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SEMI-MARKOV RELIABILITY MODELS

Abstract. Traditional technologies for reliability analysis of semi-Markov systems are limited to obtaining a stationary state probability distribution. However, when solving practical control problems in such systems, the study of transient processes is of considerable interest. This implies the subject of research - the analysis of the laws of distribution of the system states probabilities. The goal of the work is to obtain the desired distribution at any time. The complexity of the problem solving is determined by the need to obtain a result for arbitrary distribution laws of the duration of the system's stay in each state before leaving. An easy-to-implement method for the analysis of semi-Markov reliability models has been suggested. The method is based on the possibility of approximating probability-theoretic descriptions of failure and recovery flows in the system using the Erlang distribution laws of the proper order. The developed computational scheme uses the most important property of Erlang flows, which are formed as a result of sieving the simplest Poisson flow. In this case, the semi-Markov model is reduced to the Markov one, which radically simplifies the analysis of real systems.

Keywords: semi-Markov reliability models; approximation by Erlang distributions of the proper sequence.

Introduction

The elementary model of the system in the reliability theory definitions [1-3] is described as follows. The system can be in one out of two states during its functioning procedure:

E_0 means the system is functioning normally,

E_1 means the system failed and is being recovered.

Recovery failure procedures are random. To describe them, let's introduce the following:

$f_{01}(t)$ is the distribution density of the random duration of the system's stay in the state E_0 prior to entering the state E_1;

$f_{10}(t)$ is the distribution density of the random duration of the system's stay in the state E_1prior to entering the state E_0;

$H_{00}(t)$ is the conditional probability that the system at time point t would be in the state E_0, unless it was in the state E_0at the initial point;

$H_{01}(t)$ is the conditional probability that the system at time point t would be in the state E_0, unless it was in the state E_0at the initial point;

$H_{01}(t)$ is the conditional probability that the system at time point t would be in the state E_1, unless it was in the state E_0at the initial point;

$H_{10}(t)$ is the conditional probability that the system at time point t would be in the state E_0, unless it was in the state E_1at the initial point;

$H_{11}(t)$ is the conditional probability that the system at time point t would be in the state E_1, unless it was in the state E_1at the initial point.

A set of relations are obtained describing the possible dynamics of the system states.

Let $E=(E_i)$, $i=1,2,...,n$ be the set of possible system states.

A system that was in the state i at the initial point can be in the state j at point t as follows. Firstly, unless $j=i$, the system may not leave the state i until the point t, or exit this state and go back thereto by the point t. The related mathematical model is as follows:

$$G_{ij}(t)=\gamma_i(t)+\sum_{k\in E,\,k\neq i}P_{ik}\int_o^t f_{ik}(\tau)\mathrm{G}_{ki}(t-\tau)\,\mathrm{d}\,\tau. \quad (1)$$

Secondly, unless $j \neq i$, the system may find itself in this state, passing to certain intermediate state k at some point $\tau<t$. Herewith

$$G_{ij}(t)=\gamma_i(t)+\sum_{k\in E,\,k\neq i}P_{ik}\int_o^t f_{ik}(\tau)\mathrm{G}_{kj}(t-\tau)\,\mathrm{d}\,\tau. \quad (2)$$

Here P_{ik} is the probability of the system transition from the state i to the state j.

In the considered reliability theory problem, when $E = (E_0,E_1)$, the ratios (1), (2) are simplified.

$$H_{00}(t)=(1-\int_0^t f_{01}(\tau)d\tau)+\int_o^t f_{01}(\tau)H_{10}(t-\tau)\,\mathrm{d}\,\tau. \quad (3)$$

$$H_{01}(t)=\int_o^t f_{01}(\tau)H_{11}(t-\tau)\,\mathrm{d}\,\tau. \quad (4)$$

$$H_{10}(t)=\int_o^t f_{10}(\tau)H_{00}(t-\tau)\,\mathrm{d}\,\tau. \quad (5)$$

$$H_{00}(t)=(1-\int_0^t f_{10}(\tau)d\tau)+\int_o^t f_{10}(\tau)H_{01}(t-\tau)\,\mathrm{d}\,\tau. \quad (6)$$

The resulting system of integral equations (3)-(6) is solved using Laplace transformations. As it is known, the Laplace transformation of the function $u(t)$ is a function

$$L(u(\mathrm{t}))=\int_0^\infty u(t)e^{-st}dt=\frac{1}{s}L(u(t))=\frac{1}{s}u^*(s). \quad (7)$$

Applying the transformation (7) to the ratios (3)-(6), their Laplace image is obtained:

$$H^*{}_{00}(\mathrm{s})=\frac{1}{s}(1-f^*{}_{01}(\mathrm{s}))+f^*{}_{01}(s)H^*{}_{10}(s), \quad (8)$$

$$H^*{}_{01}(\mathrm{s})=f^*{}_{01}(s)H^*{}_{11}(s), \quad (9)$$

$$H^*{}_{10}(\mathrm{s})=f^*{}_{10}(s)H^*{}_{00}(s), \quad (10)$$

$$H^*{}_{11}(\mathrm{s})=\frac{1}{s}(1-f^*{}_{10}(\mathrm{s}))+f^*{}_{10}(s)H^*{}_{01}(s). \quad (11)$$

The resulting equations must be solved by expressing the unknown functions $H^*_{00}(s)$, $H^*_{01}(s)$, $H^*_{10}(s)$, $H^*_{11}(s)$ through Laplace images of known densities $f_{01}(t)$, $f_{10}(t)$.

Applying the relations (10) to (8), the following is obtained

$$H^*_{00}(s) = \frac{1}{s}(1 - f^*_{01}(s)) + f^*_{01}(s) f^*_{10}(s) H^*_{00}(s),$$

where from

$$H^*_{00}(s)(1 - f^*_{01}(s) f^*_{10}(s)) = \frac{1}{s}(1 - f^*_{01}(s)),$$

$$H^*_{00}(s) = \frac{1}{s}\,\frac{1 - f^*_{01}(s)}{1 - f^*_{01}(s) f^*_{10}(s)}. \qquad (12)$$

Similarly, the following is obtained

$$H^*_{10}(s) = \frac{1}{s}\,\frac{(1 - f^*_{01}(s)) f^*_{10}(s)}{1 - f^*_{01}(s) f^*_{10}(s)}, \qquad (13)$$

$$H^*_{11}(s) = \frac{1}{s}\,\frac{1 - f^*_{10}(s)}{1 - f^*_{01}(s) f^*_{10}(s)}, \qquad (14)$$

$$H^*_{10}(s) = \frac{1}{s}\,\frac{(1 - f^*_{10}(s)) f^*_{01}(s)}{1 - f^*_{01}(s) f^*_{10}(s)}, \qquad (15)$$

Thus, the solution of the system reliability analysis problem is reduced to the following two-stage procedure. One should obtain the Laplace images $f_{01}(s)$, $f_{10}(s)$ of the known densities $f_{01}(t)$, $f_{10}(t)$ *at the first stage*. The desired functions $H*00(t)$, $H*01(t)$, $H*10(t)$, $H*11(t)$ describing the dynamics of the system states are processed using the inverse Laplace transformation at the second stage.

Let's represent the results of solving this problem for a textbook instance when the system is considered Markov. Here with

$$f*_{01}(t) = \frac{\lambda}{s+\lambda}, \quad f*_{10}(t) = \frac{\mu}{s+\mu}$$

Then

$$H^*_{00}(s) = \frac{s+\mu}{s(s+\lambda+\mu)}, \qquad (16)$$

$$H^*_{10}(s) = \frac{\mu}{s(s+\lambda+\mu)}, \qquad (17)$$

$$H^*_{11}(s) = \frac{s+\lambda}{s(s+\lambda+\mu)}, \qquad (18)$$

$$H^*_{01}(s) = \frac{\lambda}{s(s+\lambda+\mu)}, \qquad (19)$$

The inverse transformations of the obtained Laplace images are tabular and are as follows:

$$H_{00}(t) = \frac{\mu}{\lambda+\mu} + \frac{\mu}{\lambda+\mu} e^{-(\lambda+\mu)t}, \qquad (20)$$

$$H_{10}(t) = \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu} e^{-(\lambda+\mu)t}, \qquad (21)$$

$$H_{11}(t) = \frac{\mu}{\lambda+\mu} + \frac{\mu}{\lambda+\mu} e^{-(\lambda+\mu)t}, \qquad (22)$$

$$H_{01}(t) = \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu} e^{-(\lambda+\mu)t}. \qquad (23)$$

The ratios (20)-(23) represent the solution to the problem of the system recovery analysis, containing comprehensive information about its state at any time point. Herewith, as expected, $H_{00}(s) + H_{01}(s) = 1$, $H_{10}(s) + H_{11}(s) = 1$.

So, the described computational procedure successfully solves the given problem. However, it should be noted that this approach has a fundamental drawback, which is as follows. This solution in almost all cases is a set of functions with numerical coefficients, whose values are not related to the values of the system parameters. That is, a point estimate of the system state for any given set of source data is provided. In addition, it is much more important to explicitly obtain the dependencies of the resulting estimates of the system functioning quality on the numerical values of its parameters for the actual practice of the system operation. Only in this case, the system analysis problem can be considered solved completely, since only in this event it becomes possible to develop any recommendations to improve the system efficiency, i.e., to solve the structural and parametric optimization problems.

Thus, the problem of searching for the method of in-depth analysis of semi-Markov systems, focused on obtaining the resulted analytical relations, as well as an explicit dependence of the probability distributions of the system states on the values of its parameters appears relevant. A possible direction of searching for an approximate solution to the problem is as follows. The space of values and parameters of the system accumulates a set of points generating an orthogonal plan. The problem is solved by a known numerical method for each point. Standard statistical processing of results of such a multifactorial orthogonal experiment in terms of the system parameters number that specify the dynamics of its functioning makes it possible to obtain the desired relations. This approach is reliable, though its constructive drawback is evident, that is the related computational procedure is knowingly cumbersome. Herewith, the complexity level of its implementation depends in an unpredictable way on the level of required accuracy of solving the problem.

Literature data analysis

The generated problem of analyzing semi-Markov systems is being actively discussed. In [1-3], a general approach to solving the problem is suggested, which reduces to solving the integral equations system. Herewith, the attention is drawn to the complexity of its implementation for many practical situations. In view of this circumstance, the general problem in a very large number of works is simplified. In [4], the problem of evaluating the queuing system efficiency along with a semi-Markov incoming flow was considered. It resulted in obtaining the probability distribution of its states. In

[5], a stationary probability distribution for the semi-Markov model of production system was also obtained. The same result was obtained in [6] for a set of computer network analysis problems. Similar research results are given in [7-13] for various options of descriptions of failure and recovery flows. A summary of well-known publications on the issue of the semi-Markov systems analysis allows for the following conclusion. For an extensive class of problems in the study of semi-Markov systems, there is no simple and convenient method for analyzing the dynamics of states of such systems for the purpose of practical implementation. This circumstance determines the issue relevance and the study purpose.

3. The **study objective** is to develop a fast approximate method of analyzing semi-Markov systems along with the controlled accuracy of the result.

4. The basic result. Method of analyzing semi-Markov models.

To solve the problem of analyzing semi-Markov systems, an approach based on a special approximation of real processes within the system is suggested. Such an approximation should meet the following requirements. Firstly, its implementing functions must be parameterized, i.e., proper selection of their parameters allows ensuring the required accuracy of descriptions of real processes within the system. Secondly, the approximating functions should allow for the simplicity of performing the direct and reverse Laplace transformations.

It is convenient to select the Erlang distribution laws of the required sequence as such functions. The related functions possess the following number of important benefits:

— they are positive for *[0,∞]* and integrable;

— changing the parameters of the Erlang distribution density allows changing the mathematical expectation, variance, asymmetry and kurtosis of the related random variable within a wide range.

However, the decisive benefit of Erlang distributions is that the events flow described by this distribution is a screened Poisson flow. In particular, unless the Poisson flow of events is screened, by selecting each *nth* event therefrom, then the random interval between these events shall be described by an Erlang distribution of the sequence *n*. The most important property of the Erlang flow to be generated by the Poisson flow allows it to be constructively used to analyze the semi-Markov models practically regardless of the type of probability distributions of the real system. The related technique is two-stage.

The distributions describing the incoming failure and recovery flow of the real system are independently approximated by Erlang distributions of the proper sequence at the first stage. Herewith, histograms of the related random variables are generated in a standard way by previous processing the source data on the duration of the system's stay in a state of normal functioning prior to the failure and the duration of recovery. These histograms are used to assess the approximation parameters of Erlang distributions via the max verisimilitude method.

The obtained pair of Erlang distributions is applied to construct the Markov approximation of a real semi-Markov system at the second stage as follows.

Let the input of the analyzed system *(E_0,E_1)* receives an Erlang flow of the sequence n_1 with the distribution density of the interval between failures $f_{01}(t)=\lambda^{n_1}t^{n-1}e^{-\lambda t}$, and the service sequence is an Erlang flow of the sequence m $f_{10}(t)=\mu^{m_2}t^{m-1}e^{-\mu t}$. This diagram is schematically shown in Fig. 1.

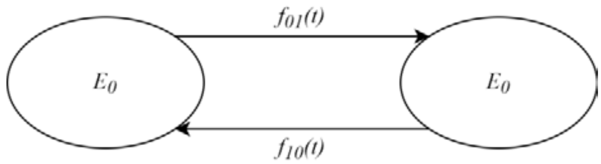

Fig. 1. System functioning diagram

Let us consider the technology of analyzing such a semi-Markov system that does not require solving an integral equations system.

Let the distribution law of the duration of the system's stay in the state *i* prior to the transition to the state *j* be given as follows: $E_{ij}(t)=P(\tau_{ij}\le t)$. Here τ_{ij} is the random duration of stay at *i* prior to transition into *j*. Then $Q_{ij}(t)=P(\tau_{ij}>t)=1-F>_{ij}(t)$ is the probability that the transition from *i* to *j* at the interval *[0,t]* did not occur.

Now let's introduce

$F_{ij}(t+\tau)=P(\tau_{ij}\le t+\tau)$ is the probability that the transition occurred at the interval *[0, t+τ]*;

$Q_{ij}(t+\tau)=P(\tau_{ij}>t+\tau)=1-F_{ij}(t+\tau)$ is the probability that the transition from *i* to *j* at the interval *[0, t+τ]* did not occur;

$Q_{ij}(t,t+\tau)$ is the probability that the transition from *i* to *j* at the interval *[t, t+τ]* did not occur.

It is clear that

$$Q_{ij}(t+\tau)=Q_{ij}(t)Q_{ij}(t,t+\tau), \qquad (24)$$

where from
$$Q_{ij}(t,t+\tau)=\frac{Q_{ij}(t+\tau)}{Q_{ij}(t)}. \qquad (25)$$

Later one shall introduce the probability of transition from *i* to *j* at the interval *[t, t+τ]*.

$$\begin{aligned} w_{ij}(t,t+\tau)&=1-Q_{ij}(t,t+\tau)=\\ &=1-\frac{Q_{ij}(t+\tau)}{Q_{ij}(t)}=\frac{Q_{ij}(t)-Q_{ij}(t+\tau)}{Q_{ij}(t)}= \\ &=\frac{(1-F_{ij}(t))-(1-F_{ij}(t+\tau))}{1-F_{ij}(t)}=\frac{F_{ij}(t+\tau)-F_{ij}(t)}{1-F_{ij}(t)}. \end{aligned} \qquad (26)$$

Now let's consider the value

$$\begin{aligned} \lambda_{ij}(t)&=\frac{dw_{ij}(t,t)}{d\tau}=\lim_{\tau\to 0}\frac{w_{ij}(t,t+\tau)}{\tau}=\\ &=\lim_{\tau\to 0}\frac{F_{ij}(t+\tau)-F_{ij}(t)}{\tau(1-F_{ij}(t))}=\lim_{\tau\to 0}\frac{F_{ij}(t+\tau)-F_{ij}(t)}{\tau}\times \\ &\times\frac{1}{1-F_{ij}(t)}=\frac{1}{1-F_{ij}(t)}\frac{dF_{ij}(t)}{dt}=\frac{f_{ij}(t)}{1-F_{ij}(t)}, \end{aligned} \qquad (27)$$

щhere $f_{ij}(t)=\dfrac{dF_{ij}(t)}{dt}$ is the distribution density of the random variable τ.

The function $\lambda_{ij(t)}$ introduced according to (27) defines the transition intensity. It follows from relation (3) that this transition intensity can be defined as the conditional probability density of transitions at time point t, provided that no transition has occurred till this point. This general approach to assess the transition intensity is inconvenient, since the transition intensity value obtained according to (27) is a function of time. In addition, when solving practical problems, it is important to know the mean value of transitions intensity. This value is easily and regularly obtained as follows. For a given density distribution $f_{ij}(t)$ of the duration of stay at i prior to leaving for j, let us calculate the mean value of this duration: $\overline{\tau}_{ij}(t)=\int_0^\infty tf_{ij}(t)dt$.

Let's set the intensity $\lambda_{ij}(t)$ of the transition from i to j by an Erlang distribution of sequence n, that is $f_n(t)=\dfrac{\lambda(\lambda t)^{n-1}}{(n-1)!}e^{-\lambda t}$. Here with

$$\overline{\tau}_{ij}(t)=\int_0^\infty tf_{ij}(t)dt=\int_0^\infty \frac{(\lambda t)^n}{(n-1)!}e^{-\lambda t}dt=\frac{\lambda^n}{(n-1)!}\int_0^\infty t^n e^{-\lambda t}dt.$$

Since $$\int_0^\infty t^n e^{-\lambda t}dt=\frac{n!}{\lambda^{n+1}},$$

then $$\overline{\tau}_{ij}(t)=\frac{\lambda^n}{(n-1)!}\int_0^\infty t^n e^{-\lambda t}dt=\frac{\lambda^n}{(n-1)!}\frac{n!}{\lambda^{n+1}}=\frac{n}{\lambda}.$$

Then the related intensity of the transition from i to j shall be equal to

$$\lambda_{ij}(t)=1/\overline{\tau}_{ij}=n/\lambda.$$

The obtained relation is quite consistent with the concept of an Erlang flow of sequence n, as a screened simplest flow wherefrom every *nth* event is extracted. It is clear that the intensity of the obtained extracted flow in this case is n times less than the intensity of the source flow. Let's go back to the formulated problem.

So, the incoming flow and the queuing flow are described respectively by Erlang distributions of sequences n_1 and n_2. Then the equivalent diagram describing the processes of such system functioning is as shown in Fig. 2.

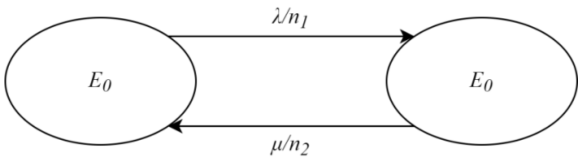

Fig. 2. Graph of states and transitions within the system

The final probability distribution of states within the system is known to be as follows:

$$P_0=\frac{\mu/n_1}{\lambda/n_1+\mu/n_2}=\frac{\mu}{\lambda\cdot n_2/n_1+\mu}=\frac{n_1\mu}{n_2\lambda+n_1\mu},$$

$$P_1=\frac{\lambda/n_1}{\lambda/n_1+\mu/n_2}=\frac{\lambda}{\lambda+\mu\cdot n_1/n_2}=\frac{n_2\lambda}{n_2\lambda+n_1\mu}.$$

Thus, the suggested technique allows to analyze single-channel semi-Markov systems, for which a satisfactory quality approximation of the incoming flow of requirements and the flow of their queuing by Erlang distributions of the appropriate sequence is obtained. The spread of the promising concept of representing models of real systems by the Erlang approximations allows applying effective technologies of state phase aggregation [14]. In addition, it should be noted, that the suggested approach can be used to study multi-threaded queuing systems with differences in priorities defined by the pairwise comparisons method [15].

Conclusions

An easy-to-implement method for approximating models for semi-Markov systems has been suggested. The computational efficiency of the method is defined by the following principal features of the analyzed systems. Firstly, there is a possibility of using the Erlang approximations to describe probability distributions that define the processes of system dynamics. Secondly, the simplicity of the obtained ratios allows to solve both the problems of analyzing the system reliability, and also the problems of their structural and parametric optimization.

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Напівмарківські моделі надійності

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А н о т а ц і я . Традиційні технології аналізу надійності напівмарківських систем обмежуються одержанням стаціонарного розподілу ймовірностей станів. Проте, під час вирішення практичних завдань управління у таких системах значний інтерес має дослідження перехідних процесів. Звідси випливає предмет дослідження – аналіз законів розподілу ймовірностей станів системи. Метою роботи є отримання розподілу на будь-який момент часу. Складність розв'язання поставленого завдання визначається необхідністю отримання результату для довільних законів розподілу тривалості перебування системи в кожному стані до догляду. Запропоновано простий у реалізації метод аналізу напівмарківських моделей надійності. Метод заснований на можливості апроксимації теоретико-імовірнісних описів потоків відмов та відновлення в системі за допомогою законів розподілу Ерланга належного порядку. Розроблена обчислювальна схема використовує найважливішу властивість потоків Ерланга, що формуються в результаті просіювання найпростішого пуассонівського потоку. При цьому напівмарківська модель редукується до марківської, що радикально полегшує процедуру аналізу реальних систем.

К л ю ч о в і с л о в а : напівмарківські моделі надійності; апроксимація розподілами Ерланга належного порядку.

Полумарковские модели надежности

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А н н о т а ц и я . Традиционные технологии анализа надежности полумарковских систем ограничиваются получением стационарного распределения вероятностей состояний. Однако, при решении практических задач управления в таких системах значительный интерес имеет исследование переходных процессов. Отсюда следует предмет исследования – анализ законов распределения вероятностей состояний системы. Целью работы является получение искомого распределения на любой момент времени. Сложность решения поставленной задачи определяется необходимостью получения результата для произвольных законов распределения продолжительности пребывания системы в каждом из состояний до ухода. Предложен простой в реализации метод анализа полумарковских моделей надежности. Метод основан на возможности аппроксимации теоретико-вероятностных описаний потоков отказов и восстановлений в системе с помощью законов распределения Эрланга надлежащего порядка. Разработанная вычислительная схема использует важнейшее свойство потоков Эрланга, формирующихся в результате просеивания простейшего пуассоновского потока. При этом полумарковская модель редуцируется к марковской, что радикально упрощает процедуру анализа реальных систем.

К л ю ч е в ы е с л о в а : полумарковские модели надежности; аппроксимация распределениями Эрланга надлежащего порядка.