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MATHEMATICAL MODEL OF COMPUTER SYSTEM RELIABILITY IN RESIDUAL CLASSES

Abstract. The **subject** of the article is the construction of a mathematical model of the reliability of a computer system (CS) that operates in a system of residual classes (RNS). This mathematical model is based on the use of structural sliding reservation. The **purpose** of the article is to increase the reliability of the CS, which operates in the system of residual classes, as well as to calculate and compare the reliability, in terms of the probability of failure-free operation, of the CS in the RNS and the ternary computing system, which operates in the positional binary number system (PBNS). **Tasks**: to analyze the influence of the number system used on the reliability of the CS; to investigate the properties of the RNS and determine their influence on the structure of the CS in the RNS; build a mathematical model of reliability and, on its basis, perform the calculation and comparative analysis of the reliability of the CS in the RNS and the tripled CS in the PBNS. **Research methods**: methods of analysis and synthesis of computer systems, number theory, coding theory in RNS, reliability theory. The following **results** are obtained. The paper shows that the provision of a given level of reliability in the design of the CS using the RNS is due to the presence of various types of redundancy at the same time: structural, informational, temporal, functional and load. In RNS, these types of redundancy can be effectively used to improve the reliability of the CS. The CS in the RNS represents a computational structure that is identical in structure and principle of operation to the sliding structural redundancy model in the PBNS in the case of a loaded (hot) mode of operation of the redundant elements. Proceeding from this, the paper presents a mathematical model of the reliability of the CS in the RNS. In this paper, the calculation and comparative analysis of the reliability of a tripled computing structure in a PBNS with an ideal majority element and a CS in an RNS with an ideal reliability automaton are carried out. **Conclusions**. As shown by the results of calculations and comparative analysis in some time intervals of operation, the probability of non-failure operation of the CS in the RNS is higher than the probability of non-failure operation of the aircraft in the PBNS with a tripled majority structure. This implies the effective use of the RNS to improve the reliability of the CS.

Keywords: number system; residual class system; positional binary number system; mathematical model; reliability of computer systems and components.

Introduction

The volumes of multidimensional settlement tasks implemented by modern computer systems (CS) in various fields of science and technology, and the tasks of management of complex technical objects requires high reliability and speed of computing facilities and systems [1-3]. This circumstance requires the adoption of effective measures to improve the reliability and speed of the CS [4]. In modern scientific and technical literature, it is noted that in a positional binary number system (PBNS), it is almost impossible to significantly increase the reliability and speed of the CS [5]. At the same time, it is known that the CS of real-time data processing, which operates in the unposition number system in residual classes (RNS) has high computational capabilities from the point of view of improving the performance of the implementation of integer modular arithmetic operations [6, 7]. It is known from the point of view of computer machine arithmetic, RNS has three basic properties [7-9]: independence, equitution and little discharge of residues whose combination determines the unposition number system into RNS. The results of preliminary studies have shown that the use of listed properties can ensure improvement of the reliability of the CS based on the use of CS [10]. However, the lack of currently mathematical model of reliability of a computer system operating in the system of residual classes does not allow to calculate and comparative analysis of the reliability of the CS in PBNS and CS in RNS [11, 12]. This, in turn, restrains the widespread use of unposition code structures to increase the reliability of the CS.

The purpose of the article is to increase the reliability of the CS, which operates in the system of residual classes, as well as to calculate and compare the reliability, in terms of the probability of failure-free operation, of the CS in the RNS and the tripled computing system that operates in the positional binary number system.

Problem statement

At the stage of creating a CS, in designing, to determine the degree of compliance with the reliability of the CS, the specified requirements or when performing a comparative analysis of the reliability of various data processing tools, various methods of calculating reliability indicators (RI) are used [13]. The results of the calculation and comparative analysis of RI CS make it possible to solve a number of scientific and practical problems. The calculation of reliability involves the choice and calculation of one or a set of RI, which determine the conditions for the functioning and operation of a specific CS. The article shows the mathematical model of the reliability of the CS, which functions in real time. For such CS, in addition to speed, it is primarily important to estimate such a property of reliability as faultlessness. In this case, it is advisable to evaluate the reliability of the CS on a single indicator of faultlessness - the probability of faultless work of CS.

The CS in RNS represents a computational structure identical to the structure and principle of the functioning of the model of the sliding structural reservation in PBNS in the case of a loaded (hot) mode of operation of the backup elements. We will show it. The structure of the

CS in RNS contains the following elements: the main computational subsystem consisting of n information computing tracts (ICT); the control computational subsystem, which in the general case contains k control (backup) CT (CCT); the reliability machine (RM) that performs the functions of defining ICT flaws, if necessary, disabling them, as well as performing CCT connection functions. The role of the main elements of the system of the redundant system in PBNS, the ICT is played in RNS. The role of the backup elements in the reserved PBNS system is identified in RNS with an CCT functioning in a loaded reserve mode. All CT CS in RNS function independently of each other and in parallel in time. The control computational subsystem, regardless of the length of the l of discharge mesh of the CS, always contains two control CT on the modules of m_{n+1}, m_{n+2}, accordingly.

In accordance with the overall theory of unposition machine arithmetic in RNS, the presence of two $k = 2$ of the control bases reliably ensures the definition of one refused ICT CS.

Taking into account the above, as well as the results of the analysis of the properties of RNS and their influence on the structure and principles of the functioning of the CS, convinces that one of the real variants of the mathematical model of reliability of the CS in RNS is advisable to consider a mathematical model similar to the model of structural sliding backup in the case of a loaded standards mode used in PBNS. In this case, the formula for determining the probability of trouble-free operation of the CS in RNS will take a type of expression:

$$P_{RNS}(t)=\sum_{i=0}^{k}\frac{L!\cdot\left[1-P_1(t)\right]^i\cdot P_1^{L-1}(t)}{i!\cdot(L-i)!}, \qquad (1)$$

where the expression $P_1(t)=e^{-\lambda_e a_{n+k}t}$ is probability of the trouble-free operation of one arbitrary CT CS data processing along the largest (least reliable) base m_{n+k} of RNS, and the value λ_1 – is the intensity of the BT CS equipment failures into RNS for the greatest base m_{n+k}.

At the calculation of value of probability of trouble-free work of CS in RNS it comfortably to use a formula

$$P_{RNS}^{(k)}(t)=\sum_{i=0}^{k}C_{k+n}^{i}P_1^{k+n-i}(t)\sum_{j=0}^{i}(-1)^j C_i^j P_1^j(t). \qquad (2)$$

The ratio (2) can be used to calculate the probability of trouble-free operation of the CS in RNS with the following assumptions:

– CT CS failures in RNS and CS in PBNS satisfy the conditions of the simplest stream; in this case, for calculating RM, in particular, the probability of trouble-free operation, the exponential law of the time distribution between failures is used, since it is sufficiently reasonable theoretically, confirmed by experimentally information about the intensity of failures of the elements of computing technology;

– the automat of reliability of the CS in RNS is ideal, that is the probability of trouble-free operation of the RM is equal to one and the switching time (disabling the ICT failed and the CCT connection) CT is zero;

– information and control CT CS are equal to reliable (the probability of the trouble-free operation of all paths is taken equal to the probability of trouble-free $P_1(t)=e^{-\lambda_e a_{n+k}t}$ CT operation, the greatest base of m_{n+k} RNS having the smallest probability of trouble-free operation, that is);

– the possibility of restoring the refusal of CT CS is not taken into account;

– the time of detection of the refused ICT CS and the connection time CCT CS is zero;

– all CT CS in RNS are loaded and all the same reliable;

– repair of refused CT is not produced or impossible.

It is known that PBNS to increase the reliability of the CS is often practically used by computing structure, consisting of three CSs with a majoritarian organ (MO). In given computing structure for certain the choice of one of three comes true capable of working CS. It comes true by means of the use of MO. We will designate through λ_e the intensity of failures of the conditional number of CS equipment, referred to a unit of the discharge grid. In this case, the probability of trouble-free operation of equipment, assigned to one binary discharge of the CS, is equal to $P_e(t)=e^{-\lambda_e t}$. For one positional l-byte, the probability of trouble-free operation is equal to $P_0(t)=e^{-\lambda_0 t}$ where $\lambda_0 = 8l\lambda_e$. Or, the probability of trouble-free operation of one CS in PBNS is equal to

$$P_0(t)=e^{-\lambda_0 t}=e^{-81\lambda_e t}.$$

It is known that probability of trouble-free operation for the trooped majority to computing structure in PBNS containing three CS and ideal MO, equal to the value

$$P_{PBNS}(t)=3P_0^2(t)-2P_0^3(t). \qquad (3)$$

Calculation and comparative analysis of reliability

As an example, we will conduct a comparative analysis of the reliability of the trooped positional computing structure in PBNS with ideal MO and CS in RNS with ideal thoroughness, applying the considered reliability model (2) for the values of the discharge mesh $l=1$ and $l=2$. For the CS in RNS, the probability of the trouble-free operation of any CT on an arbitrary base $m_i (i=\overline{1,n+2})$ is equal to $P_1(t)=e^{-\lambda_1 t}$ or $P_1(t)=e^{-\lambda_e a_{n+2}t}$, where $a_{n+2}=[log_2(m_{n+2}-1)]+1$. The probability of trouble-free operation of the CS in RNS is determined in accordance with the expression (2).

Let a single-byte CS ($l=1$, that is that the discharge mesh of the CS in PBNS contains 8 binary discharge) and the amount of CCT is two ($k=2$). In this case, the RNS can be determined as a set of information ($n=4$) the bases $m_1=3, m_2=4, m_3=5, m_4=7$ and the control ($k=2$) $m_5=11, m_6=13$ bases. At the same time

$$\prod_{i=1}^{4} m_i = 420 > 2^8 = 256$$

and $GCD\left(m_i, m_j\right) = 1 \;\; for\; i \neq j$.

Naturally, the number of ICT and CCT CS in RNS is equal, respectively, four ($n=4$) and two ($k=2$). The time of operation of the CS from the moment of inclusion before the end of operation is taken equal to one hour, that is $t=1$ hour. The intensity of failures of the part of the equipment of the CS, assigned to one binary discharge of the discharge mesh, is taken equal to $\lambda_e = 10^{-2}$ [1/hour]. Denote by $L = n + k = 4 + 2 = 6$. For this RNS, based on the source data, expression (2) to determine the probability of trouble-free operation of the P_{RNS} CS will have the following type

$$P_{RNS}(t) = \sum_{i=0}^{k} C_L^i \cdot P_1^{L-i}(t) \cdot \sum_{j=0}^{i} (-1)^j \cdot C_i^j \cdot P_1^j(t) =$$

$$= \sum_{i=0}^{k} \frac{L! P_1^{L-i}(t)}{i!(L-i)!} \cdot \sum_{j=0}^{i} (-1)^j \frac{i!}{j!(i-j)!} P_1^j(t) =$$

$$= \sum_{i=0}^{2} \frac{6! P_1^{6-i}(t)}{i!(6-i)!} \cdot \sum_{j=0}^{i} (-1)^j \cdot \frac{i!}{j!(i-j)!} \cdot P_1^j(t). \quad (4)$$

For ease of calculations, expression (3) is convenient to present in the form:

$$P_{RNS}(t) = \overset{(i=0)}{P_1}(t) + \overset{(i=\overline{0,1})}{P_1}(t) + \overset{(i=\overline{0,2})}{P_1}(t). \quad (5)$$

Let us calculate separately every term expression (4):

$$\overset{(i=0)}{P_1}(t) = \frac{6!}{0!(6-0)!} \cdot P_1^{6-0}(t) \cdot \left[\sum_{j=0}^{0} (-1)^0 \cdot \frac{0!}{0!0!} \cdot P_1^0(t) \right] =$$

$$P_1^6(t) \cdot 1 = P_1^6(t), \quad (6)$$

$$\overset{(i=\overline{0,1})}{P_1}(t) = C_6^1 \cdot P_1^5(t) \cdot \left[\sum_{j=0}^{1} (-1)^j \cdot C_i^j \cdot P_1^j(t) \right] =$$

$$= C_6^1 \cdot P_1^5(t) \cdot \left[(-1)^0 \cdot C_1^0 \cdot P_1^0(t) + (-1)^1 \cdot C_1^1 \cdot P_1^1(t) \right] =$$

$$= \frac{6!}{1!5!} \cdot P_1^5(t) \cdot \left[1 \cdot \frac{1!}{0!1!} \cdot 1 - 1 \cdot \frac{1!}{1!\cdot 0!} \cdot P_1^1(t) \right] =$$

$$= 6 \cdot P_1^5(t) \cdot \left[1 - P_1(t) \right] = 6 \cdot P_1^5(t) - 6 \cdot P_1^6(t), \quad (7)$$

$$\overset{(i=\overline{0,2})}{P_1}(t) = C_6^2 \cdot P_1^4(t) \cdot \left[\sum_{j=0}^{2} (-1)^j \cdot C_i^j \cdot P_1^j(t) \right] =$$

$$= \frac{6!}{2!\cdot 4!} \cdot P_1^4(t) \cdot [(-1)^0 \cdot C_2^0 \cdot P_1^0(t) +$$

$$+ (-1)^1 \cdot C_2^1 \cdot P_1^1(t) + (-1)^2 \cdot C_2^2 \cdot P_1^2(t)] =$$

$$= 15 \cdot P_1^4(t) \cdot [1 \cdot \frac{2!}{0!\cdot 2!} \cdot 1 - 1 \cdot \frac{2!}{1!\cdot 1!} \cdot P_1(t) + 1 \cdot \frac{2!}{2!\cdot 0!} \cdot P_1^2(t)] =$$

$$= 15 \cdot P_1^4(t) \cdot \left[1 - 2 \cdot P_1(t) + P_1^2(t) \right] =$$

$$= 15 \cdot P_1^4(t) - 30 \cdot P_1^5(t) + 15 \cdot P_1^6(t). \quad (8)$$

Based on the relation (4) - (7) we obtain the value:

$$P_{RNS}(t) = P_1^6(t) + 6 \cdot P_1^5(t) - 6 \cdot P_1^6(t) +$$

$$+ 15 \cdot P_1^4(t) - 30 \cdot P_1^5(t) + 15 \cdot P_1^6(t) =$$

$$= 10 \cdot P_1^6(t) - 24 \cdot P_1^5(t) + 15 \cdot P_1^4(t). \quad (9)$$

Where the value is the probability of trouble-free operation of one CT CS in RNS under specified source data. Thus, the value of the probability of operation of the CS in RNS (expression (8)) is equal to:

$$P_{RNS}(t) = 0{,}9989. \quad (10)$$

To carry out a comparative analysis of the reliability of the CS in RNS and computing structure in PBNS, in accordance with the expression (3), we calculate the probability of trouble-free operation for the trooped positional computing structure. For the specified source data, the probability of trouble-free operation of one CS in PBNS is equal to $P_0(t) = e^{-\lambda_0 t} = e^{-81\lambda_i t} = 0{,}9231$.

In this case, we obtain the final result of the calculation of the reliability of the computing structure in PBNS $P_{PBNS}(t) = 3P_0^2(t) - 2P_0^3(t) = 0{,}9832$.

As an example, table 1 presents some design data of reliability for the initial period of the functioning of the CS in RNS and the computing structure to PBNS, with $l=1$, for the value of $\lambda_e = 10^{-2}$.

Table 1 – **Some design data of reliability for the initial period of the functioning of the CS in RNS and the computing structure to PBNS, with $l=1$, for the value of $\lambda_e = 10^{-2}$**

$l=1$, $\lambda_e = 10^{-2}$ [1/hour]				
t [hour]	*PBNS*		*RNS*	
	$P_0(t)$	$P_{PBNS}(t)$	$P_1(t)$	$P_0(t)$
1/15	0.9993	0.9999	0.9973	0.9999
1/12	0.9991	0.9999	0.9967	0.9999
1/6	0.9983	0.9999	0.9934	0.9999

Let a two-byte CS ($l=2$, that is the discharge mesh of the CS in PBNS contains 16 binary discharge) and the amount of CCT, as for any value l, is two ($k=2$). In this case, the RNS can be determined in the form of a set of information ($n=6$) the bases $m_1 = 2, m_2 = 5, m_3 = 7, m_4 = 9 m_5 = 11, m_6 = 13$ and the control ($k=2$) bases $m_7 = 17, m_8 = 19$ grounds.

At the same time

$$\prod_{i=1}^{6} m_i = 90090 > 2^{16} = 655336$$

and $GCD\left(m_i, m_j\right) = 1 \;\; for\; i \neq j$.

Naturally, the number of ICT and CCT CS in RNS is equal, respectively, six ($n=6$) and two ($k=2$). The value of L is $L=n+k=6+2=8$. For this RNS, based on the source data, expression (2) to determine the probability of trouble-free operation of the P_{RNS} CS will have the following type

$$P_{RNS}(t) = \sum_{i=0}^{k} C_L^i \cdot P^{L-i}(t) \cdot \left[\sum_{j=0}^{i} (-1)^j \cdot C_i^j \cdot P^j(t) \right] = \sum_{i=0}^{2} \frac{8!}{i! \cdot (8-i)!} \cdot P^{8-i} \cdot \left[\sum_{j=0}^{i} (-1)^j \cdot \frac{i!}{j! \cdot (i-j)!} \cdot P^j \right], \quad (12)$$

$$\overset{(i=0)}{P_1}(t) = \frac{8!}{0! \cdot (8-0)!} \cdot P^{8-0} \cdot \left[\sum_{j=0}^{0} (-1)^0 \cdot \frac{0!}{0! \cdot (0-0)!} \cdot P^0 \right] = P_1^8 \cdot 1 = P_1^8(t), \quad (13)$$

$$\overset{(i=1)}{P_1}(t) = C_8^1 \cdot P^{8-1} \cdot \left[\sum_{j=0}^{1} (-1)^j \cdot C_i^j \cdot P^j(t) \right] = \frac{8!}{1! \cdot (7)!} \cdot P^7 \cdot \left[(-1)^0 \cdot C_1^0 \cdot P^0 + (-1)^1 \cdot C_1^1 \cdot P^1 \right] = 8 \cdot P^7 \cdot [1-P] = 8 \cdot P^7 - 8 \cdot P^8, \quad (14)$$

$$\overset{(i=2)}{P_1}(t) = C_8^2 \cdot P^{8-2} \cdot \left[\sum_{j=0}^{2} (-1)^j \cdot C_i^j \cdot P^j(t) \right] = \frac{8!}{2! \cdot (6)!} \cdot P^6 \cdot [(-1)^0 \cdot C_2^0 \cdot P^0 + (-1)^1 \cdot C_2^1 \cdot P^1 + (-1)^2 \cdot C_2^2 \cdot P^2] = 28 \cdot P^6 \cdot \left[1 - 2 \cdot P^1 + P^2 \right] = 28 \cdot P^6 - 56 \cdot P^7 + 28 \cdot P^8, \quad (15)$$

$$P_{RNS}(t) = \overset{(i=0)}{P_1}(t) + \overset{(i=\overline{0,1})}{P_1}(t) + \overset{(i=\overline{0,2})}{P_1}(t) = P_1^8(t) + 8 \cdot P_1^7(t) - 8 \cdot P_1^8(t) + 28 \cdot P_1^6(t) - = P_1^8(t) + 8 \cdot P_1^7(t) - 8 \cdot P_1^8(t) + 28 \cdot P_1^6(t) - 56 \cdot P_1^7(t) + 28 \cdot P_1^8(t) = 21 \cdot P_1^8(t) - 48 \cdot P_1^7(t) + 28 \cdot P_1^6(t). \quad (16)$$

As an example, Table 2 presents some design data of reliability for the initial period of the functioning of the CS in RNS and computing structure in PBNS, with $l=2$, for the value $\lambda_e = 10^{-2}$.

The use of the properties of RNS in the design of the CS provides the presence of at the same time different types of redundancy: structural, information, temporary, functional and load. In RNS, these redundancy types can be effectively used as to increase productivity and ensure the reliability of the CS.

Table 2 – **Some design data of reliability for the initial period of the functioning of the CS in RNS and computing structure in PBNS, with $l=2$, for the value $\lambda_e = 10^{-2}$**

$l=2$, $\lambda_e = 10^{-2}$ [1/hour]				
t [hour]	PBNS		RNS	
	$P_0(t)$	$P_{PBNS}(t)$	$P_1(t)$	$P_{RNS}(t)$
1/15	0.9986	0.9999	0.9967	0.9999
1/12	0.9983	0.9999	0.9958	0.9999
1/6	0.9967	0.9999	0.9917	0.9999
1/3	0.9934	0.9998	0.9835	0.9999

The use of any type of redundancy necessarily causes the presence of structural redundancy. Structural redundancy affects the mass-overall and other characteristics of CS. In this aspect, it is important to assess the volume of equipment that ensures the increase in the reliability of the CS.

We will evaluate the number of equipment used to increase the reliability of the CS in RNS and the computing structure in PBNS. For this we will operate with the concept of a conditional relative amount of the equipment of CS. This is part of the amount of V_i equipment of the CS, which relates to one binary discharge of the discharge mesh of the CS.

In this case, the quality of the CS equipment in RNS can be represented as

$$V_{RNS} = V_i \cdot \sum_{i=1}^{n+2} \left\{ \left[\log_2(m_i - 1) \right] + 1 \right\}. \quad (17)$$

And the conditional relative number of equipment computing structure in PBNS is equal

$$V_{PBNS} = V_i \cdot 3 \cdot 8 \cdot l. \quad (18)$$

For the one-to-one considered above ($l=1$) computing structure in PBNS formula (18) will take a view

$$V_{PBNS} = V_i \cdot 24 \quad (19)$$

and formula (17) will take the form:

$$V_{RNS} = V_i \cdot \sum_{i=1}^{6} \left\{ \left[\log_2(m_i - 1) \right] + 1 \right\} = 2+2+3+3+4+4 = 14. \quad (20)$$

The calculated data of the relative amount of CS conditional equipment in RNS and computing structure in PBNS for l -byte bitmaps are presented in Table 3. An analysis of the calculated data showed that the use of RNS provides a given level of reliability (by probability of trouble-free operation) of the CS with a smaller number of additional equipment introduced (that is at a lower cost) than the method of majority trolationing CS used in PBNS.

Table 3 – **The calculated data of the relative amount of CS conditional equipment in RNS and computing structure in PBNS for l -byte bitmaps**

A*	Information n base of RNS	B*	C*	D*	E*	F*
1	$m_1=3,\ m_2=4,\ m_3=5,\ m_4=7$	$m_5=11, m_6=13$	4	18	24	25
2	$m_1=2,\ m_2=5,\ m_3=7, m_4=9,\ m_5=11,\ m_6=13$	$m_7=17, m_8=19$	5	29	48	40
3	$m_1=3,\ m_2=4,\ m_3=5, m_4=7,\ m_5=11,\ m_6=13, m_7=17,\ m_8=19$	$m_9=23, m_{10}=29$	5	34	72	53
4	$m_1=2,\ m_2=3,\ m_3=5, m_4=7, m_5=11,\ m_6=13, m_7=17,$ $m_8=19,\ m_9=23, m_{10}=29$	$m_{11}=31,$ $m_{12}=37$	6	48	96	50
8	$m_1=3,\ m_2=4,\ m_3=5,\ m_4=7,\ m_5=11,\ m_6=13,\ m_7=17, m_8=19,$ $m_9=23, m_{10}=29, m_{11}=31, m_{12}=37, m_{13}=41, m_{14}=43, m_{15}=47, m_{16}=53$	$m_{17}=59,$ $m_{18}=61$	6	81	192	58

A* – The length of l discharge mesh CS; B* – Control $k=2$ base of RNS; C* – The values of λ_k ; D* – The number of CS conditional equipment in RNS; E* – Number of conditional equipment computing structure in PBNS; F* – The winning in the number of additional equipment [%].

Conclusions

The article proposes a variant of the mathematical model of the reliability of the CS functioning in RNS, based on the use of the model of structural sliding reservation in PBNS.

It is shown that the proposed mathematical model adequately reflects the structure and principles of the functioning of the CS in RNS. A comparative analysis of reliability is carried out, according to the probability of trouble-free operation, the computing structure in PBNS with the ideal MO and the CS in RNS with ideal RM. It should be noted that when evaluating the efficiency of use of RNS, the complexity of the construction and functioning of the MO in PBNS and RM in RNS has not been evaluated.

In the article, when calculating the reliability of the CS, it was believed that the MO and RM absolutely reliably (the probability of trouble-free operation of MO and RM is a unit, and disabling the ICT refused and the CCT connection is carried out instantly). If necessary, in the general case, the effect of MO and RM on the reliability of the CS is easy to consider. The practice of building computing structure has shown that a decrease in the reliability of the CS due to the accounting of the unreliability of MO and RM is not so great, and sometimes it can be neglected.

The article shows that the effect of ensuring a given level of reliability of the CS functioning in RNS is achieved at less hardware costs (with a smaller number of additionally introduced equipment), that is that a lower cost than the widely used in the practice of creating computing structure in PBNS, the method of trolation of the same type of CS. This is due to the fact that, first, in the proposed mathematical model, the basic properties of RNS are mostly taken into account: independence, equality and independence of residues defining the unposition code structure (UCS). Analysis and accounting of the main properties of RNS ensures the presence and use of different types of redundancy in the CS simultaneously: structural, information and functional.

The use of RNS allows you to present a single CS in the form of individual, independent parts of the CS (CT), parallel in time. Based on the properties of RNS, each UCS residue contain information about the entire code structure.

The UCS residue is processed by the corresponding ICT of the CS on its specific module. This corresponds to the fact that the entire CS is redundant to RNS, and the ICT is reserved - separate parts of the CS (minor elements are reserved). In accordance with the theory of reliability of radio-electronic equipment to increase the reliability of the system, it is advisable to reserve smaller nodes and blocks of this system. This is due to the fact that the intensity of failures of the CS subsystems is always less than the intensity of failures of the entire computer system.h

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Математична модель надійності комп'ютерної системи у залишкових класах

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Анотація. **Предметом** статті є побудова математичної моделі надійності комп'ютерної системи (КС), що функціонує у системі залишкових класів (СЗК). Ця математична модель будується на основі використання структурного ковзного резервування. **Метою** статті є підвищення надійності КС, яка функціонує в системі залишкових класів, а також проведення розрахунку та порівняльного аналізу надійності, за ймовірністю безвідмовної роботи, КС у СЗК та тройованої обчислювальної системи (ОС), яка функціонує у позиційній двійковій системі числення (ПДСЧ). **Завдання**: провести аналіз впливу системи числення, що використовується, на надійність КС; дослідити властивості СЗК та визначити їх вплив на структуру КС у СЗК; побудувати математичну модель надійності та на її основі провести розрахунок та порівняльний аналіз надійності КС у СЗК та тройованої ОС у ПДСЧ. **Методи** дослідження: методи аналізу та синтезу комп'ютерних систем, теорія чисел, теорія кодування у СЗК, теорія надійності. Отримано такі **результати**. У роботі показано, що забезпечення заданого рівня надійності при проектуванні КС з використанням СЗК обумовлено наявністю одночасно різних видів надмірності: структурної, інформаційної, часової, функціональної та навантажувальної. У СЗК зазначені види надмірності можна ефективно використовувати для підвищення надійності КС. КС у СЗК представляє собою обчислювальну структуру, яка ідентична за структурою та принципом функціонування моделі ковзного структурного резервування в ПДСЧ у разі навантаженого (гарячого) режиму функціонування резервних елементів. Виходячи з цього, у роботі представлена математична модель надійності КС у СЗК. У роботі проведено розрахунок та порівняльний аналіз надійності тройованої обчислювальної структури у ПДСЧ з ідеальним мажоритарним елементом та КС у СЗК з ідеальним автоматом надійності. **Висновки**. Як показали результати розрахунків та порівняльного аналізу в деяких часових інтервалах функціонування, ймовірність безвідмовної роботи КС у СЗК вище за ймовірність безвідмовної роботи ОС у ПДСЧ з тройованою мажоритарною структурою. Це обумовлює ефективне використання СЗК для підвищення надійності КС.

Ключові слова: система числення; система залишкових класів; позиційна двійкова система числення; математична модель; надійність комп'ютерних систем та компонентів.