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# ANALYSIS OF MARCOVIAN SYSTEMS WITH A GIVEN SET OF SELECTED STATES

**Abstract.** Analysis of stationary Marcovian systems is traditionally performed using systems of linear Kolmogorov differential equations. Such systems make it possible to determine the probability of the analyzed system being in each of its possible states at an arbitrary time. This standard task becomes more complicated if the set of possible states of systems is heterogeneous and some special subset can be distinguished from it, in accordance with the specifics of the system functioning. Subject of the study is technology development for such systems analysis. In accordance with this, the purpose of the work is to find the distribution law of the random duration of such a system's stay on a set of possible states until it falls into a selected subset of these states. Method for solving the problem is proposed based on splitting the entire set of possible states of the system into two subsets. The first of them contains a selected subset of states, and the second contains all the other states of the system. Now a subset of states is allocated from the second subset, from which a direct transition to the states of the first subset is possible. Next, a system of differential equations describing the transitions between the formed subsets is formed. The solution of this system of equations gives the desired result – distribution of the random duration of the system's stay until the moment of the first hit in the selected subset of states. The method allows solving a large number of practical problems, for example, in the theory of complex systems reliability with many different failure states. In particular, finding the law of the uptime duration distribution, calculating the average duration of uptime.

**Keywords:** Marcovian systems; subset of special states; analysis of inhomogeneous systems states probabilities dynamics.

## Introduction

Traditional technologies for queuing systems analyzing are based on the use of Marcovian models of these systems [1, 2]. Computational scheme for solving the corresponding tasks is constructed as follows [3–5].

A set of $Z$ discrete possible states of the system is introduced. State of this system at each moment of time $t$ is determined by the $\{P_k(t)\}$ probability distribution of its being in states $k \in Z$ at that moment of time. Evolution of the system states in the process of its functioning is determined by the intensity matrix, $\Lambda(t) = (\lambda_{ij}(t))$, where $\lambda_{ij}(t)$ is the intensity of the system transition from state $i$ to state $j \in Z$. Using this matrix $\Lambda(t)$, it is possible to write a system of Kolmogorov differential equations [4–6] with respect to unknown functions $P_k(t)$. This system has the form:

$$\dot{P}_k(t) = \frac{dP_k(t)}{dt} = \sum_{i \in Z \setminus k} \lambda_{ik}(t) P_i(t) - P_k(t) \sum_{i \in Z \setminus k} \lambda_{ki}(t). \quad (1)$$

Define the indicator

$$R_{ij} = \begin{cases} 1, \text{ if direct transition from i to j is possible}, \\ 0, \text{ if direct transition from i to j is impossible}. \end{cases}$$

Now introduce a stationary Marcovian process of the system functioning, for which they do $\lambda_{ij}(t) = \lambda_{ij}$ not depend on $t$. Next, for an arbitrary $k \in Z$ enter:

$Z_k^+$ - set of system states from which a direct transition to state $k$ is possible,

$$Z_k^+ = \{ j : j \in Z, R(j,k) = 1 \};$$

$Z_k^-$ - set of states of the system into which a direct transition from state $k$ is possible,

$$Z_k^- = \{ j : j \in Z, R_{k_{ij}} = 1 \}.$$

In this case, equation (1) is simplified to the form:

$$\frac{dP_k(t)}{dt} = \sum_{i \in Z_k^+} \lambda_{ik} P_i(t) - P_k(t) \sum_{i \in Z_k^+} \lambda_{ki}, \; k \in Z. \quad (2)$$

The resulting system of differential equations (2) is solved using the Laplace transform. As is known, the Laplace transform of the function *u(t)* is called the function [3-6]

$$L(u(t)) = F(s) = \int_0^\infty u(t) \cdot e^{-st} dt. \quad (3)$$

At the same time, it is clear that

$$L(\dot{u}(t)) = \int_0^\infty e^{-st} \dot{u}(t) dt = u(t) \cdot e^{-st} \Big|_0^\infty + s \int_0^\infty u(t) \cdot e^{-st} dt = sL(u(t)) - u(0). \quad (4)$$

Using the Laplace transform (3), (4) for the system of differential equations (2), we obtain

$$s\pi_k(s) = P_k(0) = \sum_{i \in Z_k^+} \lambda_{ik} \pi_i(s) - \pi_k(s) \sum_{i \in Z_k^-} \lambda_{ki}, \quad k \in Z, \pi_k(s) = L(P_k(t)). \quad (5)$$

After the reduction of such terms in the equations of system (5), we have the following system of algebraic equations

$$\begin{aligned} b_{00}\pi_0(s) + b_{01}\pi_1(s) + \ldots + b_{0n}\pi_n(s) &= c_0, \\ b_{10}\pi_0(s) + b_{11}\pi_1(s) + \ldots + b_{1n}\pi_n(s) &= c_1, \\ \ldots\ldots \\ b_{n0}\pi_0(s) + b_{n1}\pi_1(s) + \ldots + b_{nn}\pi_n(s) &= c_n, \end{aligned} \quad (6)$$

 

where *(n+1)* is the number of elements of the set *Z*.

According to Kramer's rule, write down the system solution:

$$\pi_i(s) = \frac{\Delta_i(s)}{\Delta(s)}, \tag{7}$$

where $\Delta_i(s)$ , $\Delta(s)$ are the determinants of the corresponding matrices. The desired set $P_i(t)$ is found using the inverse Laplace transform [2].

Note that when solving many practical problems of evaluating the system effectiveness, it becomes necessary to determine the duration of its stay on some specially selected subset of the set of possible states of this system [7-9]. Consider the problem of finding the distribution law of the corresponding random variable.

We introduce $Z$ – the set of possible states of the system. Divide this set into two subsets $Z_0$ and $Z_1$, $Z_0 \cup Z_1 = Z_1, Z_0 \cap Z_1 = \varnothing$.

Let $P_i(0)$ be the probability of the system being in a state $i \in Z_0$ at time $t_0$, $\sum_{i \in Z_0} P_i(0) = 1$. introduce $T$ – the random duration of wandering through states belonging to $Z_0$ before leaving for a subset of $Z_1$. Now, from the subset $Z_1$, we select a subset $Z_{01}$ containing only those elements $Z_1$ into which the system can move from $Z_0$ directly, that is

$$Z_{01} = \{ j : j \in Z_1, \textit{exists } i \in Z_0, R(i, j) = 1 \}.$$

Thus, the random duration $T$ of the system's wandering through the states $Z_0$ before the transition to $Z_1$ is equal to the time spent in $Z_0$ before entering $Z_{01}$. To find the distribution law of a random variable $T$, assume that all states of the subset $Z_{01}$ are absorbing, that is, $\lambda_{ij} = 0, j \in Z_{01}, i \in Z$. Now let $F(\mathrm{t}) = P(\tau < T)$ be the desired distribution function of a random variable T. Introduce

*Pj(t)* **-** the probability of the system being in a state $j \in Z_{01}$ at time *t*. Then

$$F(\mathrm{t}) = \sum_{j \in Z_{01}} P_j(t).$$

Hence the distribution density of the random variable $T$ is

$$f(t) = \frac{dF(t)}{dt} = \sum_{j \in Z_{01}} \dot{P}_j(t). \tag{8}$$

Now, using (2), given that $\lambda_{ij} = 0, j \in Z_{01}, i \in Z$, we have

$$\dot{P}_j(t) = \sum_{j \in Z_0} \lambda_{ij} P_j(t),\ j \in Z_{01}. \tag{9}$$

Substituting (9) into (8), we get

$$f(t) = \sum_{j \in Z_{01}} \sum_{j \in Z_0} \lambda_{ij} P_j(t). \tag{10}$$

Thus, the task of finding the desired distribution density of a random variable $T$ is reduced to calculating a set of probabilities. $P_i(t), i \in Z_0$. These probabilities can be found by integrating a system of differential equations (2), taking into account that the transition from the states of the subset $Z_0$ is possible only to the states $Z_0$ and the states of the subset $Z_{01}$, all states of which are absorbing. From here we have

$$\begin{aligned} &\dot{P}_i(t) = \\ &= \sum_{j \in Z_0} \lambda_{ji} P_j(t) - P_i(t) \cdot \sum_{j \in Z_0} \lambda_{ij} - P_j(t) \sum_{j \in Z_{01}} \lambda_{ij} = \\ &= \sum_{j \in Z_0} \lambda_{ji} P_j(t) - P_i(t) \sum_{j \in Z_0 \cup Z_{01}} \lambda_{ij},\ i \in Z_0. \end{aligned} \tag{11}$$

System of differential equations (11) should be solved taking into account the initial conditions, for example, $\left\{ P_{i_0}(0) = 1,\ P_i(0) = 0,\ i \neq i_0 \right\}$. Computational scheme for solving this problem is described in detail [7].

Consider an example of solving the simplest problem of analyzing a system using the described methodology.

Let the service system receive a random stream of two types of applications, for processing which two corresponding devices $A_1$ and $A_2$ *are used*. At the same time, the first of these devices is quite reliable, and the second fails with an intensity $\mu$.

The system that serves applications of the first type with device $A_1$, when an application of the second type is received with an intensity of $\lambda_{12}$, transmits it to device $A_2$ *for service*. Similarly, when an application of the first type with intensity $\lambda_{21}$ is received, it begins to be serviced by device $A_1$. Let's introduce the system states:

$E_1$ – device $A_1$ serves the first type of application,

$E_2$ – device $A_2$ serves the second type of application,

$E_3$ – recovery status of the failed device $A_2$.

It is required to find the distribution law of the duration of system's stay on the set of operating states $E_1$ and $E_2$ before the first failure.

Let

$P_1(t)$ is the probability of the system being in state $E_1$ at time *t*,

$P_2(t)$ is the probability of the system being in state $E_2$ at time *t*.

Initial conditions: $P_1(0) = 1, P_2(0) = 0$.

The graph of states and transitions and the corresponding infinitesimal matrix have the form

$$\Lambda = \begin{pmatrix} 0 & \lambda_{12} & 0 \\ \lambda_{21} & 0 & \mu \\ 0 & 0 & 1 \end{pmatrix}.$$

Introduce a set of states $Z_0 = \{E_1, E_2\}$ and the set of states $Z_{01} = \{E_3\}$.

At this the state $E_3 \in Z_{01}$, will be considered absorbing, which is shown in Fig.1 and in the matrix $\Lambda$.

Write down the differential equations with respect to the desired functions $P_1(t)$ and $P_2(t)$.

$$\dot{P}_1(t) = \lambda_{21} P_2(t) - P_1(t) \lambda_{12},$$
$$\dot{P}_2(t) = \lambda_{12} P_1(t) - P_2(t)(\lambda_{21} + \mu).$$

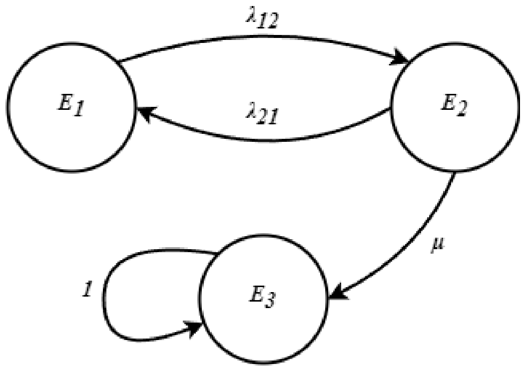


**Fig. 1.** Graph of States and Transitions

Converting these equations by Laplace, we get

$$s\pi_1(s) - P_1(0) = \lambda_{21}\pi_2(s) - \pi_1(s)\lambda_{12},$$
$$s\pi_2(s) - P_2(0) = \lambda_{12}\pi_1(s) - \pi_2(s)(\lambda_{21} + \mu).$$

Give similar terms taking into account the initial conditions.

$$\begin{aligned}(s+\lambda_{12})\pi_1(s) - \lambda_{21}\pi_2(s) = 1,\\ \lambda_{12}\pi_1(s) - (s+\lambda_{21}+\mu)\pi_2(s) = 0.\end{aligned} \quad (12)$$

Further

$$\pi_2(s) = \frac{\lambda_{12}}{s+\lambda_{21}+\mu}\pi_1(s). \quad (13)$$

Substituting (13) into (12), we have

$$(s+\lambda_{12})\pi_1(s) - \frac{\lambda_{12}\lambda_{21}}{s+\lambda_{21}+\mu}\pi_1(s) =$$
$$= \pi_1(s)\left[s+\lambda_{12} - \frac{\lambda_{12}\lambda_{21}}{s+\lambda_{21}+\mu}\right] =$$
$$= \pi_1(s)\frac{(s+\lambda_{12})(s+\lambda_{21}+\mu) - \lambda_{12}\lambda_{21}}{s+\lambda_{21}+\mu} = \pi_1(s)\times$$
$$\times\frac{s^2 + s\lambda_{21} + s\mu + s\lambda_{12} + \lambda_{12}\lambda_{21} + \lambda_{12}\mu - \lambda_{12}\lambda_{21}}{s+\lambda_{21}+\mu} =$$
$$= \pi_1(s)\frac{s^2 + s(\lambda_{21}+\lambda_{12}+\mu) + \lambda_{12}\mu}{s+\lambda_{21}+\mu} = 1.$$

From here

$$\pi_1(s) = \frac{s+\lambda_{21}+\mu}{s^2 + s(\lambda_{21}+\lambda_{12}+\mu) + \lambda_{12}\mu}; \quad (14)$$

$$\pi_2(s) = \frac{\lambda_{12}}{s+\lambda_{21}+\mu}\pi_1(s) =$$
$$= \frac{\lambda_{12}}{s+\lambda_{21}+\mu}\cdot\frac{s+\lambda_{21}+\mu}{s^2 + s(\lambda_{21}+\lambda_{12}+\mu) + \lambda_{12}\mu} =$$
$$= \frac{\lambda_{12}}{s^2 + s(\lambda_{21}+\lambda_{12}+\mu) + \lambda_{12}\mu}.$$

In order to perform the inverse Laplace transform, we find the roots of the polynomial in the denominator of the ratio (14).

$$s^2 + s(\lambda_{21}+\lambda_{12}+\mu) + \lambda_{12}\mu = 0; \quad (15)$$

$$s_{1,2} = \frac{-(\lambda_{21}+\lambda_{12}+\mu) \pm \sqrt{(\lambda_{21}+\lambda_{12}+\mu)^2 - 4\lambda_{12}\mu}}{2} =$$
$$= \frac{-(\lambda_{21}+\lambda_{12}+\mu) \pm \sqrt{D}}{2}. \quad (16)$$

Find the discriminant:

$$D = \lambda_{12}{}^2 + \lambda_{21}{}^2 + \mu^2 + 2\lambda_{12}\lambda_{21} + 2\lambda_{12}\mu + 2\lambda_{21}\mu -$$
$$-4\lambda_{12}\mu = \lambda_{12}{}^2 + \lambda_{21}{}^2 + \mu^2 + 2\lambda_{12}\lambda_{21} + 2\lambda_{21}\mu -$$
$$-2\lambda_{12}\mu = (\lambda_{12}+\lambda_{21})^2 + \mu^2 + 2\mu(\lambda_{21}-\lambda_{12}).$$

Then

$$s_{1,2} = \frac{-(\lambda_{21}+\lambda_{12}+\mu) \pm \sqrt{\begin{array}{c}(\lambda_{12}+\lambda_{21})^2 + \mu^2 + \\ +2\mu(\lambda_{21}-\lambda_{12})\end{array}}}{2}. \quad (17)$$

The ratio (17) is not convenient for further analysis. In this regard, we obtain an approximate, but significantly simpler formula for calculating the roots of equation (15). We have

$$s_{1,2} =$$
$$= \frac{-(\lambda_{21}+\lambda_{12}+\mu) \pm \sqrt{(\lambda_{21}+\lambda_{12}+\mu)^2 - 4\lambda_{12}\mu}}{2} =$$
$$= \frac{-(\lambda_{21}+\lambda_{12}+\mu) \pm (\lambda_{21}+\lambda_{12}+\mu)}{2}\times$$
$$\times\frac{\sqrt{1 - 4\lambda_{12}\mu\big/(\lambda_{21}+\lambda_{12}+\mu)^2}}{2} = \quad (18)$$
$$= \frac{-(\lambda_{21}+\lambda_{12}+\mu) \pm (\lambda_{21}+\lambda_{12}+\mu)(\ )}{2}\times$$
$$\times\frac{1 - 2\lambda_{12}\mu\big/(\lambda_{21}+\lambda_{12}+\mu)^2}{2}.$$

From here

$$s_1 = \frac{-(\lambda_{21}+\lambda_{12}+\mu) + (\lambda_{21}+\lambda_{12}+\mu)}{2} -$$
$$- \frac{2\lambda_{12}\mu}{2(\lambda_{21}+\lambda_{12}+\mu)} = -\frac{\lambda_{12}\mu}{\lambda_{21}+\lambda_{12}+\mu}; \quad (19)$$

$$s_2 = \frac{-(\lambda_{21}+\lambda_{12}+\mu) - (\lambda_{21}+\lambda_{12}+\mu)}{2}\times$$
$$\times\frac{1 - 2\lambda_{12}\mu\big/(\lambda_{21}+\lambda_{12}+\mu)^2}{2} = -\frac{\lambda_{21}+\lambda_{12}+\mu}{2} + \quad (20)$$
$$+ \frac{-(\lambda_{21}+\lambda_{12}+\mu) + 2\lambda_{12}\mu/(\lambda_{21}+\lambda_{12}+\mu)}{2} =$$
$$= -(\lambda_{21}+\lambda_{12}+\mu) + 2\lambda_{12}\mu\big/(\lambda_{21}+\lambda_{12}+\mu).$$

Now let's perform the inverse Laplace transform. We have

$$\pi_1(s) = \frac{s+\lambda_{21}+\mu}{s^2 + s(\lambda_{21}+\lambda_{12}+\mu) + \lambda_{12}\mu} = \frac{s+\lambda_{21}+\mu}{(s-s_1)(s-s_2)} =$$
$$= \frac{\alpha_1}{s-s_1} + \frac{\alpha_2}{s-s_2} = \frac{\alpha_1(s-s_1) + \alpha_2(s-s_2)}{(s-s_1)(s-s_2)} =$$

$$= \frac{s(\alpha_1+\alpha_2)-\alpha_1 s_2-\alpha_2 s_1}{(s-s_1)(s-s_2)}. \quad (21)$$

The unknown parameters $\alpha_1$ and $\alpha_2$ are found by equating the coefficients for the same degrees $s$ on the left and right in the ratio (21). In this case, we obtain a system of equations

$$\begin{aligned}&\alpha_1+\alpha_2=1,\\&-\alpha_1 s_2-\alpha_2 s_1=\lambda_{21}+\mu.\end{aligned} \quad (22)$$

From here

$$\begin{aligned}&\alpha_2=1-\alpha_1;\\&-\alpha_1 s_2-(1-\alpha_1)s_1=-\alpha_1 s_2-s_1+\alpha_1 s_1=\\&=\alpha_1(s_1-s_2)-s_1=\lambda_{21}+\mu;\end{aligned} \quad (23)$$

$$\alpha_1=\frac{\lambda_{21}+\mu+s_1}{s_1-s_2}; \quad (24)$$

$$\begin{aligned}&\alpha_2=(1-\alpha_1)=1-\frac{\lambda_{21}+\mu+s_1}{s_1-s_2}=\\&=\frac{s_1-s_2-\lambda_{21}-\mu-s_1}{s_1-s_2}=-\frac{\lambda_{21}+\mu+s_2}{s_1-s_2};\end{aligned} \quad (25)$$

Substituting (23) and (24) into (21), we get

$$\pi_1(s)=\frac{\lambda_{21}+\mu+s_1}{s_1-s_2}\cdot\frac{1}{s-s_1}-\frac{\lambda_{21}+\mu+s_2}{s_1-s_2}\cdot\frac{1}{s-s_2}. \quad (26)$$

Similarly, we define the ratio for the calculation of $\pi_2(s)$. We have

$$\begin{aligned}&\pi_2(s)=\\&-\frac{\lambda_{12}}{s^2+s(\lambda_{21}+\lambda_{12}+\mu)+\lambda_{12}\mu}=\frac{\lambda_{12}}{(s-s_1)(s-s_2)}=\\&=\frac{\beta_1}{s-s_1}+\frac{\beta_2}{s-s_1}=\frac{\beta_1(s-s_2)-\beta_2(s-s_1)}{(s-s_1)(s-s_2)}=\\&=\frac{s(\beta_1+\beta_2)-\beta_1 s_2-\beta_2 s_1}{(s-s_1)(s-s_2)}.\end{aligned} \quad (27)$$

Further,
$$\begin{cases}\beta_1+\beta_2=0,\\-\beta_1 s_2-\beta_2 s_1=\lambda_{12}.\end{cases}$$

From here:
$$\beta_2=-\beta_1;$$

$$-\beta_1 s_2-\beta_2 s_1=\beta_1(s_1-s_2)=\lambda_{12};$$

$$\beta_1=\frac{\lambda_{12}}{s_1-s_2},\ \beta_2=-\frac{\lambda_{12}}{s_1-s_2}. \quad (28)$$

Using (27) and (28), we obtain

$$\pi_2(s)=\frac{\lambda_{12}}{s_1-s_2}\frac{1}{s-s_1}-\frac{\lambda_{12}}{s_1-s_2}\frac{1}{s-s_2}. \quad (29)$$

Now, using the correspondence tables of originals, we find the desired functions $P_1(t)$ and $P_2(t)$. We have

$$\begin{aligned}&P_1(s)=\frac{\lambda_{21}+\mu+s_1}{s_1-s_2}\cdot e^{-s_1 t}-\frac{\lambda_{21}+\mu+s_2}{s_1-s_2}\cdot e^{-s_2 t},\\&P_2(s)=\frac{\lambda_{12}}{s_1-s_2}\cdot e^{-s_1 t}-\frac{\lambda_{12}}{s_1-s_2}\cdot e^{-s_2 t}.\end{aligned} \quad (30)$$

Now obtain the desired distribution laws for the random duration of the system's stay in the operating states $E_1$ and $E_2$ before going into the failure state $E_3$.

$$\begin{aligned}P(t)=P_1(t)+P_2(t)=&\frac{\lambda_{12}+\lambda_{21}+\mu+s_1}{s_1-s_2}\cdot e^{-s_1 t}-\\&-\frac{\lambda_{12}+\lambda_{21}+\mu+s_2}{s_1-s_2}\cdot e^{-s_2 t}.\end{aligned} \quad (31)$$

We check the correctness of the obtained relations by calculating the values of $P_1(0)$ and $P_2(0)$, which should be equal to one and zero, respectively. We have

$$\begin{aligned}&P_1(0)=(\lambda_{21}+\mu+s_1-\lambda_{21}-\mu-s_2)/(s_1-s_2)=1,\\&P_2(0)=(\lambda_{12}-\lambda_{12})/(s_1-s_2)=0,\end{aligned}$$

which is exactly what was required. Finally, using (30), we determine the distribution law of the system's stay duration on the set $Z_0=\{E_1,E_2\}$ to failure. We have

$$\begin{aligned}&P(t)=P_1(t)+P_2(t)=\frac{\lambda_{21}+\mu+s_1}{s_1-s_2}\cdot e^{-s_1 t}-\\&-\frac{\lambda_{21}+\mu+s_2}{s_1-s_2}\cdot e^{-s_2 t}+\frac{\lambda_{12}}{s_1-s_2}\cdot e^{-s_1 t}-\frac{\lambda_{12}}{s_1-s_2}\cdot e^{-s_2 t}=\\&=\frac{\lambda_{12}+\lambda_{21}+\mu+s_1}{s_1-s_2}\cdot e^{-s_1 t}-\frac{\lambda_{12}+\lambda_{21}+\mu+s_2}{s_1-s_2}\cdot e^{-s_2 t}=\\&=\frac{\lambda_{21}+\mu+s_1}{s_1-s_2}\left(e^{-s_1 t}-e^{-s_2 t}\right)+\\&+\left(\frac{s_1}{s_1-s_2}e^{-s_1 t}-\frac{s_2}{s_1-s_2}\cdot e^{-s_2 t}\right).\end{aligned}$$

The obtained ratio makes it possible to calculate the probability of the system being in a set of operable states for any moment of time $t$.

Let us now determine the average duration of system functioning before falling into a set of outflow states. Introduce $f(t)=\frac{df(t)}{dt}$, $\bar{T}$ - average time to failure. Then

$$\begin{aligned}&\bar{T}=\int_0^\infty tf(t)dt=\lim_{A\to\infty}\int_0^A tf(t)dt=\\&=\lim_{A\to\infty}\left[tP(t)\Big|_0^A-\int_0^A P(t)dt\right]=\lim_{A\to\infty}\left[AP(A)-\int_0^A P(t)dt\right]=\\&=\lim_{A\to\infty}\left[\int_0^A (P(\mathrm{A})-P(t))dt\right]=\int_0^\infty (1-P(t)dt.\end{aligned}$$

The problem is solved. The natural direction of further research is related to solving the problems of multithreaded systems analyzing. A possible approach to solving such problems is proposed in [10].

## Conclusions

Thus, a solution is obtained to the problem of analyzing an inhomogeneous Marcovian system that resides on a set of possible states before falling into some selected set of special states (for example, failures). The method determines the distribution law of the system's stay random duration on a set of states until the moment of the first hit in one of the sets of special states.

The obtained distribution law allows to calculate the basic numerical characteristics of a random process of wandering through a set of states before entering the selected set of states.

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### Аналіз марківських систем із заданою множиною виділених станів

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**Анотація.** Аналіз стаціонарних марківських систем зазвичай виконується з допомогою систем лінійних диференціальних рівнянь Колмогорова. Такі системи дозволяють визначити ймовірність перебування аналізованої системи у кожному з можливих станів у довільний момент часу. Ця стандартна задача ускладнюється, якщо множина можливих станів систем є неоднорідною і з неї можна виділити, відповідно до специфіки функціонування системи, певну особливу підмножину. Предмет дослідження полягає у розробці технології аналізу таких систем. Відповідно до цього мета роботи – відшукання закону розподілу випадкової тривалості перебування такий системи на множині можливих станів до моменту потрапляння у виділену підмножину цих станів. Запропоновано метод вирішення поставленої задачі, заснований на розбитті всієї множини можливих станів системи на дві підмножини. Перша містить виділену підмножину станів, а друга – решта станів системи. Тепер із другої підмножини виділяється субпідмножина станів, з яких можливий безпосередній перехід у стани першої підмножини. Далі формується система диференціальних рівнянь, що описують переходи між сформованими підмножинами. Розв'язання цієї системи рівнянь дає шуканий результат – розподіл випадкової тривалості перебування системи до моменту першого потрапляння у виділену підмножину станів. Метод дозволяє вирішувати велику кількість практичних завдань, наприклад, у теорії надійності складних систем з безліччю різних відмовних станів. Зокрема, знаходження закону розподілу тривалості безвідмовної роботи, розрахунок середньої тривалості безвідмовної роботи.

**Ключові слова:** марківські системи; підмножина особливих станів; аналіз динаміки ймовірностей станів неоднорідних систем.