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SIMULATION OF RANDOM EXTERNAL DISTURBANCE ACTING ON THE CAR BODY IN THE URGENT BRAKING MODE

Abstract. Topicality. When creating ESP (Electronic Stability Program) car stability systems, the main task is to determine the values of the varied parameters of the EBD (Electronic Brake Distribution) electronic block, which ensure the required accuracy of stabilization of the car body in urgent braking mode relative to the given direction of movement. **The purpose of the article.** To solve the problem of parametric synthesis of the EBD block, it is necessary to create a simulation model of external disturbances acting on the car body from the side of the road surface. Well-known simulation models are created for car movement modes on a random surface with a constant speed. At the same time, random external disturbances are stationary in nature. The urgent braking mode is characterized by an intense decrease in the current speed of the car. **Results.** The paper proposes a method of constructing a random function of an external disturbance acting on the car body in urgent braking mode. It is proved that this perturbation is non-stationary in nature; the forming dynamic link, the input of which is supplied with single "white noise", and the output of which is an external disturbance, is an oscillating dynamic link, the gain and time constants of which depend on the current value of the car's speed during urgent braking.

Keywords: car course stability system; forming dynamic link; single "white noise"; non-stationary random function.

Introduction

Problem statement. The urgent braking mode is one of the most dangerous driving modes of the car, as a result of which some of the car's wheels are locked, as a result of which the car loses its directional stability. Anti-lock braking systems (ABS) have become widely used to prevent the wheels from locking by reducing the braking force on the locked wheels. The effectiveness of modern ABS systems depends on the state of the car's driving surface and does not always meet traffic safety requirements. Therefore, ESP (Electronic Stability Program) stability control systems appeared in modern cars at the end of the 20th century, the main task of which is to help the driver in critical situations, primarily in urgent braking mode. In those cases, when the anti-lock ABS system is unable to ensure stable movement of the car, the ESP system comes into play. It should be noted that the possibilities of ESP for correcting skidding and stabilizing the car body in a critical situation are not unlimited. And if the speed of the car at which the skid occurs is excessively high, or the grip between the wheels and the road surface is excessively low, even the ESP system can be helpless. The ESP system significantly reduces the risks of skidding and an emergency situation on the road, but does not eliminate them.

In works [1, 2], the problems of parametric synthesis of continuous and digital ESP systems are solved under the conditions of the requirement of maximum speed, maximum margin of stability and maximum accuracy. In the listed works, a mathematical model of the disturbed movement of a car during emergency braking was developed, which has the following form:

$$\dot{v}_x(t) = -\frac{2k_\Gamma}{M} p_0 - g f_0 - g\Delta f(t); \tag{1}$$

$$\dot{y}(t) = -v_x(t)\psi(t); \tag{3}$$

$$\ddot{\psi}(t) = -\frac{Bk_\Gamma}{I}\Delta p(t) - \frac{2H_m M}{I} v_x(t)\dot{\psi}(t) f_0 - -(2H_m M/I) v_x(t)\dot{\psi}(t)\Delta f(t); \tag{2}$$

where $v_x(t)$ is the current speed of the car; $\psi(t)$ – angle of deviation of the main longitudinal central axis of inertia of the car relative to the given direction of movement; $y(t)$ – lateral shift of the center of mass of the car body relative to the given trajectory of movement; M – mass of the car; g – acceleration of gravity; I – the moment of inertia of the car body relative to the main vertical central axis of inertia; B – track width; H_m – the distance from surface of movement of the car to its center of mass; f_0 – the static component of the car's movement resistance coefficient, which depends on the type of road surface and wheel tread; $\Delta f(t)$ – the dynamic component of the traffic resistance coefficient, which depends on the micro-profile of the road surface and is a random function of time; p_0 – the maximum value of brake fluid pressure in the main brake cylinder; k_Γ is the proportionality coefficient between the braking force on one of the sides of the car and the pressure of the brake fluid in the line of the corresponding side.

To analyze the dynamic processes of disturbed car movement and the parametric synthesis of the ESP system, it is necessary to add a simulation model of random external disturbance to the mathematical model of disturbed car movement. The development of such a model determines the purpose and content of the proposed article.

Main material

Development of a simulation model. The works [3, 4] obtained expressions for the correlation function and spectral density of random irregularities of the surface of movement of the car moving at a constant speed v_0:

$$R_f(\tau) = D_f e^{-\alpha v_0 \tau} \cos\beta\tau ;$$

$$S_f(\omega) = D_f \times \times\left\{ \frac{2\alpha v_0 \left[v_0^2\left(\alpha^2+\beta^2\right)+\omega^2 \right]}{\omega^4 + 2\omega^2 v_0^2\left(\alpha^2-\beta^2\right)+v_0^4\left(\alpha^2+\beta^2\right)^2} \right\}, \quad (5)$$

where D_f is the dispersion of the height of irregularities; α, β are correlation coefficients, the numerical values of which for different road surfaces are given in Table 1.

Table 1 – **Values of correlation coefficients and dispersions of irregularities for different types of road surfaces**

Type of road surface	α , m^{-1}	β , m^{-1}	D_f , m^2	f_0
Asphalt concrete	0,22	0,44	$5{,}5 \cdot 10^{-3}$	$0{,}01 \div 0{,}02$
Cobblestone	0,32	0,64	$8{,}0 \cdot 10^{-3}$	$0{,}02 \div 0{,}03$
Unpaved road	0,47	0,94	$11{,}6 \cdot 10^{-3}$	$0{,}03 \div 0{,}05$

In the process of emergency braking, the current speed of the car $v_x(t)$ is reduced from the initial value v_0 to zero. The spectral density of a random variable $\Delta f(t)$ is described by the formula

$$S_f\left[\omega, v_x(t)\right] = D_f \times \times\left\{ \frac{2\alpha v_x(t)\left[v_x^2(t)\left(\alpha^2+\beta^2\right)+\omega^2 \right]}{\omega^4 + 2\omega^2 v_x^2(t)\left(\alpha^2-\beta^2\right)+v_x^4(t)\left(\alpha^2+\beta^2\right)^2} \right\}. \quad (6)$$

In Fig. 1 shows spectral density curves (6), which are constructed at different values of the car's current speed $v_x(t)$ during emergency braking on asphalt concrete.

On each of the curves, we highlight three characteristic points

$$S_f\left[0, v_x(t)\right],\ S_f\left[\omega_p, v_x(t)\right] \text{ and } \omega_p\left[v_x(t)\right],$$

the values of which are given in Table 2.

Table 2 – **Dependence of the characteristic points of the spectral density curves on the current speed of movement**

Speed of movement $v_x(t)$, m·s^{-1}	$S_f\left[0, v_x(t)\right]$, m^{2}s	$\omega_p\left[v_x(t)\right]$, s^{-1}	$S_f\left[\omega_p, v_x(t)\right]$, m^{2}s
25	$0{,}396 \cdot 10^{-3}$	10,92	$0{,}963 \cdot 10^{-3}$
20	$0{,}495 \cdot 10^{-3}$	8,74	$1{,}207 \cdot 10^{-3}$
15	$0{,}66 \cdot 10^{-3}$	6,56	$1{,}604 \cdot 10^{-3}$
10	$0{,}99 \cdot 10^{-3}$	4,37	$2{,}407 \cdot 10^{-3}$
5	$1{,}98 \cdot 10^{-3}$	2,19	$4{,}813 \cdot 10^{-3}$

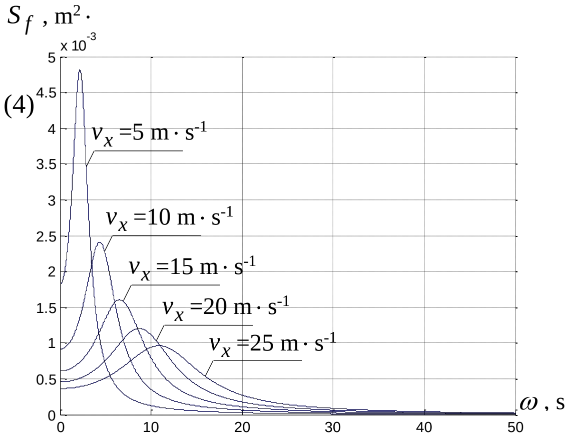

Fig. 1. Spectral densities of external disturbance at different values of car speed

Consider a non-stationary dynamic link with a transfer function $W(s,t)$, where s is a complex variable of the Laplace transform. It is known [5] that the spectral densities of the input $x(t)$ and output $y(t)$ signals of the dynamic link are related by the ratio

$$S_y(\omega,t) = R^2(\omega,t) S_x(\omega), \quad (7)$$

where $R(\omega,t)$ is the amplitude-frequency characteristic of the non-stationary dynamic link.

Assume that the input signal of the dynamic link is "white noise" $x(t)=\xi(t)$ whose spectral density is constant and equal to unity

$$S_\xi(\omega) = 1. \quad (8)$$

Then the relation (7) will take the form

$$S_f\left[\omega, v_x(t)\right] = R^2\left[\omega, v_x(t)\right]. \quad (9)$$

According to fig. 1, it can be concluded that the non-stationary dynamic link with spectral density (9) is oscillatory, the transfer function of which is written in the form

$$W\left[s, v_x(t)\right] = \frac{K\left[v_x(t)\right]}{T_1^2\left[v_x(t)\right]s^2 + T_2\left[v_x(t)\right]s + 1}. \quad (10)$$

In relation (10), we replace $s = j\omega$. As a result, we get

$$R^2\left[\omega, v_x(t)\right] = K^2\left[v_x(t)\right] \Big/ \Big/ \left\{1-\omega^2 T_1^2\left[v_x(t)\right]\right\}^2 + \omega^2 T_2^2\left[v_x(t)\right] = = S_f\left[\omega, v_x(t)\right]. \quad (11)$$

In (11) we put $\omega = 0$. As a result, we have

$$K^2\left[v_x(t)\right] = S_f\left[0, v_x(t)\right] = D_f \frac{2\alpha}{v_x(t)\left(\alpha^2+\beta^2\right)},$$

or

$$K\left[v_x(t)\right] = \sigma_f \sqrt{\frac{2\alpha}{v_x(t)\left(\alpha^2+\beta^2\right)}}, \quad (12)$$

where σ_f is the mean squared deviation of the random variable $\Delta f(t)$.

The time constants of the oscillating dynamic link (10) are determined from the relations

$$1-\omega_p^2 T_1^2\left[v_x(t)\right]=0;\; S_f\left[\omega_p, v_x(t)\right]= =\frac{1}{\omega_p^2 T_2^2\left[v_x(t)\right]}. \qquad (13)$$

From relations (13) we have

$$T_1^2\left[v_x(t)\right]=\frac{1}{\omega_p^2}; \qquad (14)$$

$$T_2\left[v_x(t)\right]=\frac{1}{\omega_p\sqrt{S_f\left[\omega_p, v_x(t)\right]}}. \qquad (15)$$

Analysis of Table 2 allows us to write down:

$$\omega_p\left[v_x(t)\right]=k_v v_x(t); \qquad (16)$$

$$S_f\left\{\omega_p\left[v_x(t)\right], v_x(t)\right\}=\frac{k_s}{v_x(t)}, \qquad (17)$$

where the coefficients k_v and k_s are equal to: k_v =0,44 m^{-1}; k_s =24,065 m^3.

Taking into account formulas (16) and (17), the time constants $T_1^2\left[v_x(t)\right]$ and $T_2\left[v_x(t)\right]$, determined by formulas (14) and (15), are written in the form

$$T_1^2\left[v_x(t)\right]=\frac{1}{k_v^2 v_x^2(t)};\; T_2\left[v_x(t)\right]=\frac{1}{k_v\sqrt{k_s v_x(t)}}. \qquad (18)$$

In work [4], the dynamic link, the input of which is supplied with single "white noise", and the output is a random signal with the given correlation function and spectral density, is called a forming dynamic link.

Let's write the differential equation of the perturbed motion of the forming oscillatory dynamic link in the form

$$T_1^2\left[v_x(t)\right]\cdot\Delta\ddot{f}(t)+T_2\left[v_x(t)\right]\cdot\Delta\dot{f}(t)+ +\Delta f(t)=K\left[v_x(t)\right]\cdot\xi(t), \qquad (19)$$

where the gain and time constants are determined by formulas (12) and (18).

We will call the non-stationary differential equation (19) a simulation model of external disturbances acting on the car body from the side of the road surface during urgent braking.

In fig. 2 shows the solutions of the differential equation (19) for different implementations of "white noise" $\xi_j(t)$, $\left(j=\overline{1,3}\right)$. The analysis of the given random functions $\Delta f_j(t)$, $\left(j=\overline{1,3}\right)$ indicates their non-stationary nature. Both the mathematical expectation $m_f(t)$ and the dispersion $D_f(t)$ of random functions $\Delta f_j(t)$ depend on the current time $t\in\left[0,t_s\right]$, where t_s is the braking time. Let's write equation (19) relative to the higher derivative

$$\Delta\ddot{f}(t)=-\frac{1}{T_1^2\left[v_x(t)\right]}\Delta f(t)- -\frac{T_2\left[v_x(t)\right]}{T_1^2\left[v_x(t)\right]}\Delta\dot{f}(t)+\frac{K\left[v_x(t)\right]}{T_1^2\left[v_x(t)\right]}\xi(t). \qquad (20)$$

The system of differential equations (1)–(3) and (20) form a stochastic mathematical model of the disturbed motion of the stabilization object.

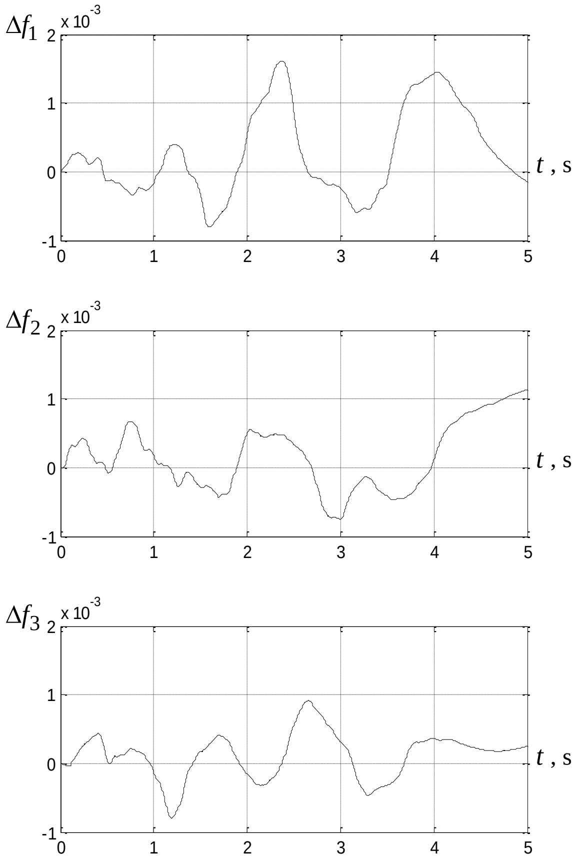

Fig. 2. Random external disturbances in different implementations of single "white noise"

Conclusions

The mode of urgent braking of the car is characterized by a continuous reduction of the current speed of the car, therefore the previously developed methods of modeling external disturbances acting on the body of the car during movement at a constant speed are not suitable in this case;

- it is proved that random external disturbances acting on the car body during urgent braking are non-stationary;
- the forming dynamic link, the input of which is supplied with single "white noise", and the output of which is an external disturbance $\Delta f(t)$, is an oscillating dynamic link, the gain K and time constants T_1 and T_2 of which depend on the current value of the car's speed during urgent braking.

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Імітаційне моделювання випадкових зовнішніх збурень, що діють на корпус автомобіля в режимі термінового гальмування

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Анотація. Актуальність. При створенні систем курсової стійкості автомобіля ESP (Electronic Stability Program) основною задачею є визначення значень варійованих параметрів електронного блоку розподілу гальмівних зусиль EBD (Electronic Brake Distribution), які забезпечують потрібну точність стабілізації корпусу автомобіля в режимі термінового гальмування відносно заданого напрямку руху. **Мета статті.** Для вирішення задачі параметричного синтезу блоку EBD необхідно створити імітаційну модель зовнішніх збурень, що діють на корпус автомобіля з боку дорожнього покриття. Відомі імітаційні моделі створені для режимів руху автомобіля по випадковій поверхні з постійною швидкістю руху. При цьому випадкові зовнішні збурення носять стаціонарний характер. Режим термінового гальмування характеризується інтенсивним зменшенням поточної швидкості руху автомобіля. **Результати дослідження.** В роботі запропоновано метод побудови випадкової функції зовнішнього збурення, що діє на корпус автомобіля в режимі термінового гальмування. Доведено, що це збурення носить нестаціонарний характер; формуюча динамічна ланка, на вхід якої подається одиночний «білий шум», а на вихід – зовнішнє збурення, є коливальною динамічною ланкою, коефіцієнт підсилення та постійні часу якої залежать від поточного значення швидкості автомобіля під час різкого гальмування.

Ключові слова: система курсової стійкості автомобіля; формуюча динамічна ланка; одиничний «білий шум»; нестаціонарна випадкова функція.