Information systems modeling

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MATHEMATICAL APPARATUS FOR MODELING OF THE PROPAGATION THE MAGNETIC FIELD ELECTRIC MACHINES WITH A GIVEN ACCURACY

Abstract. The problem of modeling the propagation local magnetic fields and spatially dispersed sources is large errors compared to field measurements. An important aspect of adequate modeling is the use of the correct mathematical apparatus. It is shown that in order to obtain reliable models of the propagation magnetic fields around electrical machines (generators, electric motors of different power, geometric dimensions and poles), it is advisable to apply the Gauss equation for a scalar potential. The solution of the equation in polar coordinates makes it possible to take into account not only the fundamental, but also other harmonics of the magnetic field (dipole, quadrupole, octupole). This allows, depending on the number of spatial harmonics taken into account, to obtain a model with the required accuracy (error) for predicting the magnetic field strength at any point around the machine. It is considered in the paper that an electronic machine is an object of base radius R_0. The presented approach makes it possible to unambiguously determine the location of zero field points at a distance from the source (for a quadrupole source and zero field lines, for an octupole source). The results of modeling and their verification by full-scale measurements for the most common four-pole machines (quadrupole source) are presented. The main task of modeling the propagation the magnetic field of such sources is to ensure the required accuracy based on the goals of modeling. It is shown that the modeling accuracy and the presence of zero field points are due to different field levels near the electrical machine housing for different harmonics. The dipole harmonic at the cabinet is 20% of its own harmonic. But it falls more slowly with distance. This necessitates taking into account a different number of harmonics depending on the value of the ratio R_0/R, R is the distance to the point of determining the field strength from the source. Therefore, with the ratio $R_0/R=2/3$, the eighth harmonic is essential. At R0/R=1/5, already the fourth spatial harmonic can be neglected. Such data allow you to choose a rational number of harmonics. This reduces the amount of calculations and simplifies the process of modeling the propagation of the magnetic field around the source.

Keywords: modeling, calculation apparatus; electric machine; spatial harmonic; dipole.

Introduction

Electric machines are widely used in industrial, household and educational premises. These are electric generators, electric motors for various purposes and power. Determining the spatial distributions of their magnetic fields makes it possible to assess the possible impact on people, energy losses, etc. At the stages of designing the placement equipment in rooms, designing ventilation and heat supply systems, designing educational laboratories, such an assessment is possible by modeling the distribution of fields. Obtaining reliable data requires the presence of correct mathematical functions. A feature of modeling the propagation of ultra-low magnetic fields (power frequencies, its harmonics and interharmonics) is the need to ensuшre the required accuracy of the final result. That is, in functions of creating software for the implementation of modeling, it is necessary to lay an allowable error. This error may vary depending on the tasks. For the purposes of environmental safety, it can be larger, given the distribution of fields in areas where people stay periodically. In industrial and educational premises with limited space, it is necessary to accurately determine the zones of safe stay of people. The required accuracy of modeling can be ensured by taking into account a sufficient number of spatial harmonics of the magnetic field. To date, from an applied point of view, this has not been worked out enough. In particular, different patterns of extinction of individual harmonics of the field are not taken into account. Enough harmonics have not been determined to calculate the magnetic field depending on the relative size of the electrical machine (the ratio of the actual size of the machine to the distance of determining the magnetic field strength).

These issues require consideration and the provision of unambiguous recommendations for modeling the propagation of a magnetic field with sufficient accuracy.

Analysis of recent research and publications

Much attention is paid to modeling the propagation of magnetic fields in electrical equipment. This is mainly aimed at reducing energy losses and concerns the distribution of the magnetic field inside the electrical device [1]. Such modeling requires minimal errors and is performed using the Comsol package. For the purposes of ensuring the safety of people, such a high accuracy is not necessary even for critical infrastructure facilities [2]. Studies on taking into account the spatial harmonics of the magnetic field of electrical machines [3] have shown that this approach is promising. It is advisable to model the propagation of a magnetic field in polar coordinates, and consider the change in the field in separate directions or angles. In most cases, models of the most common dipole-type field sources are considered [4, 5]. But most electrical machines are sources of quadrupole and even octupole magnetic fields. This greatly complicates the process of modeling the propagation such fields. Consideration of such sources was started in [6], but the field configurations in it are sketchy, and some provisions require clarification and deepening. The study [7] shows that any source of a magnetic field can be considered as a combination of magnetic dipoles with different magnetic moments. The works [8, 9] used the synthesis of the magnetic field of technical objects based on spatial harmonics. They mainly refer to the fields inside the minimum sphere enclosing the object. But for the purposes of human safety and the electromagnetic compatibility of electrical and electronic equipment, it is required modeling the propagation of an external magnetic field at the required distances. In this case, it is necessary to take into account the relative sizes of the sources in order to determine the required number of spatial harmonics.

Problem statement. The purpose of the work is to provide a convenient mathematical tool for modeling the propagation of the magnetic field of electrical machines, taking into account the required number of spatial harmonics, which ensures the necessary modeling accuracy.

Presentation of the main material

The main source of errors in modeling the propagation of magnetic fields of electrical equipment is taking into account the field of only industrial frequency. In real conditions, there are always harmonic fields of the fundamental frequency in space (mainly the third and other multiples of three).

This is due to the non-linearity of the current-voltage characteristics of most equipment, including the electric drive. In electrical machines of any design, it is always possible to distinguish harmonics corresponding to different harmonics depending on the number of poles of the machine: two, four, eight.

This corresponds to dipole, quadrupole and octupole harmonics (n=1, 2, 3).

The most common and powerful are four-pole electric machines. These include, for example, all generators of power plants and a powerful electric drive.

An accurate determination of changes in the level of the magnetic field with distance is expedient using the Gauss equation for a scalar magnetic potential. In spherical coordinates $R,\ \theta,\ \varphi$, the source magnetic field distribution function has the form:

$$U_M = R_0 \times \sum_{n=1}^{\infty} \left(R_0/R\right)^{n+1} \times$$
$$\times \sum_{m=0}^{n} \left(a_{nm}\cos m\varphi + b_{nm}\sin m\varphi\right) \times P_n^m \times \cos\varphi,$$

where R_0 – the radius of the sphere of the determination the potential, a_{nm}, b_{nm} – constant coefficients $P_n^m \cos\varphi$ – Legendre polynomial.

In doing so, $R \geq R_0$, the coordinates a, b are the amplitudes of the spherical harmonics of the magnetic field strength in the sphere R_0.

The strength (induction) of the magnetic field is determined from the above equation based on the fundamental relationships:

$$\boldsymbol{H} = -gradU_M, \quad \boldsymbol{B} = \mu_0 \boldsymbol{H};$$

$$H_r = \sum_{n=1}^{\infty} (n+1) \times \left(R_0/R\right)^{n+2} \times$$
$$\times \sum_{m=0}^{n} \left(a_{nm} \times \cos m\varphi + b_{nm} \times \sin m\varphi\right) \times P_n^m \times \cos\theta;$$

$$H\varphi = \sum_{n=1}^{\infty} \left(R_0/R\right)^{n+2} \times$$
$$\times \sum_{m=0}^{n} \left(a_{mn} \times \sin m\varphi + b_{mn} \times \cos m\varphi\right) \times$$
$$P_n^m \times \cos\theta / \sin\varphi;$$

$$H_\theta = \sum_{n=1}^{\infty} \left(R_0/R\right)^{n+2} \times \sum_{m=0}^{n} \left(a_{nm} \times \cos m\varphi + b_{nm} \times \sin m\varphi\right) \times$$
$$\times \frac{1}{\sin\varphi} \begin{bmatrix} (n-m+1) \times P_{n+1}^m \cos\varphi - \\ -(n+1) \times P_n^m \times \cos^2\theta \end{bmatrix}.$$

The above relations indicate that the magnetic field strength decreases with distance, and this decrease is proportional to the increase in the harmonic index *n*.

Thus, based on the tasks set, it is advisable to consider the first spherical harmonics, which correspond to the slightest decrease in the level of the magnetic field with distance.

These are the dipole harmonic (n=1) and the quadrupole harmonic ($n = 2$).

The radial component of the magnetic field is determined from the above relations by a standard procedure using the Legendre polynomials in the usual form.

Для $n = 1$:

$$H_r^{(n=1)} = 2 \times \left(R_0/R\right)^3 \times$$
$$\times \left(a_{10}\cos\theta + a_{11}\cos\varphi\sin\theta + b_{11}\sin\varphi\sin\theta\right).$$

Для $n = 2$:

$$H_r^{(n=2)} = \frac{3}{4\pi}\left(R_0/R\right)^4 \times \left(\frac{a_{20}}{2}\left(3\cos^2\theta - 1\right) + \right.$$
$$+3\left(a_{21}\cos\varphi + b_{21}\sin\varphi\right)\sin 2\theta +$$
$$\left. +12\left(a_{22}\cos 2\varphi + b_{22}\sin 2\varphi\right)\sin^2\theta\right).$$

These radial harmonics decrease with distance from the field source in proportion to the third and fourth steps of the radius.

A four-pole electric machine, in this case a turbogenerator, is a source of a magnetic field a dipole-quadrupole type, that is, a source that has dipole and quadrupole spherical harmonics of the field. The magnetic field around such a source is characterized by the sum of harmonics $H_r^{(n=1)}$ and $H_r^{(n=2)}$.

In general:

$$H = \left(R_0/R\right)^3 \times a_{11} \times \cos\varphi \times \sin\varphi +$$
$$+ \left(R_0/R\right)^4 \times a_{22}\cos 2\varphi \times \sin^2\theta.$$

Thus, the dependence of the field strength on the distance for different angles of spherical coordinates will differ significantly. For example, for

$$\theta = \frac{\pi}{2}\left(\sin\theta = 1,\ \sin^2\theta = 1\right)$$

in the direction φ=0 dipole and quadrupole harmonics are added, and in the direction φ=π – are subtracted.

The result obtained is important from the point of view of ensuring the electromagnetic safety of personnel located near power generators.

This is explained by the fact that at φ=π there is a point where H=0, that is, within this angle, the total levels of the fields are insignificant.

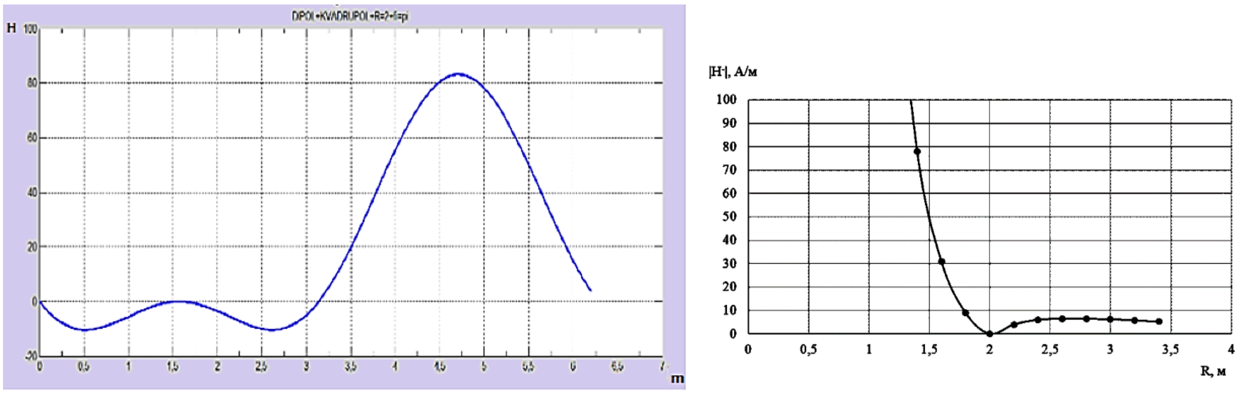
Considering the change in the field strength by φ=0 and φ=π, assuming R_0=1, we obtain the relation:

$$H_1 = \frac{a_{22}}{R^4} + \frac{a_{11}}{R^3},$$

$$H_2 = \frac{a_{22}}{R^4} - \frac{a_{11}}{R^3}.$$

The result obtained indicates that, under the condition φ=π, as a result of different rates of decrease in the strength of the dipole and quadrupole components of the magnetic field with distance, there is a point where H = 0. Modeling of the spatial distribution the magnetic field a four-pole electric machine using the Matlab package for φ=π, R=2 is shown in Fig. 1, a.

An experimental verification of the change in the magnetic field of a four-pole electric machine with distance is shown in Fig. 1, b.



a b

Fig. 1. Change in the magnetic field strength near a four-pole electric machine: a – modeling the value of the magnetic field strength in the plane around the electric machine (3.14 corresponds to π; 6.28, respectively 2π);
b – measured change in magnetic field strength with distance from the electrical machine

For a dipole-octupole field, its zero value can be on closed lines. In the plane θ=π/2, there is only one component of the magnetic field H_0 with exponents n = 1,3:

$$H_0 = H_{010} + H_{030} = \left(\frac{R_0}{R}\right)^3 \times \left[\frac{3}{2}\left(\frac{R_0}{R}\right)^2 a_{30} - a_{10}\right].$$

That is, in the plane θ in the angle 0<φ<2π along the entire circle H_0=0 on a distance:

$$\frac{R}{R_0} = \sqrt{\frac{3a_{30}}{2a_{10}}}.$$

These ratios, in addition to clarifying the safe zones, allow you to correctly select the places for monitoring the electromagnetic environment.

Accounting for a large number of spatial field harmonics allows you to calculate the field strength at any point around the electric machine and indicate them graphically.

But it is important to choose a sufficient number of harmonics to determine the strength of the magnetic fields with the required accuracy.

Minimizing the number of harmonics taken into account reduces the volume of calculations and simplifies the field propagation modeling processes.

As shown in [7], in the external magnetic field of multipole electrical machines, in addition to the intrinsic spatial harmonic with the exponent n = p, there is the first (dipole) harmonic. At the surface of an electric machine, the dipole harmonic is approximately 20% of the magnetic field strength of the natural harmonic. But its level decreases more slowly than the main one, therefore it gradually compares with it, and prevails at a greater distance. This is the reason for the appearance of zero field points.

In the presence of a dipole harmonic, the maximum value of the field strength H_φ is achieved at φ=0, θ=π/2:

$$H_\varphi = \frac{1}{4\pi} \times \left[b_{nn}\left(\frac{R_0}{R}\right)^{n+2} + b_{11}\left(\frac{R_0}{R}\right)^3\right].$$

As shown in [7], the relative level of higher spatial harmonics:

$$K = \frac{H_{nn}}{H_{11}} = \frac{b_{nn}}{b_{11}} \times \left(\frac{R_0}{R}\right)^{n-1},$$

and given that: $b_{11} = 0,2b_{nn}$,

$$K = 5\left(\frac{R_0}{R}\right)^{n-1}.$$

That is, it is possible to calculate the relative level of higher spatial harmonics for any ratios R_0/R (Table 1).

Table 1 – **The relative level of the higher spatial harmonics of the magnetic field K for different relative distances from the electric machine**

R_0/R	*K*						
	n=2	*n*=3	*n*=4	*n*=5	*n*=6	*n*=7	*n*=8
2/3	3,33	2,22	1,48	0,99	0,66	0,44	0,29
1/2	2,50	1,25	0,63	0,31	0,16	0,08	0,04
1/3	1,67	0,56	0,19	0,06	0,02	-	-
1/4	1,25	0,31	0,08	0,02	-	-	-
1/5	1,00	0,20	0,04	-	-	-	-

As can be seen from the table, for smaller relative dimensions of the electric machine a smaller number of spatial harmonics should be taken into account. For example, for R_0/R=2/3 even the eighth harmonic gives about 30% of the first, and for the second R_0/R=1/5 the fifth harmonic becomes insignificant.

This makes it possible to rationalize the processes of modeling the spatial propagation of magnetic fields of electrical machines with the necessary accuracy (permissible error) for determining the field strengths at any point.

Conclusions

1. For modeling of the propagation magnetic fields around multi-pole electrical machines, it is necessary to take into account the presence of spatial harmonics the magnetic field. To create software, it is advisable to use the solution of the Gauss equation for a scalar potential in spherical coordinates. Verification of the modeling results, taking into account two harmonics, proved the possibility of obtaining results similar to the experimental ones.
2. The use of the above calculation apparatus allows one to determine the points of zero magnetic field around electrical machines and their actual location, which is not symmetrical in a full turn around the machine. By increasing the number of machine poles to eight, zero field lines can be defined around it.
3. It is shown that in order to ensure the required accuracy when modeling the field propagation (permissible error), one should take into account the relative dimensions the electric machine (the ratio of the conditional radius of the machine to the distance of determining the magnetic field strength). At smaller relative sizes, a smaller number of spatial harmonics is taken into account. This simplifies the process of modeling the propagation the field depending on its objectives.

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Математичний апарат моделювання поширення магнітного поля електричних машин із заданою точністю

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Анотація. Проблемою моделювання поширення магнітних полів локальних та розсерджених у просторі джерел є великі похибки порівняно натурними вимірюваннями. Важливим аспектом адекватного моделювання є застосування коректного математичного апарату. Показано, що для отримання достовірних моделей поширення магнітних полів навколо електричних машин (генераторів, електродвигунів різної потужності, геометричних розмірів та полюсності) доцільно застосовувати рівняння Гауса для скалярного потенціалу. Розв'язок рівняння у полярних координатах дозволяє врахувати не тільки основну, а і інші гармоніки магнітного поля (дипольну, квадрупольну, октупольну). Це дозволяє у залежності кількості врахованих просторових гармонік отримати модель з потрібною точністю (похибкою) прогнозування напруженості магнітного поля у будь-якій точці навколо машини. У роботі вважається, що електрична машина є об'єктом базового радіусу R_0. Наведений підхід дозволяє однозначно визначити розташування точок нульового поля на певній відстані від джерела (для квадрупольного джерела та ліній нульового поля, для октупольного джерела). Наведено результати моделювання та його верифікації натурними вимірюваннями для найбільш поширених чотириполюсних машин (квадрупольне джерело). Головною задачею моделювання поширення магнітного поля таких джерел є забезпечення потрібної точності, виходячи з цілей моделювання. Показано, що точність моделювання та наявність точок нульового поля обумовлені різними рівнями поля біля корпусу електричної машини для різних гармонік. Дипольна гармоніка біля корпусу складає 20 % власної гармоніки. Але вона спадає повільніше з відстанню. Це обумовлює необхідність врахування різної кількості гармонік у залежності від значення співвідношення R_0/R, R – відстань до точки визначення напруженості поля від джерела. Тому за співвідношення $R_0/R=2/3$ суттєвою є восьма гармоніка. За $R_0/R=1/5$ вже четвертою просторовою гармонікою можна нехтувати. Такі дані дозволяють обрати раціональну кількість гармонік. Це зменшує обсяги обчислень і спрощує процес моделювання поширення магнітного поля навколо джерела.

Ключові слова: моделювання; розрахунковий апарат; електрична машина; просторова гармоніка; диполь.

Математический аппарат моделирования распространения магнитного поля электрических машин с заданной точностью

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Проблемой моделирования распространения магнитных полей локальных и рассредоточенных в пространстве источников являются большие погрешности по сравнению с натурными измерениями. Важным аспектом адекватного моделирования является применение корректного математического аппарата. Показано, что для получения достоверных моделей распространения магнитных полей вокруг электрических машин (генераторов, электродвигателей разной мощности, геометрических размеров и полюсности) целесообразно применять уравнение Гаусса для скалярного потенциала. Решение уравнения в полярных координатах позволяет учесть не только основную, но и другие гармоники магнитного поля (дипольную, квадрупольную, октупольную). Это позволяет в зависимости от количества учтенных пространственных гармоник получить модель с нужной точностью (погрешностью) прогнозирования напряженности магнитного поля в любой точке вокруг машины. В работе считается, что электронная машина является объектом базового радиуса R_0. Приведенный подход позволяет однозначно определить расположение точек нулевого поля на определённом расстоянии от источника (для квадрупольного источника и линий нулевого поля, для октупольного источника). Приведены результаты моделирования и их верификации натурными измерениями для наиболее распространенных четырёхполюсных машин (квадрупольный источник). Главной задачей моделирования распространения магнитного поля таких источников является обеспечение требуемой точности, исходя из целей моделирования. Показано, что точность моделирования и наличие точек нулевого поля обусловлены разными уровнями поля у корпуса электрической машины для разных гармоник. Дипольная гармоника у корпуса составляет 20% собственной гармоники. Но она спадает медленнее с расстоянием. Это обуславливает необходимость учета разного количества гармоник в зависимости от значения соотношения R_0/R, R – расстояние до точки определения напряженности поля от источника. Поэтому при соотношении $R_0/R=2/3$ существенной является восьмая гармоника. При $R_0/R=1/5$ уже четвертой пространственной гармоникой можно пренебрегать. Такие данные позволяют выбрать оптимальное количество гармоник. Это уменьшает объемы вычислений и упрощает процесс моделирования распространения магнитного поля вокруг источника.

Ключевые слова: моделирование; расчетный аппарат; электрическая машина; пространственная гармоника; диполь.