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EVALUATION MODEL OF THE RECOVERY PROCESSES OF NON-MARKOVIAN SYSTEMS, CONSIDERING THE ELEMENTS UNRELIABILITY UNDER ARBITRARY DISTRIBUTION LAWS

Abstract. The subject of the study is the reliability of recoverable non–Markovian systems, functioning of which is described by arbitrary distribution laws. The purpose of the article is to develop a mathematical model of the functioning of modern computer systems under arbitrary laws of the distribution of stay duration in each of the states, taking into account the recovery system and the provision of spare elements. The main task is to develop an adequate model of the system functioning process, taking into account the non-Markovian character of the processes occurring in the system, its possible large dimension, and the presence of a hierarchical recovery system. Based on this model, a method for calculating the density of the system recovery time distribution has been developed. At the same time, a universal four-parameter distribution is proposed to describe random processes occurring in the system. Using this approximation, the calculation of the desired parameter of the recovery flow is performed by solving the Volterra integral equation with a difference kernel.

Keywords: restoration of non-Markovian systems; mathematical model of reliability; density of the recovery time distribution.

Introduction

Problem Statement. In the last decade, due to the wide spread of the Internet, the so–called "cloud technologies" have been rapidly developing, structurally implemented as complex software and hardware complexes representing Multi-Position Distributed Systems (MPDS). From the hardware point of view, these are complex, spatially and functionally distributed multi-level hierarchical structures consisting of many functional subsystems - groups of radio electronic and computer systems (RECS), united by means of network communication and control. The main features of the MPDS in comparison with single-position systems are as follows: presence of structural and information redundancy arising from the interaction of RECS groups, use of spatial-temporal methods of multidimensional processing of measurement information, large amounts of data (*BigData* technology), unified synchronization of connections between the systems included in MPDS, variety of principles of RECS technical implementation and computational means of ensuring the "*cloud*" storage, information processing and transmission, high requirements for the reliability of elements and subsystems, etc.

In the process of functioning, elements and modules of all subsystems often fail. The concept of failure is understood as a complete or partial loss of operability by object, which is a consequence of environmental factors impact (temperature, humidity, vibration), internal physical-chemical processes, as well as continuous operation in loaded modes, violations of operating modes, maintenance, software failures, etc.

In order to guarantee the high reliability of such complex long-term MPDS for their hardware component, the following methods of reliability increasing are currently used [1-4, 6, 9, 11]:

- redundancy of unreliable or especially important elements and their blocks;
- reducing the failure rate of elements and the entire system;
- reduction of continuous operation time in especially loaded modes;
- reduction of the average recovery time, etc...

In order to guarantee high reliability and efficiency of MPDS, based on the possibilities of practical implementation and the degree of efficiency, the most significant for the hardware component of MPDS is the formation of self-control system and spare standard elements provision (SCSSSEP). At the same time, each individual RECS is given a set of standard spare parts containing a certain number of replacement elements of all necessary types. To replenish the spare parts SPTA, a group (SPTA-G) is used, which is attached to the entire MPDS both to replenish the spare parts and to ensure reliability for certain types absent in the nomenclature of single spare parts. In addition, a common repair body (RB) is distinguished in the structure of the entire SCSSSEP, the functioning of which consists in eliminating failures by identifying and replacing (repairing) hierarchically smaller failed structural elements in faulty elements [4-6, 11].

To solve the problem of optimal synthesis of restoration and maintenance system SCSSSEP of MPDS, it is necessary to develop methods for analyzing the effectiveness of RECS functioning together with SCSSSEP, using the parameters of the structure and configuration of the spare parts and providing a target function of the form

$$P = f(\bar{X}),$$

where P is the efficiency indicator of MPDS, $\bar{X}$ is the vector of SCSSSEP parameters. To construct mathematical models of the functioning of both individual RECS and MPDS, the theory of random processes and queuing theory are used today in the vast majority of cases, focused mainly on the exponential distribution of failure and recovery flows [1-6, 9, 12-17]. Several types of indicators are recommended to evaluate the effectiveness of modern RECS, taking into account the reliability. The main ones are [1,3,11]:

- probability of uptime for a given time $t_0 : P(t_0)$;
- average operating time to failure: $\overline{T_O}$;
- average recovery time: $\overline{T_R}$;
- system availability coefficient for reliability K_R ;
- operational readiness coefficient for time t_0: $R(t_0)$.

It should be noted that the traditionally used models for evaluating the MPDS effectiveness, taking into account the reliability of its elements, are focused on assumptions about the exponential distribution of the uptime of all types of elements and their recovery time, and about the relatively small dimension of tasks. This ensures the formation of elegant and simple mathematical models, but, of course, imposes certain limitations on the level of accuracy of the results obtained using these models. Analysis of the problems RECS with SCSSSEP effectiveness evaluation shows [1-6,9,11], that only particular problems of this problem can be solved by existing analytical methods using sufficiently strict restrictions on the number of variables, the laws of distribution of failure flows and recoveries, the structure and options for SCSSSEP construction, maintenance processes using the assumption that maintenance updates the system completely.

Thus, the tasks of analyzing the effectiveness of MPDS functioning, taking into account the structure, composition, options for strategies for replenishment, control and maintenance of spare parts kits, as well as the synthesis of hierarchical spare parts of the optimal structure and quantitative composition, despite a large number of developments and publications on this topic, remain relevant. This is due to the lack of a sufficiently general methodology for assessing the MPDS effectiveness, which would allow expanding the space of the analyzed parameters, would provide the possibility of using non-Markovian laws of failure flow distribution and recoveries and obtaining the final quantitative results.

The article attempts to construct a general mathematical model of RECS functioning, taking into account the recovery system. Analysis of the functioning of such a generalized system will allow to obtain the laws of RECS recovery times distribution, necessary for calculating the MPDS functioning efficiency, taking into account reliability.

Main Results

Development of a mathematical model of non-Markovian functioning systems taking into account the recovery system. Consider an MPDS consisting of *S* of one-type RECS, each of which is given its own set of spare parts. All single sets of spare parts are closed to a group set of SPTA-G. Let's assume that the average failure rate of elements of the *j*-type of the RECS set when operating under current as part of the equipment is equal to $\lambda_T^{(j)}$, and in storage mode – $\lambda_{ST}^{(j)}$. When transferring elements from storage mode to operation mode under current, their failure rate becomes equal to $\lambda_T^{(j)}$. Then, when the RECS is operating in standby mode, the failure rate of elements of the *j*-type of the RECS aggregate can be written as follows [3]:

$$\lambda^{(j)} = \lambda_T^{(j)} K_u + \lambda_{ST}^{(j)} \left(1 - K_u\right), \qquad (1)$$

where K_u the intensity coefficient of equipment operation. The cyclicity of RECS operation has a significant impact on their reliability and to account for this effect on the failure rate of RECS elements, the ratio (1) can be written as follows [3, 5]:

$$\lambda_C^{(j)} = \lambda_T^{(j)} \left(1 + \rho_j \gamma\right), \qquad (2)$$

where ρ_j is a coefficient showing how many times the failure rate of *j*-type elements increases with each turn on of the RECS equipment, γ is a coefficient characterizing the average number of turns on of the RECS equipment per hour. At this

$$\lambda^{(j)} = \lambda_C^{(j)} K_u + \lambda_{ST}^{(j)} \left(1 - K_u\right). \qquad (3)$$

The models given below characterize an arbitrary *k*-th interval of RECS operation, the index *k* will be omitted in the future for convenience of recording.

Suppose that the RECS has in its composition a set of elements that are homogeneous in characteristics. Duration of their uptime is a continuous random variable with a distribution density that $f_0\left(\underline{\varepsilon}(t), t\right)$ is a function of the conditions and operating modes of the system $\underline{\varepsilon}(t)$, as well as their lifetime *t*, and

$$\begin{aligned} f_0\left(\underline{\varepsilon}(t), t\right) &= \lambda\left(\underline{\varepsilon}(t), t\right) P\left(\underline{\varepsilon}(t), t\right) = \\ &= \lambda\left(\underline{\varepsilon}(t), t\right) \exp\left(-\int_0^t \lambda\left(\underline{\varepsilon}(x), x\right) dx\right), \end{aligned} \qquad (4)$$

where $\lambda\left(\underline{\varepsilon}(t), t\right)$ is the failure rate of the elements as a function of the conditions and modes of their operation $\underline{\varepsilon}(t)$ and the "age" *t*.

Let's consider the process of elements operation of the set of *S* RECS, taking into account the functioning of the support system in time. At the moment of t_1^{FAIL} failure of any of the RECS elements, it goes into a recovery state.

The failed element is either rejected or sent to RB, where it is restored, and a working element from the spare parts kit is installed in its place. The recovery ends at a random moment t_1^R, after which the RECS is operational until the next failure.

The density of the RECS recovery duration distribution is denoted by $f_R(t)$. During the RECS operation, random periods of uptime τ'_j and recovery time alternate τ''_j.

Such a process is called an alternating recovery process [5, 7]. Introduce $G_k(t)$ - time distribution function of the *k*-th transition from the recovery state to the working state. At this

$$G_k(t)=\int_0^t \Phi\left[(2k-1),\tau\right] f_R(t)\,dt, \qquad (5)$$

where $\Phi\left[(2k-1),\tau\right]$ - there is a probability that the system, starting from the recovery state at $t=\tau_1'$ the moment of time, has *returned* to the same state by the moment of time, having made 2 *k*-1 transitions.

On the other hand, it is clear that

$$\Phi\left[(2k-1),\tau\right]=\int_0^t G_{k-1}(\tau-u)\, f_0(u)\,du. \qquad (6)$$

Substituting expression (6) into (5), we obtain the distribution function $G_k(t)$:

$$G_k(t)=\int_0^t\left[\int_0^\tau G_{k-1}(\tau-u)\, f_R(u)\,du\right] f_0(t-\tau)\,d\tau. \qquad (7)$$

At this $\qquad G_1(t)=\int_0^t f_R(\tau)\,d\tau. \qquad (8)$

It is shown [7] that if there $F_k(t)$ is a time distribution function of the *k*-th transition from the working state to the recovery state in the alternating process, then the failure flow parameter is obtained from the ratio

$$\omega_0(t)=\frac{d}{dt}\left(\sum_{k=1}^{\infty} F_k(t)\right). \qquad (9)$$

Then, in the same alternating process, by analogy, we calculate the parameter of the recovery flow $\omega_R(t)$ by the formula

$$\omega_R(t)=\frac{d}{dt}\left(\sum_{k=1}^{\infty} G_k(t)\right)=\sum_{k=1}^{\infty} g_k(t). \qquad (10)$$

Performing the forward and reverse Laplace transformation of the expression

$$G(t)=\sum_{k=1}^{\infty} G_k(t). \qquad (11)$$

after the transformations, we find the ratio for the parameter of the recovery flow

$$\omega_R(t)= f_R(t)+ +\int_0^t\left[\int_0^\tau \omega_R(\tau-u)\, f_0(u)\,du\right] f_R(t-\tau)\,d\tau, \qquad (12)$$

or, equivalently

$$\omega_R(t)= f_R(\tau)+ +\int_0^t\left(\int_0^{t-\tau} \omega_R(u)\, f_0(t-\tau-u)\,du\right) f_R(\tau)\,d\tau. \qquad (13)$$

This relation for the given ones $f_0(t)$ $f_R(t)$ is the Volterra integral equation of the second kind with a difference kernel relative to $\omega_R(t)$.

The main indicators of RECS reliability will be considered the system readiness coefficient (the probability that at any given time *t* the RECS will be in working condition - $K_r(t)$) and the operational readiness coefficient (the RECS will work flawlessly for a given time interval **(***t*, *t*+*τ*), that is - *P*(*t*, *t*+*τ*)), which are respectively equal to:

$$K_r(t)=P_1+P_2=p(t)+\int_0^t p(t-\tau)\,\omega_R(\tau)\,d\tau,$$

$$P(t,t+\tau)=p(t+\tau)+\int_0^t \omega_R(u)\, p(t+\tau-u)\,du, \qquad (14)$$

where *p*(*t*+*τ*) is the probability that the RECS will not fail once during the time interval **(***t*+*τ*), $\int_0^t \omega_R(u) P(t+\tau-u)\,du$ is the probability that the RECS will fail for the last time at some point in time *u*(*u*<*t*), will be restored by time *t* and will not fail again until time *t*+*τ*.

Thus, in order to calculate the main indicators of RECS reliability, it is necessary to know the law of system operation duration distribution before failure (or the corresponding density - $f_0(t)$) and the parameter of the recovery flow $\omega_B(t)$, found from the integral equation (13) through the recovery time distribution density $f_R(t)$.

On the other hand, to analyze the alternating recovery process, it is necessary to know the density of the uptime $f_0(t)$ and recovery time $f_R(t)$ distribution, while equation (13) can be used with this law of change $\omega_R(t)$ to find the recovery time distribution density $f_R(t)$.

An exact analytical solution of the Volterra equation of the second kind is possible only in some special cases.

Consider the options for obtaining a numerical solution of the Volterra equation.

Option 1. In this case, the traditional approach [8] is used, which consists in decomposing the core of the resolvent equation into a series. Let's evaluate its versatility.

Let's rewrite the expression of the Volterra equation (13) as follows

$$f_R(t)=\omega_R(t)+\lambda\int_0^t K(t,\tau) f(\tau)\,d\tau. \qquad (15)$$

Here *λ*=*-1*, and

$$K(t,\tau)=\int_0^{t-\tau} \omega_R(u) f_0(t-\tau-u)\,du.$$

The kernel of the equation (resolvent) is represented as an expression

$$K(t,\tau)=\sum_{n=0}^{\infty}K_{n+1}(t,\tau),$$

where $K_n(t,\tau)$ – iterated kernels obeying the recurrence relation

$$K_1(t,\tau)=K(t,\tau),$$

$$K_{n+1}(t,\tau)= =\int_0^t K(t,\tau)K_n(t,\tau)d\tau,\quad n=1,2,\ldots. \tag{16}$$

We will present the desired solution in the form of infinite series

$$f_R(t)=f_0(t)+\lambda f_1(t)+ +\lambda^2 f_2(t)+\ldots+\lambda^n f_n(t)+\ldots \tag{17}$$

After the transformations, we get

$$f_R(t)=\omega_R(t)+ +\sum_{\gamma=1}^{\infty}\lambda^{\gamma}\int_0^t K_{\gamma}(t,\tau)\omega_R(\tau)d\tau. \tag{18}$$

In order to evaluate the effectiveness of the described method of restoring an unknown law of the distribution of recovery time, a computational experiment was conducted. During the experiment, the reliability behavior of *M* homogeneous TEZ was simulated (*M*=50 was assumed).

Random recovery time durations were formed in accordance with the test distribution density $f_R(t)$. Rayleigh's law was chosen as a test distribution law. Random operating time to failure was assumed to be exponentially distributed.

Data on failures and subsequent recoveries were processed in order to calculate the law of change in the parameter of the recovery flow $\hat{\omega}(t)$.

The empirical function obtained in this case was $\hat{\omega}_R(t)$ further used to calculate the density of the recovery time distribution.

However, the functions obtained in this case $\hat{f}(t)$ are not distribution densities (they are not normalized and have negative values in the distribution area).

Therefore, an attempt was made to modernize the traditional methodology.

The resulting empirical function is $\hat{\omega}_R(t)$ pre-smoothed. To form a smoothed estimate of the $\hat{\omega}_{SMTH}(t)$ function, the $\hat{\omega}(t)$ cubic polynomial approximation using the least squares method was used. But the $\hat{\omega}_{SMTH}(t)$ functions obtained as a result of substitution into equation (12) and its solution are $f_{SMTH}(t)$ also not normalized and are negative in the distribution domain.

Thus, significant limitations of using the traditional method of obtaining a numerical solution of the Volterra equation of the second kind are revealed.

Option 2. Using parameterization technology. Due to the fact that the traditional approach of obtaining a solution to the Volterra equation of the second kind is not universal enough, a different approach is proposed. To find the density of the recovery time distribution, we use parameterization technology. In this case, we will look for the required density in some class of distributions.

The requirements for the analytical description of such a density consist in the possibility of changing its statistical characteristics in a wide range by varying the parameters.

In practice, [3, 5, 7, 10] several different approaches are used to obtain such descriptions, but the necessary requirements are met by the distribution density function, called the φ - distribution:

$$\phi(x)=A\cdot\left[1+\theta_4\cdot(x-\theta_1)^2/(2\theta_2^2)\right]\times \times\exp\left[-(x-\theta_1)^2/(2\theta_2^2)\cdot(\rho+\theta_3\cdot\operatorname{sgn}(x-\theta_1))\right]. \tag{19}$$

Here is:

A – normalizing coefficient,

θ_1 – parameter that characterizes mathematical expectation x,

θ_2 – parameter that characterizes variance x,

θ_3 – parameter that characterizes asymmetry x,

θ_4 – parameter that characterizes kurtosis x,

ρ – parameter that defines the possibilities of $\phi-$ distribution by asymmetry.

If $\theta_3=0,\ \theta_4=0\ and\ \rho=1,$

then $\varphi(x)$ is a normal distribution. It follows from relation (19) that the values of the $\varphi-$distribution for any values of the parameters included in it are non-negative.

The multiplier A is found from the normalization condition:

$$\int_{-\infty}^{\infty}\phi(x)dx=1. \tag{20}$$

At this

$$A\cdot\int_{-\infty}^{\infty}\left(\begin{array}{c}\left[1+\theta_4\cdot(x-\theta_1)^2/(2\theta_2^2)\right]\times\\ \times\exp\left[\begin{array}{c}-(x-\theta_1)^2/(2\theta_2^2)\times\\ \times(p+\theta_3\cdot\operatorname{sgn}(x-\theta_1))\end{array}\right]\end{array}\right)dx=1. \tag{21}$$

From here, after simple but cumbersome transformations, we get

$$A=\left\{\theta_2\sqrt{\pi}\times\left[\begin{array}{c}\dfrac{1}{\sqrt{2(\rho+\theta_3)}}\left(1+\dfrac{\theta_4}{2(\rho+\theta_3)}\right)+\\ +\dfrac{1}{\sqrt{2(\rho-\theta_3)}}\left(1+\dfrac{\theta_4}{2(\rho-\theta_3)}\right)\end{array}\right]\right\}^{-1}. \tag{22}$$

For unimodality $\phi(x)$, we require that it increases on the interval $[-\infty;m]$, i.e. $\frac{d\phi(x)}{dx}>0,$ if $x\in[-\infty;m]$, and decreases on the interval $[m;+\infty]$, i.e. $\frac{d\phi(x)}{dx}<0,$ if $x\in[m;+\infty]$, where $m=\theta_1$ is the mode of distribution x. Then after analyzing the expression of the first derivative we get

$$\left[1+\theta_4\cdot(x-\theta_1)^2/\left(2\theta_2^2\right)\right]\cdot(\rho-|\theta_3|)>\theta_4. \quad (23)$$

Condition (23) should be satisfied for all $x\in(-\infty;+\infty)$; and since $\theta_4>0$ u $|\theta_3|<\rho,$ in particular, and for $x=\theta_1,$ which the multiplier $\left[1+\theta_4\frac{(x-\theta_1)^2}{2\theta_2^2}\right]$ takes the smallest value equal to 1. Hence the unimodality condition is follows:

$$\theta_4+|\theta_3|<\rho. \quad (24)$$

We will investigate the possibility of using the $\phi-$distribution to find the density of the recovery time distribution satisfying equation (12). Let's consider an optimization problem. For the given functions $\omega_B(t)$ and $f_0(t)$, describing the law of change of the recovery flow and the density of the distribution of the operating time to failure, it is necessary to find such a fixed set of parameters $(\theta_1,\theta_2,\theta_3,\theta_4)$ defining the $\phi-$distribution so as to minimize

$$J=\int_0^t\left[\begin{array}{c}\omega_B(t)-\phi(\theta_1,\theta_2,\theta_3,\theta_4)-\\ -\int_0^t\left(\int_0^{t-\tau}\omega_B(u)\,f_0(t-\tau-u)\,du\right)\times\\ \times\phi(\theta_1,\theta_2,\theta_3,\theta_4)\,d\tau\end{array}\right]^2 dt. \quad (25)$$

The objective functional (25) is the integral of the discrepancy square between the observed law of change of the recovery flow parameter and the one calculated in accordance with (13) description of this law corresponding to a fixed set of parameters $(\theta_1,\theta_2,\theta_3,\theta_4)$.

The resulting optimization problem was solved by the modified Nelder–Meade method. The Nelder–Mead method is a development of the simplex method of Spendl, Hext and Himsworth for finding the minimum of functions of n variables. In this case, a set of unknown parameters $(\theta_1,\theta_2,\theta_3,\theta_4)$ determines the optimization procedure of the function n=4 variables. The essence of the method consists in comparing the values of the objective function J. $(\theta_1,\theta_2,\theta_3,\theta_4)$ At the (n+1) vertices of the simplex and moving the simplex in the direction of the optimal point using an iterative procedure.

The original simplex D_o is introduced so that the coordinates of the vertices are determined by the table (one of the vertices is at the origin):

$$D_0=\underbrace{\begin{array}{ccccc}\left(\begin{array}{ccccc}0 & d_1 & d_2 & \ldots & d_2\\ 0 & d_2 & d_1 & \ldots & d_2\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ 0 & d_2 & d_2 & \ldots & d_1\end{array}\right)\\ \underline{\theta}_1 \quad \underline{\vartheta}_2 \quad \underline{\vartheta}_3 \quad \ldots \quad \underline{\vartheta}_{n+1}\end{array}}_{(n+1)\,\text{dots}},$$

$$d_1=\frac{t}{n\sqrt{2}}(\sqrt{n+1}+n-1),\ d_2=\frac{t}{n\sqrt{2}}(\sqrt{n+1}-1),$$

t is some selected number.

At each iteration for the current simplex, the coordinates of the center of gravity of the figure resulting from the removal of the vertex are found $\underline{\vartheta}_{n+1}$:

$$\underline{c}=\frac{1}{n}\cdot\sum\nolimits_{j=1}^{n}\underline{\vartheta}_j \quad (26)$$

Further, as is known, the simplex moves towards the optimum using three operations – reflection, stretching and compression, providing at each *kth* iteration $J_k(\theta_1,\theta_2,\theta_3,\theta_4)<J_{k-1}(\theta_1,\theta_2,\theta_3,\theta_4)$

The criterion for stopping a computational procedure has the form:

$$K=\delta\sqrt{\frac{1}{n+1}\sum_{j=1}^{n+1}\left[J\left(\underline{\vartheta}_j\right)-J(\underline{c})\right]^2}+$$
$$+(1-\delta)\sqrt{\frac{1}{n+1}\sum_{j=1}^{n+1}(\underline{\vartheta}_j-\underline{c})^T(\underline{\vartheta}_j-\underline{c})},\ \delta\in[0;1]. \quad (27)$$

The stop criterion J is composite. At the same time, its components have different weights depending on the nature of behavior of the optimized function in the vicinity of the extremum. If the optimized function changes in the "deep depression" type in the region of the extremum, then the first term makes a greater contribution to the numerical value of the criterion K, and the second one decreases rapidly. On the contrary, if the optimized function changes in the "flat plateau" type, then the first term quickly becomes small and therefore the second term contributes more to the value of the criterion K.

To prevent premature triggering of the stop criterion near the optimum and neutralize the so-called “ravine effect” for the function $J_k(\theta_1,\theta_2,\theta_3,\theta_4)$ at the *kth* iteration, an improvement technique was used, the essence of which is as follows. After the stop criterion is triggered, a new simplex is constructed above the center of gravity of the compressed simplex, the dimensions of which correspond to the original simplex. Let the coordinates of the center of gravity of the compressed simplex form a vector $\underline{\vartheta}=\begin{pmatrix}\vartheta_1\\ \vdots\\ \vartheta_n\end{pmatrix}$.

Is the coordinates of a point $\underline{\hat{A}}=\begin{pmatrix}\hat{a}_1\\ \vdots\\ \hat{a}_n\end{pmatrix}$ such that the center of gravity of a simplex with an edge length equal to *t*, using the vertex $\underline{\hat{A}}$ as the starting point, would coincide with $\underline{\hat{X}}$. The coordinate matrix of the specified simplex has the form

$$D_{\hat{A}} = \begin{pmatrix} \hat{a}_1 & \hat{a}_1 + d_1 & \hat{a}_1 + d_2 & \cdots & \hat{a}_1 + d_2 \\ \hat{a}_2 & \hat{a}_2 + d_2 & \hat{a}_2 + d_1 & \cdots & \hat{a}_2 + d_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \hat{a}_n & \hat{a}_n + d_2 & \hat{a}_n + d_2 & \cdots & \hat{a}_n + d_1 \end{pmatrix}. \quad (28)$$

The coordinates of the center of gravity of this simplex form a vector

$$\underline{C}_{\hat{A}} = \begin{pmatrix} \hat{a}_1 + \dfrac{1}{n+1}(d_1 + (n-1)d_2) \\ \hat{a}_2 + \dfrac{1}{n+1}(d_1 + (n-1)d_2) \\ \vdots \\ \hat{a}_n + \dfrac{1}{n+1}(d_1 + (n-1)d_2) \end{pmatrix}. \quad (29)$$

Now we will $\underline{\hat{A}}$ find the coordinates of the point from the equality $\underline{C}_{\hat{A}} = \underline{\vartheta}$, from where

$$\begin{aligned} \hat{a}_1 &= \hat{x}_1 - \frac{1}{n+1}(d_1 + (n-1)d_2) = \hat{x}_1 - s, \\ \hat{a}_2 &= \hat{x}_2 - \frac{1}{n+1}(d_1 + (n-1)d_2) = \hat{x}_2 - s, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \hat{a}_n &= \hat{x}_n - \frac{1}{n+1}(d_1 + (n-1)d_2) = \hat{x}_n - s, \end{aligned} \quad (30)$$

where $s = \dfrac{d_1 + (n-1)d_2}{n+1}$.

Substituting the calculated values $\hat{a}_1, \hat{a}_2, ..., \hat{a}_n$ into expression (28), we obtain the required simplex, using which the minimum search procedure continues. This procedure is considered complete if, after the next compression, the algorithm leads to a point from which the distance to the point of the previous compression does $(\theta_{1K}, \theta_{2K}, \theta_{3K}, \theta_{4K})$ not exceed some sufficiently small δ.

The results of solving the problem of finding a set $(\theta_1, \theta_2, \theta_3, \theta_4)$ are shown in Fig. 1. The resulting function $\phi(t)$ is the desired distribution density, since it satisfies the normalization condition, that is $\lim\limits_{t\to\infty} F_1(t) = \lim \int_0^t \phi(t)\,dt = 1$, as illustrated by the graph in Fig. 2. Here is also a graph of the function for comparison $F_2(t) = \int_0^t f_{SMTH}(t)\,dt$.

Thus, using the $\phi-$distribution allows to calculate the recovery time distribution density based on data on the law of change in the parameter of the failure flow.

After the density of the recovery time distribution is formed according to the data on the law of change in the parameter of the failure flow, the average recovery times of the RECS operability are calculated.

So, let the average recovery time of RECS working state in case of a failure of *j*-type element with an unlimited supply of spare parts is $T_{RO}^{(j)}$. If there is no *j*-type element in the spare parts (or it is not provided), then RECS recovery is carried out at the expense of the elements of the spare parts SPTA-G and the average recovery time of the functional state will be

$$T_R^{(j)} = T_{RO}^{(j)}(1 - P_O^{(j)}) + T_{AD} P_O^{(j)}, \quad (31)$$

where $P_O^{(j)}$ is the probability that there are no *j*-type elements in the spare parts kit at any given time, T_{AD} is the average delivery time of the replacement element from SPTA-G to the spare parts (the average RECS recovery time due to the spare parts).

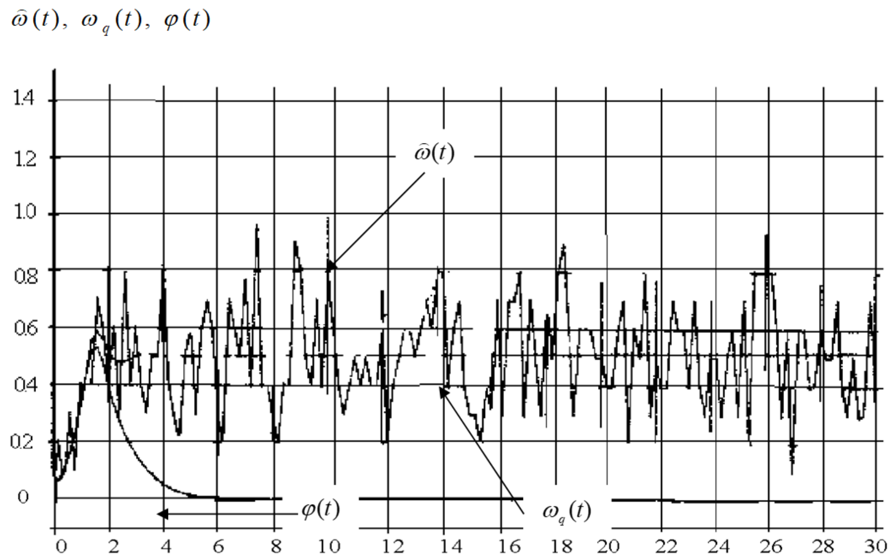

Fig. 1. Calculation $\hat{f}(t)$ Using ϕ -Distributions

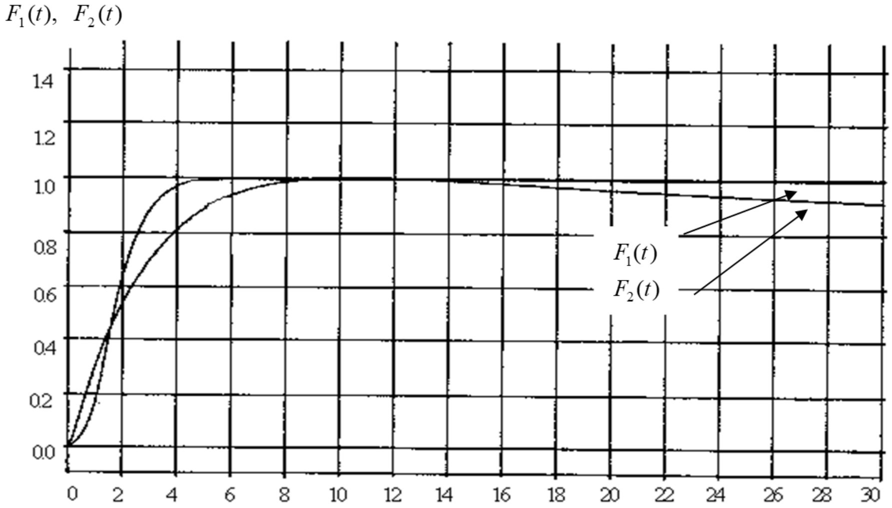

Fig. 2. Checking the Normalization of the Distribution Law

The ratio (31) is obtained under the assumption that the group set of SPTA-G is an inexhaustible source of replenishment.

In the absence of a replacement element in SPTA-G, the RECS restoration is carried out at the expense of the element returned from the repair body. Then the average recovery time of the RECS can be written as follows

$$T_R^{(j)} = T_{RO}^{(j)}\left(1 - P_O^{(j)}\right) + \\ + T_{AD} P_O^{(j)}\left(1 - P_G^{(j)}\right) + T_P^{(j)} P_O^{(j)} P_G^{(j)}, \qquad (32)$$

where $P_G^{(j)}$ - the probability that at any given time there are no *j*-type elements in the SPTA-G kit, $T_P^{(j)}$ - the average repair time in the repair body of the *j*-type element.

In conclusion, we note that the calculated recovery time density is uniquely determined by the nature of the data on the failure flow, however, the specifics of the structure and parameters of the spare parts system, heterogeneity, and multi-nomenclature of the source data are not fully taken into account [18, 19], which does not allow to consider the described methodology as a comprehensive method for analyzing the effectiveness of RECS groupings with unreliable elements in depending on the parameters and structure of SCSSSEP.

Consideration of these circumstances determines the direction of further research.

Conclusions

1. Mathematical model of the recovery process is proposed. A universal four-parameter distribution is used to describe random processes occurring in non-Markovian systems.

2. The resulting model makes it possible to determine the parameter of the recovery flow by solving the Volterra integral equation with a difference kernel.

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Модель оцінки процесів відновлення немарківських систем з урахуванням ненадійності елементів за довільних законів розподілу

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Анотація. Предмет дослідження – надійність немарківських систем, що відновлюються, функціонування яких описується довільними законами розподілу. **Метою статті** є розробка математичної моделі функціонування сучасних комп’ютерних систем за довільними законами розподілу тривалості перебування в кожному із станів з урахуванням відновлення системи та забезпеченості запасними елементами. **Основним завданням** є розробка адекватної моделі процесу функціонування системи з урахуванням немарковського характеру процесів, що відбуваються в системі, її можливої великої розмірності та наявності ієрархічної системи відновлення. На основі цієї моделі розроблено метод розрахунку щільності розподілу часу відновлення системи. Водночас запропоновано універсальний чотирипараметричний розподіл для опису випадкових процесів, що відбуваються в системі. Використовуючи цю апроксимацію, розрахунок шуканого параметру потоку відновлення виконується шляхом розв’язання інтегрального рівняння Вольтерра з різницевим ядром.

Ключові слова: відновлення немарківських систем; математична модель надійності; щільність розподілу часу відновлення.