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# MULTI-CRITERIA EVALUATION OF THE MULTIFACTOR STOCHASTIC SYSTEMS EFFECTIVENESS

**Annotation. Subject** of the research in the article is the evaluation of complex multifactor stochastic systems functioning effectiveness according to a variety of correlated criteria. The problem actuality is determined by the fact that an independent evaluation of system effectiveness for each of the mutually correlated criteria for the system under consideration is not informative**.** In well-known works in the direction of multiple correlation, a relatively simple problem of estimating the correlation between one resulting parameter and a set of influencing factors is considered, which is not enough for the analysis and management of multicriteria systems. In addition, the known results do not take into account possible significant differences in influencing factors mutual correlation values. **Purpose of Work** is to develop a methodology for a comprehensive assessment of system effectiveness according to a variety of interrelated criteria. **Tasks to be Solved**: splitting the set of system parameters into two subsets (parameters determining the effectiveness of system and parameters affecting it), forming additive convolutions of parameters included in subsets, developing a methodology for calculating the multiple correlation coefficient between the components of the selected subsets, developing a method for differentiating a scalar function from a vector argument by this argument. **Applied Methods**: nonlinear programming, multidimensional correlation analysis, method of differentiation of scalar functions by vector argument. These methods are used for forming and calculating a multiple correlation coefficient between the set of system effectiveness complex criterion components values and its control parameters set values. **Results Obtained**: proposed methodology provides the possibility of solving the problems of system management, taking into account the revealed relationship between the multi-criteria evaluation of system effectiveness and values of its controlled parameters. At the same time, an important advantage of the obtained result lies in the possibility that arises when using it to take into account the joint (group) influence of control variables on the complex criterion of system efficiency. The developed technology of scalar functions differentiation by vector argument has great practical importance which expands the arsenal of computational mathematics.

**Keywords:** multifactorial stochastic system; complex efficiency criterion; vector differentiation procedure.

## Introduction, publications analysis

One of the methods widely used in the study of multidimensional systems with interrelated parameters is the calculation of multiple correlations [1-3]. This method allows to estimate the measure of the relationship between one of the random variables and many others. The method is useful in evaluating a system of controlled parameters (indicators), as well as in solving numerous problems of evaluating the effectiveness of complex systems, optimization, management, rational allocation of resources, etc. At the same time, in particular, the task of establishing a link between the performance indicators of the system and the values of factors affecting it is of considerable interest. Typical similar problems are considered in [4-9]. Multiple correlation techniques significantly extend the arsenal of computational methods for systems analysis. However, the well-known methodological works [10-12] do not consider some fundamental features of a significant number of real stochastic systems. In these works, the possibilities of applying multiple correlation are limited to considering the special case when the system is single-criteria. The canonical works [13-15] in this direction do not take into account the possible significant differences between the correlation relationships of the factors that differently affect the efficiency of the system. Finding the analytical relationships that define these relationships is usually a difficult task. A possible direction of its solution is to identify statistical descriptions of correlations obtained using appropriate technologies for processing real observations of the system functioning process.

## Materials and methods

We introduce $x_i = \left(x_{11}, x_{12}, ..., x_{1n_1}\right)$ – a set of values of the system's performance indicators, and $x_2 = \left(x_{21}, x_{22}, x..., x_{2n_2}\right)$ – a set of values of factors that presumably affect the system's efficiency. We assume that the random values of the vector components $x_1$ and $x_2$ are pre-centered, that is

$$\mathrm{M}\left[x_{1i_1}\right] = \mathrm{M}\left[x_{2i_2}\right] = 0,\ i_1 = \overline{1, n_1},\ i_2 = \overline{1, n_2}. \quad (1)$$

We introduce a covariance matrix $K$ for the entire set of observed random variables, which we represent blockwise as follows:

$$K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix},$$

where $k_{11}$ is the covariance matrix for the random components of the vector $x_1$,

$$k_{11} = \mathrm{M}\left[\left(x_{1s_1} - \mathrm{M}\left[x_{1s_1}\right]\right)\left(x_{1s_2} - \mathrm{M}\left[x_{1s_2}\right]\right)\right],$$

$$\dim k_{11} = n_1 \times n_1;$$

$k_{12}$ – covariance matrix for components of vectors $x_1$ and $x_2$,

$$k_{12} = \mathrm{M}\left[\left(x_{1s_1} - \mathrm{M}\left[x_{1s_1}\right]\right)\left(x_{2s_2} - \mathrm{M}\left[x_{2s_2}\right]\right)\right],$$

$$\dim k_{12} = n_1 \times n_2;$$

$$k_{21} = k_{12}^{\mathrm{T}},\ \dim k_{21} = n_2 \times n_1;$$

$k_{22}$ – covariance matrix for vector components $x_2$,

$$k_{22} = \mathrm{M}\left[\left(x_{2s_1} - \mathrm{M}\left[x_{2s_1}\right]\right)\left(x_{2s_2} - \mathrm{M}\left[x_{2s_2}\right]\right)\right],$$

$$\dim k_{22} = n_2 \times n_2.$$

Now, using column vectors of weighting coefficients

$$A_1 = \begin{pmatrix} a_{11} \\ a_{12} \\ \dots \\ a_{1n_1} \end{pmatrix}, \quad A_2 = \begin{pmatrix} a_{21} \\ a_{22} \\ \dots \\ a_{2n_2} \end{pmatrix}$$

for vectors $x_1$ and $x_2$ we introduce additive convolution

$$U = A_1^{\mathrm{T}} x_1 = \left(a_{11} a_{12} \dots a_{1n_1}\right) \begin{pmatrix} x_{11} \\ x_{12} \\ \dots \\ x_{1n_1} \end{pmatrix},$$

$$V = A_2^{\mathrm{T}} x_2 = \left(a_{21} a_{22} \dots a_{2n_2}\right) \begin{pmatrix} x_{21} \\ x_{22} \\ \dots \\ x_{2n_2} \end{pmatrix}. \qquad (2)$$

At the same time, taking into account (1)

$$\mathrm{M}[U] = \mathrm{M}[V] = 0,$$

so
$$\mathrm{M}\left[A_1^{\mathrm{T}} x_1\right] = \mathrm{M}\left[A_2^{\mathrm{T}} x_2\right] = 0.$$

Next, we determine the correlation coefficient between $U$ and $V$:

$$K_{UV} = \mathrm{M}\left[A_1^{\mathrm{T}} x_1 x_2^{\mathrm{T}} A_2\right] = A_1^{\mathrm{T}} k_{12} A_2. \qquad (3)$$

We will choose the vectors $A_1$ and $A_2$ so that the sums of the weighting coefficients used in the formation of random vectors $U$ and $V$ are equal to one, that is

$$A_1^{\mathrm{T}} I_1 = A_2^{\mathrm{T}} I_2 = 1, \qquad (4)$$

$$I_1^{\mathrm{T}} = (11 \dots 1),\ \dim I_1^{\mathrm{T}} = 1 \times n_1,$$

$$I_2^{\mathrm{T}} = (11 \dots 1),\ \dim I_2^{\mathrm{T}} = 1 \times n_2.$$

Now set the task of finding vectors $A_1$ and $A_2$ maximizing (3) and satisfying constraints (4).

The resulting mathematical programming problem will be solved by the method of indefinite Lagrange multipliers [8]. The Lagrange function has the form

$$F(A_1, A_2) = \\ = A_1^{\mathrm{T}} k_{12} A_2 - \lambda_1\left(A_1^{\mathrm{T}} I_1 - 1\right) - \lambda_2\left(A_2^{\mathrm{T}} I_2 - 1\right). \qquad (5)$$

We differentiate the scalar function (5) by vector arguments $A_1$, $A_2$ and equate the results to zero. In this case, the operation of differentiating a scalar function from a vector argument by this argument is performed as follows.

Let $F(x)$ be a scalar function of the vector $x$. We introduce an auxiliary function of the scalar argument $t$

$$F(x,t) = F(x + ht),\ \ \dim h = \dim x.$$

It is clear that

$$F(x + ht)\Big|_{t=0} = F(x).$$

Now we calculate

$$\left.\frac{dF(x,t)}{dt}\right|_{t=0} = \left.\frac{dF(x+ht)}{dt}\right|_{t=0} = \\ = \left.\frac{dF(x+ht)}{d(x+ht)} \cdot \frac{d(x+ht)}{dt}\right|_{t=0} = \frac{dF(x)}{dx} \cdot h. \qquad (6)$$

Thus, it turned out that the desired derivative $\frac{dF(x)}{dx}$ is a linear operator acting on a vector $h$ in relation (6).

Let's illustrate the technique of vector differentiation with extremely simple example.

## Example

Find a set $x = (x_1, x_2)$ that minimizes the function

$$F(x_1, x_2) = x_1^2 + 2x_2^2$$

and satisfies the constraint $x_1 + x_2 = 1$.

We can solve this problem in the usual way using the method of indefinite Lagrange multipliers. The Lagrange function has the form of

$$F(x_1, x_2) = x_1^2 + 2x_2^2 - \lambda(x_1 + x_2 - 1).$$

We differentiate $F(x_1, x_2)$ by $x_1, x_2$ and equate the results to zero.

$$\frac{dF(x_1, x_2)}{dx_1} = 2x_1 - \lambda = 0,$$

$$\frac{dF(x_1, x_2)}{dx_2} = 4x_2 - \lambda = 0.$$

From here we express $x_1$ and $x_2$ through $\lambda$. We have an

$$x_1 = \frac{\lambda}{2},\ x_2 = \frac{\lambda}{4}.$$

We will find indefinite multiplier $\lambda$ using the constraint.

$$x_1 + x_2 = \frac{\lambda}{2} + \frac{\lambda}{4} = \frac{3}{4}\lambda = 1,$$

where from $\lambda = 4/3$.

Then

$$x_1 = \frac{\lambda}{2} = \frac{2}{3}, \quad x_2 = \frac{\lambda}{4} = \frac{1}{3}.$$

Solution has been received. We will now solve this problem using the vector differentiation procedure. According to the task condition, let's introduce a vector $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ and a unit vector $I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Now the formulation of the problem in terms of matrix theory has the form: find a vector $x$ that minimizes the function $F(x) = x^{\mathrm{T}} Ax$ and satisfies the constraint $x^{\mathrm{T}} I = 1$. Use the method of indefinite Lagrange multipliers.

The Lagrange function has the form

$$\phi(x) = x^{\mathrm{T}} Ax - \lambda\left(x^{\mathrm{T}} I - 1\right).$$

In accordance with the introduced technology of vector differentiation, we form an auxiliary function

$$\phi(x+ht) =$$
$$= (x+ht)^{\mathrm{T}} A(x+ht) - \lambda\left[(x+ht)^{\mathrm{T}} I - 1\right] =$$
$$= \left(x^{\mathrm{T}} + h^{\mathrm{T}} t\right) A(x+ht) - \lambda\left[\left(x^{\mathrm{T}} + h^{\mathrm{T}} t\right)\cdot I - 1\right] =$$
$$= \left(x^{\mathrm{T}} A + h^{\mathrm{T}} At\right)(x+ht) - \lambda\left(x^{\mathrm{T}} I + h^{\mathrm{T}} It - 1\right) =$$
$$= x^{\mathrm{T}} Ax + x^{\mathrm{T}} Aht + h^{\mathrm{T}} Axt + h^{\mathrm{T}} Aht^2 -$$
$$-\lambda\left(x^{\mathrm{T}} I - h^{\mathrm{T}} It - 1\right).$$

We will find

$$\left.\frac{d\phi(x+ht)}{dt}\right|_{t=0} = x^{\mathrm{T}} Ah + h^{\mathrm{T}} Ax - \lambda h^{\mathrm{T}} I =$$
$$= x^{\mathrm{T}} Ah + x^{\mathrm{T}} Ah - \lambda I^{\mathrm{T}} h = \left(2x^{\mathrm{T}} A - \lambda I^{\mathrm{T}}\right) h.$$

Hence, according to (6),

$$\frac{dF(x)}{dx} = 2x^{\mathrm{T}} A - \lambda I^{\mathrm{T}} = 0, \quad x^{\mathrm{T}} A = \frac{1}{2}\lambda I^{\mathrm{T}}.$$

We will find the unknown multiplier $\lambda$ using the constraint. We have

$$x^{\mathrm{T}} = \frac{1}{2}\lambda I^{\mathrm{T}} A^{-1},$$

$$x^{\mathrm{T}} I = \frac{1}{2}\lambda I^{\mathrm{T}} A^{-1} I = 1,$$

where from

$$\frac{1}{2}\lambda = \frac{1}{I^{\mathrm{T}} A^{-1} I}.$$

Then

$$x^{\mathrm{T}} = \frac{I^{\mathrm{T}} A^{-1}}{I^{\mathrm{T}} A^{-1} I}. \qquad (7)$$

Since

$$A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix},$$

that

$$x^{\mathrm{T}} = \frac{(1\ \ 1)\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}}{(1\ \ 1)\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}} = \frac{\begin{pmatrix} 1 & \frac{1}{2} \end{pmatrix}}{\begin{pmatrix} 1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}} =$$
$$= \frac{2}{3}\begin{pmatrix} 1 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix},$$

The resulting solution, of course, coincides with the previous one.

The obvious usefulness of the introduced vector differentiation operation is as follows. The computational scheme of the resulting matrix representation using vector differentiation technology, the final ratio (7) does not depend on the problem dimension, while the complexity of the standard solution increases with increasing vector $x$ dimension.

Let's go back to the original problem. Let's perform the introduced operation (6) on the function (5) sequentially to calculate derivatives by vector arguments $A_1$ and $A_2$.

In accordance with (5) we introduce

$$F\left[(A_1 + ht, A_2)\right] =$$
$$= (A_1 + ht)^{\mathrm{T}} k_{12} A_2 - \lambda_1\left[(A_1 + ht)^{\mathrm{T}} I_1 - 1\right] -$$
$$-\lambda_2\left(A_2^{\mathrm{T}} I_2 - 1\right) =$$
$$= \left(A_1^{\mathrm{T}} + h^{\mathrm{T}} t\right) k_{12} A_2 - \lambda_1\left[\left(A_1^{\mathrm{T}} I_1 + h^{\mathrm{T}} I_1\right) - 1\right] -$$
$$-\lambda_2\left(A_2^{\mathrm{T}} I_2 - 1\right) =$$
$$= \left(A_1^{\mathrm{T}} k_{12} A_2 + h^{\mathrm{T}} k_{12} A_2 t\right) -$$
$$-\lambda_1\left(A_1^{\mathrm{T}} I_1 + h^{\mathrm{T}} I_1 - 1\right) - \lambda_2\left(A_2^{\mathrm{T}} I_2 - 1\right).$$

Now

$$\left.\frac{dF\left[(A_1 + ht)^{\mathrm{T}}\right]}{dt}\right|_{t=0} = h^{\mathrm{T}} k_{12} A_2 - \lambda_1 h^{\mathrm{T}} I_1. \qquad (7)$$

Relation (7) describes a scalar. Therefore

$$h^{\mathrm{T}} k_{12} A_2 - \lambda_1 h^{\mathrm{T}} I_1 = \left(h^{\mathrm{T}} k_{12} A_2 - \lambda_1 h^{\mathrm{T}} I_1\right)^{\mathrm{T}} =$$
$$= A_2^{\mathrm{T}} k_{12}^{\mathrm{T}} h - \lambda_1 I_1^{\mathrm{T}} h = \left(A_2^{\mathrm{T}} k_{12}^{\mathrm{T}} - \lambda_1 I_1^{\mathrm{T}}\right) h.$$

Hence, in accordance with (6), we have

$$\frac{dF(A_1, A_2)}{dA_1} = A_2^{\mathrm{T}} k_{12}^{\mathrm{T}} - \lambda_1 I_1^{\mathrm{T}}. \qquad (8)$$

Equate (8) to zero and express $A_2^{\mathrm{T}}$ by $\lambda_1$. We have

$$A_2^{\mathrm{T}} k_{12}^{\mathrm{T}} - \lambda_1 I_1^{\mathrm{T}} = 0. \quad (9)$$

To find, $A_2^{\mathrm{T}}$ multiply (9) on the right by $K_{12}$. We will get

$$A_2^{\mathrm{T}} k_{12}^{\mathrm{T}} k_{12} - \lambda_1 I_1^{\mathrm{T}} k_{12} = 0.$$

From here

$$A_2^{\mathrm{T}} = \lambda_1 I_1^{\mathrm{T}} k_{12} \left( k_{12}^{\mathrm{T}} k_{12} \right)^{-1}.$$

We will find an unknown indefinite multiplier $\lambda_1$ using constraint (4). At the same time

$$A_2^{\mathrm{T}} I_2 = \lambda_1 I_1^{\mathrm{T}} k_{12} \left( k_{12}^{\mathrm{T}} k_{12} \right)^{-1} I_2 = 1.$$

From here

$$\lambda_1 = \frac{1}{I_1 k_{12} \left( k_{12}^{\mathrm{T}} k_{12} \right)^{-1} I_2}.$$

Then

$$A_2^{\mathrm{T}} = \frac{1}{I_1 k_{12} \left( k_{12}^{\mathrm{T}} k_{12} \right)^{-1} I_2} \cdot I_1^{\mathrm{T}} k_{12} \left( k_{12}^{\mathrm{T}} k_{12} \right)^{-1}. \quad (10)$$

Similarly to the previous one, we define $A_1$. We will perform the necessary operations without detailed additions.

$$\begin{aligned} F &= \left[ \left( A_2 + ht \right)^{\mathrm{T}}, A_1 \right] = \left( A_2 + ht \right)^{\mathrm{T}} k_{21} A_1 - \\ &- \lambda_1 \left( A_1^{\mathrm{T}} I_1 - 1 \right) - \lambda_2 \left[ \left( A_2 + ht \right)^{\mathrm{T}} I_2 - 1 \right] = \\ &= \left( A_2^{\mathrm{T}} + h^{\mathrm{T}} t \right) k_{21} A_1 - \lambda_1 \left( A_1^{\mathrm{T}} I_1 - 1 \right) - \quad (11) \\ &- \lambda_2 \left( A_2^{\mathrm{T}} I_2 + h^{\mathrm{T}} I_2 t - 1 \right) = A_2^{\mathrm{T}} k_{21} A_1 + h^{\mathrm{T}} k_{21} A_1 t - \\ &- \lambda_1 \left( A_1^{\mathrm{T}} I_1 - 1 \right) - \lambda_2 \left( A_2^{\mathrm{T}} I_2 + h^{\mathrm{T}} I_2 t - 1 \right). \end{aligned}$$

Let's perform differentiation (11) by $A_2$.

$$\left. \frac{dF \left[ \left( A_2^{\mathrm{T}} + ht \right), A_1 \right]}{dt} \right|_{t=0} = h^{\mathrm{T}} k_{21} A_1 - \lambda_2 h^{\mathrm{T}} I_2 =$$

$$\begin{aligned} &= \left( h^{\mathrm{T}} k_{21} A_1 - \lambda_2 h^{\mathrm{T}} I_2 \right)^{\mathrm{T}} = \\ &= A_1^{\mathrm{T}} k_{21}^{\mathrm{T}} h - \lambda_2 I_2^{\mathrm{T}} h = \\ &= \left( A_1^{\mathrm{T}} k_{21}^{\mathrm{T}} - \lambda_2 I_2^{\mathrm{T}} \right) h. \end{aligned}$$

Then

$$\frac{dF\left( A_1, A_2 \right)}{dA_2} = A_1^{\mathrm{T}} k_{21}^{\mathrm{T}} - \lambda I_2^{\mathrm{T}} = 0. \quad (12)$$

Further

$$A_1^{\mathrm{T}} k_{21}^{\mathrm{T}} k_{21} - \lambda_2 I_2^{\mathrm{T}} k_{21} = 0.$$

From here

$$A_1^{\mathrm{T}} = \lambda_2 I_2^{\mathrm{T}} k_{21} \left( k_{21}^{\mathrm{T}} k_{21} \right)^{-1}. \quad (13)$$

We will find $\lambda_2$

$$A_1^{\mathrm{T}} I_1 = \lambda_2 I_2^{\mathrm{T}} k_{21} \left( k_{21}^{\mathrm{T}} k_{21} \right)^{-1} I_1 = 1.$$

Then

$$\lambda_2 = \frac{1}{I_2^{\mathrm{T}} k_{21} \left( k_{21}^{\mathrm{T}} k_{21} \right)^{-1} I_1}. \quad (14)$$

From where, substituting (14) into (13), we get

$$A_1^{\mathrm{T}} = \frac{1}{I_2^{\mathrm{T}} k_{21} \left( k_{21}^{\mathrm{T}} k_{21} \right)^{-1} I_1} \times \times I_2^{\mathrm{T}} k_{21} \left( k_{21}^{\mathrm{T}} k_{21} \right)^{-1}. \quad (15)$$

Problem solution is completed.

## Conclusions

1. The obtained ratios (10) and (15) enable rational management in a complex system. The proposed approach allows us to determine to what extent the values of a set of control variables affect the values of controlled variables that determine the system efficiency.

2. These relations determine a new quality of assessment of the joint (group) influence of some variables on others.

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**Багатокритеріальна оцінка ефективності багатофакторних стохастичних систем**

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**Анотація. Предметом дослідження** у статті є оцінка ефективності функціонування складних багатофакторних стохастичних систем за багатьма корельованими критеріями. **Актуальність проблеми** визначається тим, що незалежна оцінка ефективності системи по кожному з взаємно корельованих критеріїв для системи, що розглядається, малоінформативна. У відомих роботах за напрямом множинна кореляція розглядається відносно просте завдання оцінки кореляційного зв'язку між одним результуючим параметром і набором факторів, що впливають, чого задачах аналізу та управління багатокритеріальних систем недостатньо. Крім того, відомі результати не враховують можливі суттєві відмінності значень взаємної кореляції факторів, що впливають. **Мета роботи** – розробка методики комплексної оцінки ефективності системи з безлічі взаємозалежних критеріїв. Розв'язування задач: розбиття безлічі параметрів системи на два підмножини (параметри, що визначають ефективність системи, і параметри, що впливають на неї), формування адитивних згорток параметрів, що входять до підмножини, розробка методики розрахунку коефіцієнта множинної кореляції між компонентами виділених підмножин, розробка методу від векторного аргументу з цього аргументу. **Методи, що застосовуються**: нелінійне програмування, багатовимірний кореляційний аналіз, метод диференціювання скалярних функцій за векторним аргументом. Ці методи використовуються для формування та розрахунку множинного коефіцієнта кореляції між сукупністю значень компонентів комплексного критерію ефективності функціонування системи та значеннями набору її керуючих параметрів. **Отримані результати**: запропонована методика забезпечує можливість вирішення завдань управління системою з урахуванням взаємозв'язку, що виявляється між багатокритеріальною оцінкою ефективності системи і значеннями керованих її параметрів. При цьому важлива перевага отриманого результату полягає в можливості, що виникає при його використанні, обліку спільного (групового) впливу управляючих змінних на комплексний критерій ефективності системи. Велике практичне значення має розроблена технологія диференціювання скалярних функцій за векторним аргументом, що розширює арсенал обчислювальної математики.

**Ключові слова:** багатофакторна стохастична система; комплексний критерій ефективності; процедура векторного диференціювання.