The values of the speed of the car were chosen as follows:

$$v_1=25\,\mathrm{m\cdot s^{-1}};\ v_2=20\,\mathrm{m\cdot s^{-1}};\ v_3=15\,\mathrm{m\cdot s^{-1}};$$

$$v_4=10\,\mathrm{m\cdot s^{-1}};\ v_5=5\,\mathrm{m\cdot s^{-1}};\ v_6=0\,\mathrm{m\cdot s^{-1}}.$$

Analysis of fig. 4 allows us to conclude that the value of the fixed speed of movement $v_s$ has almost no effect on the region of stability of the closed system of the first approximation, especially in the region of low frequencies.

On the constructed set $G_K^{\psi}$ using the algorithmic combined method of parametric synthesis of the stabilizer of a complex dynamic object, we find the values of the variable parameters $k_{\psi}^{*}\in G_K^{\psi}$; $k_{\dot\psi}^{*}\in G_K^{\psi}$, which deliver the global minimum of functional

$$I\left(k_{\psi},k_{\dot\psi}\right)=\int_0^{\tau}\left[\beta_1^2x_1^2(t)+\beta_2^2x_2^2(t)\right]dt \qquad (31)$$

on the solutions of the closed nonlinear system (14), (17), (18) under the condition that in relation (17) the variable parameter $k_y$ is equal to zero, and the weighting coefficients of the functional (31) are chosen according to the method described in works [22,23].

The values of the coefficients $k_{\psi}^{*}$ and $k_{\dot\psi}^{*}$, which deliver the minimum of the functional (31), are

$$k_{\psi}^{*}=198{,}9\ \mathrm{V};\quad k_{\dot\psi}^{*}=349{,}4\ \mathrm{V\cdot s}.$$

Let's close the outer circuit of the scheme shown in fig. 3 and return to the characteristic equation (26), putting in it $k_{\psi}=k_{\psi}^{*}$ and $k_{\dot\psi}=k_{\dot\psi}^{*}$. We solve the characteristic equation with respect to the variable parameter $k_y$:

$$k_y=\frac{1}{a_{\psi p}\bar{k}_u v_s}\Big\{a_{\psi p}\bar{k}_u k_{\psi}^{*}s+\left(a_{\psi p}\bar{k}_u k_{\dot\psi}^{*}+a'_{\psi\psi}a_{pp}v_s\right)s^2+\left(a'_{\psi\psi}a'_{pp}v_s+a_{pp}\right)s^3+\left(a'_{\psi\psi}v_s+a'_{pp}\right)s^4+s^5\Big\}. \qquad (32)$$

In relation (32), we make a replacement $s=j\omega$ and separate the real and imaginary parts

$$\operatorname{Re}k_y(\omega)=\frac{1}{a_{\psi p}\bar{k}_u v_s}\times\Big\{-\left(a_{\psi p}\bar{k}_u k_{\dot\psi}^{*}+a'_{\psi\psi}a_{pp}v_s\right)\omega^2+\left(a'_{\psi\psi}v_s+a'_{pp}\right)\omega^4\Big\}; \qquad (33)$$

$$\operatorname{Im}k_y(\omega)=\frac{1}{a_{\psi p}\bar{k}_u v_s}\cdot\Big\{a_{\psi p}\bar{k}_u k_{\psi}^{*}s\omega-\left(a'_{\psi\psi}a'_{pp}v_s+a_{pp}\right)\omega^3+\omega^5\Big\}. \qquad (34)$$

In Fig. 5 shows the boundaries of the regions of stability of systems of the first approximation (22) in the plane of the complex variable parameter $k_y$. The region $G_K^{y}$ is an intersection of regions $G_K^{y}(v_s), \left(s=\overline{1,q}\right)$

$$G_K^{y}=G_K^{y}(v_1)\cap G_K^{y}(v_2)\cap\ldots\cap G_K^{y}(v_q), \qquad (35)$$

each of which is a segment of the real axis, located on its negative part between the points of intersection of the latter and constructed using relations (33) and (34) as the boundary of the stability region.

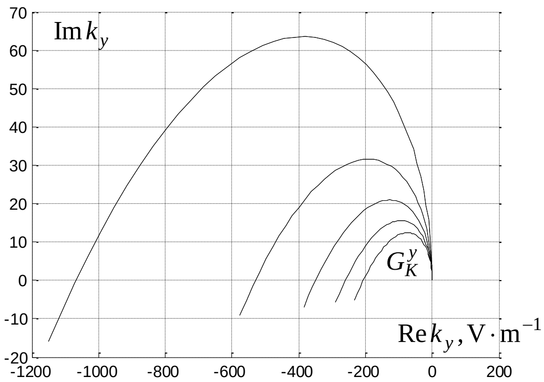


**Fig. 5.** Stability regions $G_K^{y}(v_s)$

Unlike regions $G_K^{\psi}(v_s), \left(s=\overline{1,q}\right)$, regions $G_K^{y}(v_s)$ are significantly dependent on values $v_s$ and increase as values $v_s$ decrease.

Using the algorithmic combined method of the parametric synthesis of the stabilizer on the set $G_K^{y}$ we choose the point $k_y^{*}\in G_K^{y}$ that delivers the global minimum of the functional

$$I\left(k_{\psi}^{*},k_{\dot\psi}^{*},k_y\right)=\int_0^{\tau}\left[\beta_1^2x_1^2(t)+\beta_2^2x_2^2(t)+\beta_3^2x_5^2(t)\right]dt, \qquad (36)$$

which calculated on the mathematical model of the closed dynamic system (14), (17), (18). The value of parameter $k_y^{*}\in G_K^{y}$ is $k_y^{*}=-113{,}1B\cdot\mathrm{m}^{-1}$. The set of allowable values of the variable parameters of the stabilizer $G_K$ is a union of the sets $G_K^{\psi}$ and $G_K^{y}$

$$G_K=G_K^{\psi}\cup G_K^{y},$$

each of which is the region of stability of the closed system of the first approximation in the plane of variable parameters of the stabilizer.

In Fig. 6 shows the processes of stabilization of the car body relative to the given direction of movement during emergency braking at the calculated values of the variable parameters of the stabilizer, which ensure the minimum of the functional (31) by the internal circuit of corner stabilization and the minimum of the functional (36) by the two-circuit system, calculated on the solutions of the closed system (14), (17), (18).

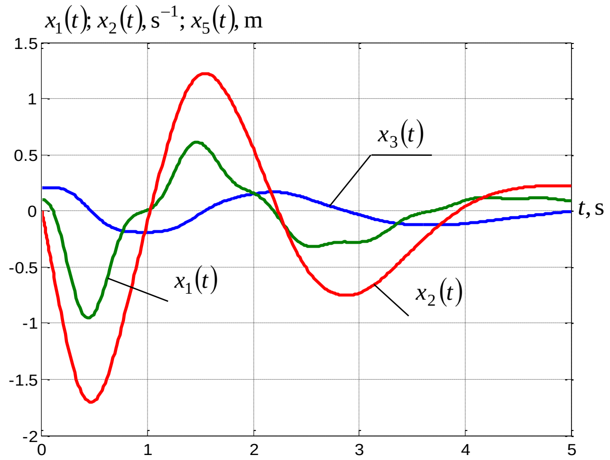


**Fig. 6.** Stabilization processes of the car body during emergency braking

## Conclusions and recommendations

As a result of the research, the conclusions were drawn and recommendations were offered:

• an effective method of choosing the values of variable parameters of stabilizers of complex dynamic objects is the combined algorithmic method of parametric synthesis, based on the application of main coordinates, which in the greatest extent characterize the dynamic properties of the object being stabilized;

• one of the most difficult tasks when applying the combined algorithmic method of parametric synthesis of stabilizers of complex dynamic objects is the task of determining the set of allowable values of the variable parameters of the stabilizer, which is used to find the minimum value of the additive integral quadratic functional, which is calculated based on the solutions of the mathematical model of the disturbed motion of the closed stabilization system;

• it is proved that as a set of allowable values of variable stabilizer parameters of a complex dynamic object, the region of stability of a linear closed system of the first approximation, which is built in the space of varied stabilizer parameters, can be used;

• considered an example of choosing a set of allowable values of variable parameters of the car's course stability system, on which the values of the parameters that ensure the high quality of the stabilization processes of the car body relative to the given direction of movement during emergency braking are selected.

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**Вибір множини допустимих значень варійованих параметрів стабілізатора складного динамічного об'єкта**

Є. Є. Александров, Т. Є. Александрова, І. В. Костяник, Я. Ю. Моргун

**Анотація. Актуальність.** Однією з найважливіших задач при застосуванні обчислювальних методів параметричного синтезу регуляторів складних динамічних об'єктів є задача визначення множини допустимих значень варійованих параметрів регулятора, де проводиться обчислення цільової функції на рішеннях математичної моделі збуреного руху динамічного об'єкта з її подальшою мінімізацією. **Метою роботи** є побудова множини допустимих значень варійованих параметрів стабілізатора складного динамічного об'єкта при застосуванні алгоритмічного комбінованого методу параметричного синтезу стабілізаторів складних динамічних об'єктів, сутність якого полягає у безпосередньому обчисленні інтегрального квадратичного функціонала на рішеннях замкненої динамічної системи з подальшим знаходженням його глобального мінімуму шляхом послідовної комбінації двох алгоритмів – алгоритму побудови сітки Соболя і алгоритму Нелдера-Міда. **Результати.** За допомогою алгоритму побудови сітки Соболя стартова точка обчислювального процесу приводиться до вузла сітки Соболя, який знаходиться в малому околі точки глобального мінімуму. На другому етапі оптимізації знайдений вузол сітки Соболя вибирається стартовою точкою для застосування методу Нелдера-Міда, який реалізується програмним продуктом Optimization Toolbox пакету MATLAB або програмним продуктом Minimize пакету MATHCAD і виводить обчислювальний процес в точку глобального мінімуму. **Висновок.** В роботі доведено теорему, згідно з якою в якості такої множини може прийматися область стійкості замкненої системи першого наближення, а також наведено приклад рішення задачі параметричного синтезу стабілізатора системи курсової стійкості автомобіля в процесі його термінового гальмування.

**Ключові слова:** складний динамічний об'єкт; математична модель збуреного руху замкненої динамічної системи; параметричний синтез стабілізатора; варійовані параметри.