Adaptive control methods

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ON THE STABILITY OF THE STABILIZED MOTION OF A CARRIER ROCKET WITH A LIQUID-PROPELLANT JET ENGINE AND AN ONBOARD DIGITAL COMPUTER IN THE STABILIZATION LOOP

Abstract. The problem of choosing the values of the variable parameters of the digital stabilizer of the cosmic stage of a carrier rocket with a liquid-propellant jet engine and an onboard digital computer in the stabilization loop, which ensures stable movement of the stage along the entire active section of the flight trajectory, is considered. The effect of the stabilizer quantization period on the stability region of a closed-loop stabilization system is considered. It is recommended to choose the intersection of stability regions corresponding to uniformly distributed moments of time along the active section of the stage flight trajectory as acceptable values for the variable parameters of the stabilizer of a non-stationary stabilization object.

Keywords: cosmic stage of carrier rocket, digital stabilizer, variable stabilizer parameters, active section of flight trajectory; region of stability in the space of variable parameters.

Introduction

Problem statement. On December 17, 1956, the Decree of the Council of Ministers (CM) of the USSR "On the creation of an intercontinental ballistic rocket (ICBR) R-16 (8K64)" was issued. The development of the R-16 ICBR was entrusted to OKB-586 (Dnepropetrovsk), headed by Chief Designer M.K. Yangel, who went to the USSR Council of Ministers with a proposal to create an experimental design bureau for rocket control systems in Kharkov, headed by Chief Designer B.M. Konoplev. It was planned to entrust this design bureau, under the general scientific supervision of Academician of the Academy of Sciences of the USSR B.N. Petrov, with the development of a fully autonomous control system for the R-16 rocket. On April 11, 1959, the Decree of the Council of Ministers of the USSR on the organization of the Special Design Bureau No. 692 (Kharkov) was issued. OKB-692 was instructed to take over the functions of the head integrated research and development enterprise for the development of control systems for rockets created in OKB-586.

The first task of OKB-692 was the development of the control system for the R-16 rocket, which was replete with a number of tragic events, the main of which was the unauthorized launch of the second-stage liquid-propellant rocket engine in preparing the rocket for launch on October 24, 1960.

The disaster led to the death of 92 people, including Commander-in-Chief of the USSR Rocket Forces, Chief Marshal of Artillery M. I. Nedelin, Chief Designer of OKB-692 B. M. Konoplev, Deputy Chief Designers of OKB-586 V. A. Kontsevoi and L. A. Berlin. More than 50 people received severe burns, poisoning with toxic fuel components and injuries of varying severity.

By decision of the Government of the USSR, work on the creation of the R-16 rocket was continued subject to a radical revision of the organization of work on the design, production and development of rocket technology in OKB-586 and OKB-692.

The subsequent several launches of the R-16 rocket ended in accidents due to the loss of stability of the second stage of the rocket. The telemetric data of the second stage flight pattern led to the assumption that the loss of its stability is associated with the influence of fluctuations of the free surfaces of the fuel and oxidizer in the stage fuel tanks on the movement of the rocket body.

The problem of stability of the R-16 rocket was solved by a group of OKB-692 scientists consisting of A. I. Gudimenko, Ya. E. Aizenberg, V. N. Romanenko and V. S. Stoletny with the involvement of scientists NII-88 (Moscow) B. I. Rabinovich and G. N. Mikishev, as well as NII-4 (Moscow) G. S. Narimanov and M. M. Bordyukov. As a result, a school of scientists in the field of dynamics was created in OKB-692, later supplemented by V. V. Sorokobatko, V. A. Bataev, and V. G. Sukhorebry, who developed the theoretical basis for designing systems for stabilizing rocketry objects with cavities filled with fuel and oxidizer [1].

As a result, a mathematical model of perturbed motion was created for the two-stage rocket R-16, which takes into account the fluctuations of the fuel and oxidizer in the tanks of the operating stage, on the basis of which the values of the parameters of the stabilization machine were chosen, ensuring the stability of the rocket throughout the entire flight path [2]. The R-16 rocket as part of the 8K64 rocket complex was put into service in 1961 and, together with the 8K67 and 8K69 complexes, until 1977 formed the basis of the USSR nuclear rocket defense.

The theoretical foundations of mathematical modeling of the perturbed movement of ICBR, developed in the process of creating the R-16 rocket, were used to create all subsequent types of rockets: R-36 and R-36P with analog-type stabilizers, as well as R-36M, R-36M UTTKh and R-36M2 with on-board digital computers in

the stabilization circuit. On the basis of R-36 rockets, as part of the 8K69 rocket complex, cosmic rocket complexes 11K69 (Cyclone-2 and Cyclone-3) were created, and the Cyclone-3 carrier rocket is a three-stage with a third (cosmic) stage C5M. The mathematical model of the angular perturbed motion of the C5M cosmic stage (CS) is given in [3,4] and has the following form in the yaw channel:

$$
\begin{aligned}
&\ddot{\psi}(t) = a'_{\psi\psi}\,\dot{\psi}(t) + a''_{\psi\beta_1}\,\ddot{\beta}_1(t) + \\
&\qquad + a''_{\psi\beta_2}\,\ddot{\beta}_2(t) + a_{\psi\delta}\delta(t); \\
&\ddot{\beta}_1(t) + 2\xi_1\omega_1\dot{\beta}_1(t) + \omega_1^2\beta_1(t) = a''_{\beta_1\psi}\,\ddot{\psi}(t); \\
&\ddot{\beta}_2(t) + 2\xi_2\omega_2\dot{\beta}_2(t) + \omega_2^2\beta_2(t) = a''_{\beta_2\psi}\,\ddot{\psi}(t),
\end{aligned}
\tag{1}
$$

where $\psi(t)$ is the angle of rotation of the longitudinal axis of the stage relative to the plane of the orbit; $\beta_1(t)$, $\beta_2(t)$ are the angles of deviation of the free surface of the fuel and oxidizer, respectively, from the unperturbed position; $\delta(t)$ is the angle of deviation of the axis of the marching liquid-propellant jet engine from the longitudinal axis of the stage in the channel of yaw; $a'_{\psi\psi}$, $a_{\psi\delta}$ are time-varying coefficients characterizing the angular motion of the "solidified" cosmic stage; $a''_{\psi\beta_1}$, $a''_{\psi\beta_2}$ are the coefficients of influence of fluctuations of fuel and oxidizer on the angular motion of the rocket; $a''_{\beta_1\psi}$, $a''_{\beta_2\psi}$ are the coefficients of influence of the angular motion of the stage on fluctuations of the liquid in the fuel and oxidizer tanks; ξ_1, ξ_2 – damping coefficients of the vibrations of the fuel and oxidizer in the tanks of the stage; ω_1, ω_2 are the natural frequencies of free vibrations of the fuel and oxidizer in the tanks of the stage.

Further, when presenting the main material of the article, the mathematical model (1) will be used as an example, which is associated with the following considerations [5]:

- At present, the Cyclone-4 cosmic rocket complex with the third cosmic stage and onboard computer in the stabilization loop is being developed;
- CS C5M has a short length, which allows neglecting the vibrations of the body of the CS;
- the shape of the fuel and oxidizer tanks of CS C5M is such that in the mathematical model of perturbed motion (1) it is sufficient to take into account only the first tone of oscillations of the free surfaces of the fuel and oxidizer;
- the flight of the C5M CS, which is the third stage of the Cyclone-3 carrier rocket, takes place in the upper layers of the atmosphere, where random external disturbances are of a stationary nature.

The listed features of C5M CS determine the simplicity of its mathematical model (1) while maintaining the basic dynamic characteristics of a solid body with cavities filled with liquid. The R-16, R-36 and R-36P rockets have analog automatic stabilization devices, the electronic control unit of which implements a proportional-differential stabilization law, which for the angular motion of the yaw channel is written as

$$
u_\psi(t) = k_\psi \tilde{u}_\psi(t) + k_{\dot{\psi}} \tilde{u}_{\omega\psi}(t),
\tag{2}
$$

where k_ψ and $k_{\dot{\psi}}$ are variable coefficients of the stabilization automaton; $\tilde{u}_\psi(t)$ is the output signal of the gyroscopic unit (GU) of the yaw channel of the triaxial gyro-stabilized platform (TGSP); $\tilde{u}_{\omega\psi}(t)$ is the output signal of the gyroscopic angular velocity sensor (GAVS) of the yaw channel.

Neglecting the intrinsic dynamics of the GU and GAVS, the stabilization law (2) generated by the analog stabilization automaton can be written as

$$
u_\psi(t) = k_\psi k_\Gamma \psi(t) + k_{\dot{\psi}} k_{\mathcal{Д}} \omega_\psi(t),
\tag{3}
$$

where k_Γ and $k_{\mathcal{Д}}$ are the gains of the GU and GAVS, respectively

The stability of a closed-loop stabilization system is ensured by an appropriate choice of variable parameters k_ψ, $k_{\dot{\psi}}$ relation (3) using the Nyquist stability criterion, according to which the amplitude-phase-frequency characteristic (APFC) of an open-loop stabilization system should not cover the critical point (–1.0) [2] . Further, for building the APFC of the open-loop stabilization system of the carrier rocket, as well as the stability regions in the planes $(k_\psi, k_{\dot{\psi}})$ of variable stabilizer parameters (3) at NPO KHARTRON (former OKB-692) and at Yuzhnoye Design Bureau (former OKB-586) began to use the MathCAD application package [6].

In the third-generation rockets R-36M (15A14), R-36M UTTKh (15A18) and R-36M2 (15A18M), the stabilization automaton contains an onboard computer in the stabilization circuit. On November 18, 2021, the space agencies of Ukraine and Canada signed a joint statement on partnership, and on November 19, the laying of the first stone of the launch site of the Ukrainian-Canadian project to create a spaceport on the Atlantic coast of New Scotland for the Cyclone-4M carrier rocket, developed and manufactured by state-owned enterprises of Ukraine KB Yuzhnoye and PO Yuzhny Machine-Building Plant.

Cyclone-4M is a two-stage carrier rocket designed to launch the third (cosmic) stage with a mass of 3700 kg into an orbit with a height of 450 km or a CS with a mass of 3000 kg into an orbit with a height of 1200 km. Stabilization of all three stages is carried out by a digital stabilization automaton with an onboard computer in the control loop. The onboard computer performs high-frequency interference filtering of the output signals of the GU and GAVS after their conversion into lattice functions $\tilde{u}_\psi[nT]$ and $\tilde{u}_{\omega\psi}[nT]$ the analog-to-code converter (ACC). Filtering is carried out by implementing on-board computer algorithms for digital recursive Butterworth filters [7] of the second and third orders [8]. In the onboard computer, a stabilization algorithm is also formed in the form

$$
u_\psi[nT] = k_\psi \hat{u}_\psi[nT] + k_{\dot{\psi}} \hat{u}_{\omega\psi}[nT],
\tag{4}
$$

where $\hat{u}_\psi[nT]$ and $\hat{u}_{\omega\psi}[nT]$ are lattice functions corresponding to the output signals of the GU and GAVS and filtered from high-frequency noise by digital recursive Butterworth filters, and

$$Z\{\hat{u}_\psi[nT]\}=W_Б(z)Z\{\tilde{u}_\psi[nT]\};$$
$$Z\{\hat{u}_{\omega\psi}[nT]\}=W_Б(z)Z\{\tilde{u}_{\omega\psi}[nT]\}, \quad (5)$$

where $Z\{\}$ is the Z-transformation of the lattice function $\{\}$; $W_Б(z)$ is the discrete transfer function of the low-frequency Butterworth filter. If the onboard computer implements second-order filters, then their discrete transfer function looks like this:

$$W_Б(z)=\frac{a_0+a_1z^{-1}+a_2z^{-2}}{b_0+b_1z^{-1}+b_2z^{-2}}. \quad (6)$$

If the onboard computer implements third-order filters, then their discrete transfer function is written

$$W_Б(z)=\frac{a_0(1+3z^{-1}+3z^{-2}+z^{-3})}{b_0+b_1z^{-1}+b_2z^{-2}+b_3z^{-3}}. \quad (7)$$

We will assume that Butterworth filters effectively suppress high-frequency noise, primarily determined by the intrinsic dynamics of the GU and GAVS sensors. Then one can write

$$\hat{u}_\psi[nT]=k_Г\psi[nT]; \quad \hat{u}_{\omega\psi}[nT]=k_Д\omega_\psi[nT].$$

Stabilization algorithm (4) in this case can be reduced to the form

$$u_\psi[nT]=k_\psi k_Г\psi[nT]+k_{\dot\psi}k_Д\omega_\psi[nT]. \quad (8)$$

The control signal in the form of a lattice function (8) is fed to the input of the code-to-analogue converter (CAC), which converts the lattice function into a piecewise constant function in accordance with the algorithm

$$u_\psi(t)=\begin{cases}u_\psi[nT] & \text{при } nT<t\le(n+1)T;\\ u_\psi[(n+1)T] & \text{при } (n+1)T<t\le(n+2)T.\end{cases} \quad (9)$$

A closed-loop stabilization system can be represented as a connection with feedback of the continuous part of the system (CPS) and its discrete part (DPS). The perturbed motion of the CPS is described by a system of differential equations (1), and the operation of the DPS is described by relations (4)–(9) in lattice functions, so the direct use of the MathCAD software package to construct the APFC of an open-loop stabilization system and the stability regions of a closed-loop system is difficult.

The purpose of this article is to bring the mathematical model of a closed-loop stabilization system of a carrier rocket with a liquid-propellant rocket jet engine and an on-board computer in the stabilization loop, using the example of the C5M cosmic stage of the Cyclone-3 carrier rocket to a form convenient for solving the problem of parametric synthesis of the system using the MathCAD software package, as well as the study of the influence on the stability of a closed system of stabilizing the oscillations of the fuel and oxidizer in the fuel tanks of the CS and period of quantization of the onboard computer.

Main material

Let us write the system of differential equations (1) in the vector-matrix form

$$A\ddot{X}(t)=B\cdot\dot{X}(t)+C\cdot X(t)+D\cdot\delta(t), \quad (10)$$

where the corresponding vectors and matrices are

$$X(t)=\begin{bmatrix}\psi(t)\\ \beta_1(t)\\ \beta_2(t)\end{bmatrix}; \quad A=\begin{bmatrix}1 & -a''_{\psi\beta_1} & -a''_{\psi\beta_2}\\ -a''_{\beta_1\psi} & 1 & 0\\ -a''_{\beta_2\psi} & 0 & 1\end{bmatrix};$$

$$B=\begin{bmatrix}a'_{\psi\psi} & 0 & 0\\ 0 & -2\xi_1\omega_1 & 0\\ 0 & 0 & -2\xi_2\omega_2\end{bmatrix};$$

$$C=\begin{bmatrix}0 & 0 & 0\\ 0 & -\omega_1^2 & 0\\ 0 & 0 & -\omega_2^2\end{bmatrix}; D=\begin{bmatrix}a_{\psi\delta}\\ 0\\ 0\end{bmatrix}.$$

The differential equation (10) can be solved with respect to the vector of highest derivatives

$$\ddot{X}(t)=A^{-1}B\cdot\dot{X}(t)+A^{-1}C\cdot X(t)+A^{-1}D\cdot\delta(t). \quad (11)$$

The matrix A^{-1} is defined by the following formula

$$A^{-1}=$$

$$=\frac{1}{\det A}\begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix}^T=\frac{1}{\det A}\begin{bmatrix}A_{11} & A_{21} & A_{31}\\ A_{12} & A_{22} & A_{32}\\ A_{13} & A_{23} & A_{33}\end{bmatrix},$$

where $\det A=1-a''_{\beta_2\psi}a''_{\psi\beta_2}-a''_{\beta_1\psi}a''_{\psi\beta_1}$;

$$A_{11}=1; A_{12}=a''_{\beta_1\psi}; A_{13}=a''_{\beta_2\psi}; A_{21}=a''_{\psi\beta_1};$$

$$A_{22}=1-a''_{\beta_2\psi}a''_{\psi\beta_2}; A_{23}=a''_{\beta_2\psi}a''_{\psi\beta_1};$$

$$A_{31}=a''_{\psi\beta_2}; A_{32}=a''_{\beta_1\psi}a''_{\psi\beta_2}; A_{33}=1-a''_{\beta_1\psi}a''_{\psi\beta_1}.$$

Let us introduce into consideration the state vector of the stabilization object

$$Y(t)=\begin{bmatrix}y_1(t)\\ y_2(t)\\ y_3(t)\\ y_4(t)\\ y_5(t)\\ y_6(t)\end{bmatrix}=\begin{bmatrix}\psi(t)\\ \dot\psi(t)\\ \beta_1(t)\\ \dot\beta_1(t)\\ \beta_2(t)\\ \dot\beta_2(t)\end{bmatrix}.$$

As a result, the system of differential equations (11) can be written in the Cauchy normal form

$$\dot{Y}(t)=P\cdot Y(t)+Q\cdot\delta(t), \quad (12)$$

where the matrices P and Q are equal

$$P=\begin{bmatrix}0 & 1 & 0 & 0 & 0 & 0\\ 0 & p_{22} & p_{23} & p_{24} & p_{25} & p_{26}\\ 0 & 0 & 0 & 1 & 0 & 0\\ 0 & p_{42} & p_{43} & p_{44} & p_{45} & p_{46}\\ 0 & 0 & 0 & 0 & 0 & 1\\ 0 & p_{62} & p_{63} & p_{64} & p_{65} & p_{66}\end{bmatrix}; \quad Q=\begin{bmatrix}0\\ q_2\\ 0\\ q_4\\ 0\\ q_6\end{bmatrix}.$$

The elements of the matrices P and Q at the same time constitute

$$p_{22} = \frac{A_{11}a'_{\psi\psi}}{\det A}; \quad p_{23} = -\frac{A_{21}\omega_1^2}{\det A}; \quad p_{24} = -\frac{2A_{21}\xi_1\omega_1}{\det A};$$

$$p_{25} = -\frac{A_{31}\omega_2^2}{\det A}; \quad p_{26} = -\frac{2A_{31}\xi_2\omega_2}{\det A};$$

$$p_{42} = \frac{A_{12}a'_{\psi\psi}}{\det A}; \quad p_{43} = -\frac{A_{22}\omega_1^2}{\det A}; \quad p_{44} = -\frac{2A_{22}\xi_1\omega_1}{\det A};$$

$$p_{45} = -\frac{A_{32}\omega_2^2}{\det A}; \quad p_{46} = -\frac{2A_{32}\xi_2\omega_2}{\det A};$$

$$p_{62} = \frac{A_{13}a'_{\psi\psi}}{\det A}; \quad p_{63} = -\frac{A_{23}\omega_1^2}{\det A}; \quad p_{64} = -\frac{2A_{23}\xi_1\omega_1}{\det A};$$

$$p_{65} = -\frac{A_{33}\omega_2^2}{\det A}; \quad p_{66} = -\frac{2A_{33}\xi_2\omega_2}{\det A};$$

$$q_2 = \frac{A_{11}a_{\psi\delta}}{\det A}; \quad q_4 = \frac{A_{12}a_{\psi\delta}}{\det A}; \quad q_6 = \frac{A_{13}a_{\psi\delta}}{\det A}.$$

Neglecting the own dynamics of the cosmic stage executive device, we write the control algorithm (8) in the form

$$\delta[nT] = k_{HO}\{k_\psi k_\Gamma y_1[nT] + k_\psi k_Д y_2[nT]\}, \quad (13)$$

where k_{HO} is the gain of the executive device.

Let us introduce the notations

$$k_{HO}k_\psi k_\Gamma = k_1; \quad k_{HO}k_\psi k_Д = k_2.$$

As a result, algorithm (13) takes the following form

$$\delta[nT] = k_1 y_1[nT] + k_2 y_2[nT].$$

Algorithm (14) will be written in the vector-matrix form

$$\delta[nT] = K \cdot Y[nT], \quad (15)$$

where the matrix of variable stabilizer parameters has the form

$$K = [k_1 \quad k_2 \quad 0 \quad 0 \quad 0 \quad 0].$$

Let us write a vector-matrix difference equation relating the initial state of system (12) with its final state at each quantization period [9]:

$$Y[(n+1)T] = \Phi(T) \cdot Y[nT] + H(T) \cdot \delta[nT], \quad (16)$$

where the matrices $\Phi(T)$ and $H(T)$ are defined by the series

$$\Phi(T) = \sum_{i=0}^{\infty} \frac{P^i T^i}{i!}; \quad H(T) = \sum_{i=0}^{\infty} \frac{P^i T^{i+1}}{(i+1)!} Q. \quad (17)$$

Modern onboard computers of carrier rocket have a fairly short quantization period T, therefore, with a sufficient degree of accuracy, we can put:

$$\Phi = E + P \cdot T; \quad H = Q \cdot T. \quad (18)$$

Let us substitute relation (15) and (18) into the right side of equation (16). As a result, we obtain the difference equation of the perturbed motion of the closed stabilization system

$$Y[(n+1)T] = [E + (P + QK)T] \cdot Y[nT]. \quad (19)$$

The characteristic equation of a closed discrete system (19) is written as

$$M(z) = \det[(P + QK)T + E \cdot (1 - z)] = 0. \quad (20)$$

Substituting the matrices P, Q, K into equation (20) and expanding the determinant (20), we obtain:

$$M(z, k_1, k_2) = (1 - z)^6 + m_1(k_2)(1 - z)^5 +$$
$$+ m_2(k_1, k_2)(1 - z)^4 + + m_3(k_1, k_2)(1 - z)^3 + \quad (21)$$
$$+ m_4(k_1, k_2)(1 - z)^2 + m_5(k_1, k_2)(1 - z) + m_6(k_1) = 0.$$

The coefficients of the characteristic equation (21) are determined by the following relations:

$$m_1(k_2) = m_{11} + m_{12}k_1;$$
$$m_2(k_1, k_2) = m_{21} + m_{22}k_2 + m_{23}k_1;$$
$$m_3(k_1, k_2) = m_{31} + m_{32}k_2 + m_{33}k_1;$$
$$m_4(k_1, k_2) = m_{41} + m_{42}k_2 + m_{43}k_1; \quad (22)$$
$$m_5(k_1, k_2) = m_{51} + m_{52}k_2 + m_{53}k_1;$$
$$m_6(k_1) = m_{63}k_1,$$

and

$$m_{11} = (p_{22} + p_{44} + p_{66})T; \quad m_{12} = q_2 T;$$

$$m_{21} = \begin{pmatrix} p_{44}p_{66} + p_{22}p_{44} + p_{22}p_{66} - p_{26}p_{62} - \\ - p_{64}p_{46} - p_{24}p_{42} - p_{65} - p_{43} \end{pmatrix} T^2;$$

$$m_{22} = -[(p_{44} + p_{66})q_2 + p_{24}q_4 + p_{26}q_6 - p_{65} - p_{43}]T^2;$$

$$m_{23} = -q_2 T^2;$$

$$m_{31} = (p_{22}p_{44}p_{66} + p_{24}p_{46}p_{62} + p_{26}p_{64}p_{42} -$$
$$- p_{44}p_{26}p_{62} - p_{22}p_{64}p_{46} - p_{24}p_{66}p_{42} -$$
$$- p_{44}p_{65} - p_{22}p_{65} + p_{62}p_{25} + p_{64}p_{45} -$$
$$- p_{22}p_{43} - p_{66}p_{43} + p_{42}p_{23} + p_{63}p_{46})T^3;$$

$$m_{32} = (p_{44}p_{66}q_2 + p_{24}p_{46}q_6 + p_{26}p_{64}q_4 -$$
$$- p_{44}p_{26}q_6 - p_{64}p_{46}q_2 - p_{24}p_{66}q_4 -$$
$$- p_{65}q_2 + p_{25}q_6 + p_{64}p_{45} - p_{43}q_2 + p_{23}q_4)T^3;$$

$$m_{33} = -[(p_{44} + p_{66})q_2 - p_{26}q_6 - p_{24}q_4]T^3;$$

$$m_{41} = -(p_{22}p_{44}p_{65} + p_{24}p_{45}p_{62} + p_{25}p_{64}p_{42} -$$
$$- p_{44}p_{25}p_{62} - p_{22}p_{64}p_{45} -$$
$$- p_{24}p_{65}p_{42} + p_{22}p_{43}p_{66} + p_{23}p_{46}p_{62} +$$
$$+ p_{26}p_{63}p_{42} - p_{43}p_{26}p_{62} -$$
$$- p_{42}p_{66}p_{23} - p_{22}p_{63}p_{46} - p_{43}p_{65} + p_{63}p_{45})T^4;$$

$$m_{42} = -(p_{44}p_{65}q_2 + p_{24}p_{45}q_6 + p_{25}p_{64}q_4 -$$
$$- p_{44}p_{25}q_6 - p_{64}p_{45}q_2 - p_{24}p_{65}q_4 +$$
$$+ p_{43}p_{66}q_2 + p_{23}p_{46}q_6 + p_{63}p_{26}q_4 -$$
$$- p_{43}p_{26}q_6 - p_{66}p_{23}q_4 - p_{63}p_{46}q_2)T^4;$$

$$m_{43} = -(p_{44}p_{66}q_2 + p_{24}p_{46}q_6 + p_{26}p_{64}q_4 -$$
$$- p_{44}p_{26}q_6 - p_{24}p_{66}q_4 - p_{64}p_{46}q_2 -$$
$$- p_{65}q_2 + p_{25}q_6 - p_{43}q_2 + p_{23}q_4)T^4;$$

$$m_{51} = (p_{22}p_{43}p_{65} + p_{23}p_{45}p_{62} + p_{25}p_{63}p_{42} - \\ - p_{43}p_{25}p_{62} - p_{42}p_{23}p_{65} - p_{22}p_{63}p_{45})T^5;$$

$$m_{52} = (p_{43}p_{65}q_2 + p_{23}p_{45}q_6 + p_{25}p_{63}q_4 - \\ - p_{43}p_{25}q_6 - p_{23}p_{65}q_4 - p_{63}p_{45}q_2)T^5;$$

$$m_{53} = (p_{44}p_{65}q_2 + p_{24}p_{45}q_6 + p_{25}p_{64}q_4 - \\ - p_{44}p_{25}q_6 - p_{64}p_{45}q_2 - p_{24}p_{65}q_4 + \\ + p_{43}p_{66}q_2 + p_{23}p_{46}q_6 + p_{63}p_{26}q_4 - \\ - p_{43}p_{26}q_6 - p_{63}p_{46}q_2 - p_{66}p_{23}q_4)T^5;$$

$$m_{63} = -(p_{43}p_{65}q_2 + p_{23}p_{45}q_6 + p_{25}p_{63}q_4 - \\ - p_{43}p_{25}q_6 - p_{23}p_{65}q_4 - p_{63}p_{45}q_2)T^6 \quad .$$

In the characteristic equation (21), we will make the replacement

$$z = \frac{1+w}{1-w} \quad (23)$$

and write a new characteristic equation with respect to a new complex variable w associated with the complex variable z by a bilinear W - transformation (23):

$$\begin{pmatrix} 64 + 32m_1(k_2) + 16m_2(k_1,k_2) + 8m_3(k_1,k_2) + \\ + 4m_4(k_1,k_2) + 2m_5(k_1,k_2) + m_6(k_1) \end{pmatrix} w^6 - \\ - \begin{pmatrix} 32m_1(k_2) + 32m_2(k_1,k_2) + 24m_3(k_1,k_2) + \\ + 16m_4(k_1,k_2) + 10m_5(k_1,k_2) + 6m_6(k_1) \end{pmatrix} w^5 + \\ + \begin{pmatrix} 16m_2(k_1,k_2) + 24m_3(k_1,k_2) + 24m_4(k_1,k_2) + \\ + 20m_5(k_1,k_2) + 15m_6(k_1) \end{pmatrix} w^4 - \quad (24) \\ - \begin{pmatrix} 8m_3(k_1,k_2) + 16m_4(k_1,k_2) + \\ + 20m_5(k_1,k_2) + 20m_6(k_1) \end{pmatrix} w^3 + \\ + (4m_4(k_1,k_2) + 10m_5(k_1,k_2) + 15m_6(k_1))w^2 - \\ - (2m_5(k_1,k_2) + 6m_6(k_1))w + m_6(k_1) = 0.$$

It is known [10] that a closed discrete stabilization system is stable if all roots of its characteristic equation (20) in the complex plane Z are located inside a circle of unit radius. Bilinear transformation (23) conformally maps a circle of unit radius of the complex plane Z into the imaginary axis of the complex plane W, and a closed discrete stabilization system is stable if all roots of its characteristic equation

$$M\left(\frac{1+w}{1-w}, k_1, k_2\right) = 0. \quad (25)$$

in the complex plane W are to the left of the imaginary axis. Thus, to study the stability of a closed discrete system with a transformed characteristic equation (25), the stability criteria for continuous systems can be used.

In the characteristic equation (24), we will make the replacement $w = j\omega$, select the real and imaginary parts, and set them equal to zero

$$R_1(\omega,T)k_1 + R_2(\omega,T)k_2 = R_3(\omega,T); \\ R_4(\omega,T)k_1 + R_5(\omega,T)k_2 = R_6(\omega,T), \quad (26)$$

where

$$R_1(\omega,T) = -(16m_{23} + 8m_{33} + 4m_{43} + 2m_{53} + m_{63})\omega^6 + \\ + (16m_{23} + 24m_{33} + 24m_{43} + 20m_{53} + 15m_{63})\omega^4 - \\ - (4m_{43} + 10m_{53} + 15m_{63})\omega^2 + m_{63};$$

$$R_2(\omega,T) = -(32m_{12} + 16m_{22} + 8m_{32} + 4m_{42} + 2m_{52})\omega^6 + \\ + (16m_{22} + 24m_{32} + 24m_{42} + 20m_{52})\omega^4 - \\ - (4m_{42} + 10m_{52})\omega^2;$$

$$R_3(\omega,T) = \begin{pmatrix} 64 + 32m_{11} + 16m_{21} + \\ + 8m_{31} + 4m_{41} + 2m_{51} \end{pmatrix} \omega^6 - \\ - (16m_{21} + 24m_{31} + 24m_{41} + 20m_{51})\omega^4 + \\ + (4m_{41} + 10m_{51})\omega^2;$$

$$R_4(\omega,T) = -\begin{pmatrix} 32m_{23} + 24m_{33} + 16m_{43} + \\ + 10m_{53} + 6m_{63} \end{pmatrix} \omega^4 + \\ + (8m_{33} + 16m_{43} + 20m_{53} + 20m_{63})\omega^2 - (2m_{52} + 6m_{63});$$

$$R_5(\omega,T) = -\begin{pmatrix} 32m_{12} + 32m_{22} + 24m_{32} + \\ + 16m_{42} + 10m_{52} \end{pmatrix} \omega^4 + \\ + (8m_{32} + 16m_{42} + 20m_{52})\omega^2 - 2m_{53};$$

$$R_6(\omega,T) = (32m_{11} + 32m_{21} + 24m_{31} + 16m_{41} + 10m_{51})\omega^4 - \\ - (8m_{31} + 16m_{41} + 20m_{51})\omega^2 + 2m_{51}.$$

Let us write down the solutions of the system of algebraic equations (26) using the Cramer rule

$$k_1(\omega,T) = \frac{\Delta_1(\omega,T)}{\Delta(\omega,T)}; \quad k_2(\omega,T) = \frac{\Delta_2(\omega,T)}{\Delta(\omega,T)}, \quad (27)$$

$$\Delta(\omega,T) = \begin{vmatrix} R_1(\omega,T) & R_2(\omega,T) \\ R_4(\omega,T) & R_5(\omega,T) \end{vmatrix} = \\ = R_1(\omega,T)R_5(\omega,T) - R_4(\omega,T)R_2(\omega,T); \quad (28)$$

$$\Delta_1(\omega,T) = \begin{vmatrix} R_3(\omega,T) & R_2(\omega,T) \\ R_6(\omega,T) & R_5(\omega,T) \end{vmatrix} = \\ = R_3(\omega,T)R_5(\omega,T) - R_6(\omega,T)R_2(\omega,T); \quad (29)$$

$$\Delta_2(\omega,T) = \begin{vmatrix} R_1(\omega,T) & R_3(\omega,T) \\ R_4(\omega,T) & R_6(\omega,T) \end{vmatrix} = \\ = R_1(\omega,T)R_6(\omega,T) - R_4(\omega,T)R_3(\omega,T). \quad (30)$$

Calculation results and conclusions

The values of the coefficients of the mathematical model (1) change over time. Their values for different values of the moments of time of the active part of the flight trajectory of the C5M cosmic stage are given in Table 1 [5]. For the indicated times, we calculate **det** A; A_{11}; A_{12}; A_{13}; A_{21}; A_{22}; A_{23}; A_{31}; A_{32} and A_{33}, and then the elements of the matrices P and Q of the differential equation (12). The next stage of calculations is the calculation of quantities m_{11}; m_{12}; m_{21}; m_{22}; m_{23}; m_{31}; m_{32}; m_{33}; m_{41}; m_{42}; m_{43}; m_{51}; m_{52}; m_{53} and m_{63}, and then functions $R_1(\omega,T)$; $R_2(\omega,T)$; $R_3(\omega,T)$;

$R_4(\omega,T)$; $R_5(\omega,T)$ and $R_6(\omega,T)$. And, finally, the last stage of calculations is the construction of the stability regions of the closed stabilization system using relations (27) when ω changes from zero to infinity and for different values of the quantization period T for different moments of the current time t.

Table 1 – **Values of the coefficients of the model (1)**

t, s	$a'_{\psi\psi}$, s^{-1}	$a_{\psi\delta}$, s^{-2}	$a''_{\psi\beta_1}$	$a''_{\psi\beta_2}$	$a''_{\beta_1\psi}$	$a''_{\beta_2\psi}$	ω_1^2, s^{-2}	ω_2^2, s^{-2}	$2\xi_1\omega_1$	$2\xi_2\omega_2$
0	-0,0119	-0,643	0,054	-0,046	0	0	14,8	14,2	0,172	0,468
36	-0,0116	-0,643	0,093	0,001	0,477	0,082	14,7	12,1	0,255	0,319
72	-0,0124	-0,721	0,107	-0,024	0,297	-0,813	15,4	31,6	0,359	0,690
108	-0,0084	-0,487	0,413	0,156	0,764	0,296	8,2	9,02	0,357	0,397

On Fig. 1 shows the stability regions of the closed-loop stabilization system corresponding to 36 second of flight of the C5M cosmic stage for various values of the quantization period of the onboard digital computer: curve 1 corresponds to $T = 0,005$ s; curve 2 – $T = 0,01$ s; curve 3 – $T = 0,02$ s. An analysis of the constructed curves indicates that an increase in the quantization period of the onboard digital computer leads to a decrease in the stability region of a closed discrete system.
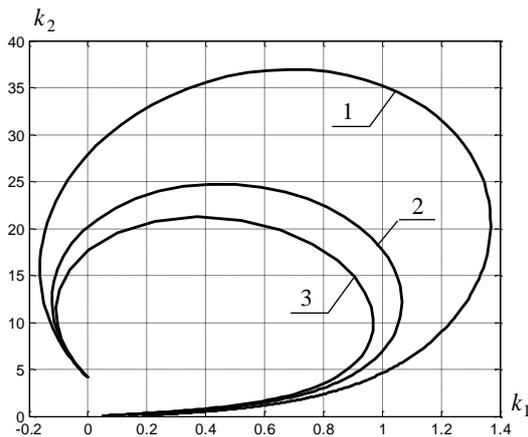

Fig. 2. Stability region of the closed stabilization system at $T = 0,01$ s: 1 – $t = 0$ s; 2 – $t = 36$ s; 3 – $t = 72$ s; 4 – $t = 108$ s

Fig. 1. Stability region of the closed stabilization system at $t = 36$ s: 1 – $T = 0,005$ s; 2 – $T = 0,01$ s; 3 – $T = 0,02$ s

On Fig. 2 shows the stability regions of the C5M CS closed-loop stabilization system corresponding to different moments of the active section of the flight trajectory:
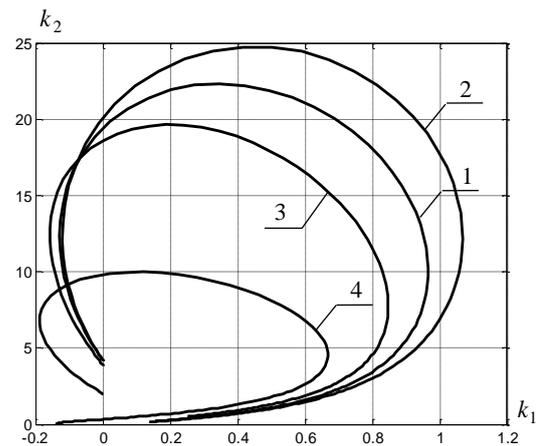
curve 1 corresponds to the moment $t = 0$ s; curve 2 – $t = 36$ s; curve 3 – $t = 72$ s; curve 4 – $t = 108$ s.

The quantization period of the onboard digital computer for all four options is the same and equal to $T = 0,01$ s. An analysis of the constructed curves leads to the conclusion that the stable motion of the cosmic stage along the entire active flight trajectory ensures the choice of the values of the variable parameters of the stabilizer k_1 and k_2 within the region of their allowable values, which is the intersection of the stability regions constructed for individual points of the active part of the flight trajectory.

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Про стійкість руху, що стабілізується, ракети-носія з рідинним реактивним двигуном і бортовою цифровою обчислювальною машиною в контурі стабілізації

Є. Є. Александров, Т. Є. Александрова, І. В. Костяник, Я. Ю. Моргун

Анотація. Розглядається задача вибору значень варійованих параметрів цифрового стабілізатора космічного ступеня ракети-носія з рідинним реактивним двигуном (РРД) і бортовою цифровою обчислювальною машиною в контурі стабілізації, що забезпечує стійкий рух ступеня уздовж усієї активної ділянки траєкторії польоту. Розглянуто вплив величини періоду квантування стабілізатора на область стійкості замкненої системи стабілізації. У якості області допустимих значень варійованих параметрів стабілізатора нестаціонарного об'єкта стабілізації рекомендується обирати перетин областей стійкості, що відповідають рівномірно розподіленим моментам часу уздовж активної ділянки траєкторії польоту ступеня.

Ключові слова: космічний ступінь ракети-носія, цифровий стабілізатор, варійовані параметри стабілізатора, активна ділянка траєкторії польоту; область стійкості в просторі варійованих параметрів.

Об устойчивости стабилизируемого движения ракеты-носителя с жидкостным реактивным двигателем и бортовой цифровой вычислительной машиной в контуре стабилизации

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Аннотация. Рассматривается задача выбора значений варьируемых параметров цифрового стабилизатора космической ступени ракеты-носителя с жидкостным реактивным двигателем (ЖРД) и бортовой цифровой вычислительной машиной в контуре стабилизации, обеспечивающего устойчивое движение ступени вдоль всего активного участка траектории полета. Рассмотрено влияние величины периода квантования стабилизатора на область устойчивости замкнутой системы стабилизации. В качестве допустимых значений варьируемых параметров стабилизатора нестационарного объекта стабилизации рекомендуется выбирать пересечение областей устойчивости, соответствующих равномерно распределенным моментам времени вдоль активного участка траектории полета ступени.

Ключевые слова: космическая ступень ракеты-носителя, цифровой стабилизатор, варьируемые параметры стабилизатора, активный участок траектории полета; область устойчивости в пространстве варьируемых параметров.