Methods of information systems synthesis

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TO THE QUESTION OF CONSTRUCTING THE REGION OF ALLOWABLE VALUES OF VARIABLE PARAMETERS OF A DIGITAL STABILIZER OF A MOVABLE OBJECT

Abstract. Solving the problems of analysis and synthesis of closed digital systems for stabilization of movable objects is associated with significant difficulties. One of the possible ways to solve the problem is the transition from a mathematical model of a continual-discrete closed stabilization system to an approximate mathematical model of a discrete closed system using infinite matrix series containing the own matrix and the control matrix of the continuous part of the system, as well as the quantization period of the discrete part. Using the example of a closed digital stabilization system for a space stage of a solid-propellant carrier rocket flying in an airless space with a marching engine turned on, the problem of constructing stability regions of a closed digital stabilization system in the plane of variable parameters of a digital stabilizer was solved and a comparative analysis of these regions was carried out for various numbers of members of matrix series taken into account and different values of the digital stabilizer quantization period.

Keywords: continual-discrete stabilization system; digital stabilizer; stability region of a closed discrete system; stabilizer quantization period.

Introduction

Problem statement. Let perturbed motion of a movable object is described by vector-matrix differential equation

$$\dot{X}(t) = A \cdot X(t) + B \cdot U(t), \qquad (1)$$

where $X(t)$ is the n-dimensional state vector of the object; $U(t)$ is the m-dimensional control vector; A is the object's own matrix of size $n \times n$; B is the control matrix of size $n \times m$.

Assume that the digital stabilizer realizes stabilization algorithm

$$U[nT] = K \cdot X[nT], \qquad (2)$$

where K is the matrix of constants of algorithm (2) of size $m \times n$, moreover some columns of the matrix K are zero, corresponding to the unmeasured components of the vector $X(t)$.

The problem of parametric synthesis of the digital stabilizer (2) consists in finding the values of the matrix elements K that ensure the stability of the closed system (1), (2) and deliver the required quality of the stabilized processes to the closed system (1), (2).

To solve the problem of parametric synthesis of digital stabilizers of complex high-dimensional objects, an algebraic method is used, based on the use of Optimization Toolbox software package MATLAB or Minimize software MATHCAD [1,2], with the help of which the solution is found in the region G_k representing the stability region of the closed system (1), (2) in the space of variable constants of the algorithm (2). For this, in accordance with the work [3], from the ordinary vector-matrix differential equation (1) one passes to the vector-matrix equation in finite differences

$$X[(n+1)T] = \Phi \cdot X[nT] + H \cdot U[nT], \qquad (3)$$

in which the matrices Φ of size $n \times n$ and H of size $n \times m$ respectively, are determined by the formulas

$$\Phi = \sum_{i=0}^{\infty} \frac{1}{i!} A^i T^i ; \qquad (4)$$

$$H = \sum_{i=0}^{\infty} \left[\frac{1}{(i+1)!} A^i T^{i+1} \right] B . \qquad (5)$$

The number of taken into account members of the matrix series (4) and (5) depends on the value of the quantization period T.

Modern onboard digital computers, used for information processing in complex movable technical objects, perform, in addition to generating control signals, many other functions. First of all, these functions are associated with digital filtering of the output signals of sensors noisy with high-frequency interference, analysis of the technical state of various systems and assemblies that make up the object, solution of navigation problems and decision-making problems by the crew members of a moving object. The combination of these problems leads to the fact that the value of the quantization period of the on-board computer is limited from below T^* by the value below which this value cannot be chosen. In modern on-board computers of movable military objects, this value is T^* = (0,002 – 0,01) c. Let us substitute relation (2) into the right side of equation (3). As a result, we obtain the vector-matrix difference equation of the closed digital stabilization system

$$X[(n+1)T] = [\Phi + H \cdot K] \cdot X[nT]. \qquad (6)$$

Then the characteristic equation of the closed digital stabilization system is written in the form

$$\det\left[\Phi + H \cdot K - E \cdot z\right] = 0\,, \tag{7}$$

where z is complex variable, Z is transformation of the lattice function.

Using the characteristic equation (7) for a given value of the quantization period T^*, it is possible to construct the stability region of a closed digital stabilization system G_k in the space of variable stabilizer constants (2) [4]. It is clear that the region G_k depends not only on the value T^*, but also on the number of considered members of the matrix series (4) and (5).

The purpose of this article is to study the influence of the number of taken into account members of the matrix series (4) and (5) on the stability region G_k of a closed digital stabilization system of a moving object at various values of the quantization period of the on-board computer.

Main material

This purpose is carried out on the example of constructing stability regions of a closed digital stabilization system for a cosmic stage of a solid-propellant carrier rocket flying in an airless space with a marching engine turned on. The equations of angular disturbed motion of such a stage have the following form [5, 6]:

$$\begin{aligned} &\ddot{\psi}(t) = a_{\psi\delta} \cdot \delta(t); \\ &T_1^2 \ddot{\delta}(t) + T_2 \dot{\delta}(t) + \delta(t) = k \cdot u(t), \end{aligned} \tag{8}$$

where $\psi(t)$ is the yaw angle of the stage; $\delta(t)$ is the angle of deviation of the axis of the marching engine from the longitudinal axis of the rocket; $u(t)$ is the control signal generated by the stabilization system; k is the coefficient of proportionality.

As a first step, we assume that stage stabilizer is analog one, realizing the stabilization law in the form of the relation

$$u(t) = k_1 \psi(t) + k_2 \dot{\psi}(t). \tag{9}$$

Let us introduce into consideration the state vector of the stabilization object

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = \begin{bmatrix} \psi(t) \\ \dot{\psi}(t) \\ \delta(t) \\ \dot{\delta}(t) \end{bmatrix}$$

and write the differential equation of the perturbed motion of the stabilized object in the form

$$\dot{X}(t) = A \cdot X(t) + B \cdot u(t)\,, \tag{10}$$

where matrices A and B are equal

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_{\psi\delta} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\dfrac{1}{T_1^2} & -\dfrac{T_2}{T_1^2} \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k}{T_1^2} \end{bmatrix}.$$

Relation (9) can be written in vector-matrix form

$$u(t) = K \cdot X(t)\,, \tag{11}$$

where the matrix of the gain coefficients K is equal

$$K = \begin{bmatrix} k_1 & k_2 & 0 & 0 \end{bmatrix}.$$

Then, taking into account (10) and (11), the equation of the perturbed motion of the closed stabilization system is written

$$\dot{X}(t) = \left[A + B \cdot K\right] X(t)\,, \tag{12}$$

and the characteristic equation of the closed system (12) is written in the form

$$\det\left[A + B \cdot K - E \cdot s\right] = 0\,, \tag{13}$$

where s is complex variable of Laplace transformation.

Substituting the matrices A, B and K, into the characteristic equation (13), and disclosing the determinant, we have

$$s^4 + \frac{T_2}{T_1^2} s^3 + \frac{1}{T_1^2} s^2 - a_{\psi\delta} \frac{k}{T_1^2} k_2 s - a_{\psi\delta} \frac{k}{T_1^2} k_1 = 0\,. \tag{14}$$

In the characteristic equation (14), we make a substitution $s = j\omega$, select the real and imaginary parts in the obtained relation, equate them to zero, and solve the resulting system of two algebraic equations with respect to the variable gains k_1 and k_2:

$$k_1 = \frac{T_1^2 \omega^2}{a_{\psi\delta} k} \left(\omega^2 - \frac{1}{T_1^2} \right); \quad k_2 = -\frac{T_2 \omega^2}{a_{\psi\delta} k}. \tag{15}$$

Relations (15) will be used below when comparing the stability regions of continuous and discrete stabilizers.

Let's move on to considering a digital stabilizer that implements the stabilization algorithm (2) for two options. In the first variant, the matrices (4) and (5) are represented in the form:

$$\Phi = E + AT; \quad H = BT, \tag{16}$$

and in the second – in the form:

$$\Phi = E + AT + \frac{1}{2} A^2 T^2; \quad H = BT + \frac{1}{2} ABT^2. \tag{17}$$

For the first variant, the characteristic equation of the closed stabilization system (7) with the substitution of matrices (16) into it is written

$$\begin{aligned} &\begin{vmatrix} 1-z & T & 0 & 0 \\ 0 & 1-z & a_{\psi\delta} T & 0 \\ 0 & 0 & 1-z & T \\ \dfrac{k}{T_1^2} k_1 T & \dfrac{k}{T_1^2} k_2 T & -\dfrac{1}{T_1^2} T & 1 - z - \dfrac{T_2}{T_1^2} T \end{vmatrix} = \\ &= (1-z)^4 + A_1^{\mathrm{I}} (1-z)^3 + A_2^{\mathrm{I}} (1-z)^2 + \\ &+ A_3^{\mathrm{I}} (1-z) + A_4^{\mathrm{I}} = 0, \end{aligned} \tag{18}$$

moreover

$$A_1^{\mathrm{I}} = -\frac{T_2}{T_1^2}T;\quad A_2^{\mathrm{I}} = \frac{1}{T_1^2}T^2;\quad A_3^{\mathrm{I}} = a_{\psi\delta}\frac{k}{T_1^2}k_2T^3;\quad A_4^{\mathrm{I}} = -a_{\psi\delta}\frac{k}{T_1^2}k_1T^4. \tag{19}$$

For the second variant, equation (7), upon substitution of matrices (17), takes the following form

$$\begin{bmatrix} 1-z & T & \frac{1}{2}a_{\psi\delta}T^2 & 0 \\ 0 & 1-z & a_{\psi\delta}T & a_{\psi\delta}T^2/2 \\ \frac{1}{2}\frac{k}{T_1^2}k_1T^2 & \frac{1}{2}k_2T^2 & 1-z-\frac{T^2}{2T_1^2} & T-\frac{T_2}{2T_1^2}T^2 \\ \left(\frac{k}{T_1^2}T-\frac{T_2k}{2T_1^4}T^2\right)k_1 & \left(\frac{k}{T_1^2}T-\frac{T_2k}{2T_1^4}T^2\right)k_2 & -\frac{T}{T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right) & 1-z-\frac{T_2}{T_1^2}T-\frac{1}{2T_1^2}\left(1-\frac{T_2^2}{T_1^2}\right)T^2 \end{bmatrix} = \tag{20}$$

$$= (1-z)^4 + A_1^{\mathrm{II}}(1-z)^3 + A_2^{\mathrm{II}}(1-z)^2 + A_3^{\mathrm{II}}(1-z) + A_4^{\mathrm{II}} = 0,$$

moreover

$$\begin{gathered} A_1^{\mathrm{II}} = -\frac{T}{T_1^2}\left[T_2 + T\left(1-\frac{T_2^2}{T_1^2}\right)\right]; \\ A_2^{\mathrm{II}} = \frac{T^3}{2T_1^4}\left[T_2 + T\left(1-\frac{T_2^2}{T_1^2}\right)\right] + \frac{T_2^2}{T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right)^2 - \\ -a_{\psi\delta}k\frac{T^4}{4T_1^2}k_1 - a_{\psi\delta}k\frac{T^3}{2T_1^2}k_2 - \\ -a_{\psi\delta}k\frac{T^3}{2T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right)k_2; \\ A_3^{\mathrm{II}} = a_{\psi\delta}k\frac{T^4}{2T_1^4}\left[T_2 + \frac{T}{2}\left(1-\frac{T_2^2}{T_1^2}\right)\right]k_2 + \\ +a_{\psi\delta}k\frac{T^3}{T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right)^2 k_2 + a_{\psi\delta}k\frac{T^4}{2T_1^2}k_1 + \\ +a_{\psi\delta}k\frac{T^5}{4T_1^4}\left[T_2 + \frac{T}{2}\left(1-\frac{T_2^2}{T_1^2}\right)\right]k_1 + \\ +a_{\psi\delta}k\frac{T^4}{2T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right)^2 k_1; \\ A_4^{\mathrm{II}} = -a_{\psi\delta}k\frac{T^5}{2T_1^4}\left[T_2 + \frac{T}{2}\left(1-\frac{T_2^2}{T_1^2}\right)\right]k_1 - \\ -a_{\psi\delta}k\frac{T^4}{T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right)^2 k_1. \end{gathered} \tag{21}$$

For small values T, determinant (20) degenerates into determinant (18), and the values of the coefficients of equation (20) determined by relations (21) approach the values of the coefficients of equation (18) determined by relations (19).

A closed digital stabilization system is stable if all roots of its characteristic equation (18) or (20) are located inside a circle of radius that equal one, of the complex plane z (Schur-Kohn criterion [7]).

To construct the stability regions of a closed digital stabilization system in the plane of variable constants (k_1, k_2), we use the W-transformation method [8, 9], according to which the bilinear transformation

$$z = \frac{1+w}{1-w} \tag{22}$$

defines a conformal mapping of a circle of unit radius of the complex plane z to the imaginary axis of the complex plane w. Replacing (22) in characteristic equations (18) and (20), we obtain new characteristic equations with respect to a complex variable w, in relation to which we can use all the provisions of the theory of stability of continuous dynamical systems, including the D-partition method for constructing stability regions of systems in the space of variable parameters.

In accordance with formulas (21), the coefficients of the characteristic equation (20) A_2^{II} and A_3^{II} represent in the form:

$$A_2^{\mathrm{II}} = A_{20}^{\mathrm{II}} + A_{21}^{\mathrm{II}}k_1 + A_{22}^{\mathrm{II}}k_2;\quad A_3^{\mathrm{II}} = A_{31}^{\mathrm{II}}k_\psi + A_{32}^{\mathrm{II}}k_2,$$

where

$$\begin{gathered} A_{20}^{\mathrm{II}} = \frac{T^3}{2T_1^4}\left[T_2 + T\left(1-\frac{T_2^2}{T_1^2}\right)\right] + \frac{T_2^2}{T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right)^2; \\ A_{21}^{\mathrm{II}} = -a_{\psi\delta}k\cdot T^4/\left(4T_1^2\right); \\ A_{22}^{\mathrm{II}} = -a_{\psi\delta}k\cdot T^3/\left(2T_1^2\right)\cdot\left(2-T\cdot T_2/\left(2T_1^2\right)\right); \\ A_{31}^{\mathrm{II}} = \frac{1}{2}a_{\psi\delta}k\left\{\begin{array}{c} \frac{T^5}{2T_1^4}\left[T_2+\frac{T}{2}\left(1-\frac{T_2^2}{T_1^2}\right)\right] + \\ +\frac{T^4}{T_1^2}\left(2-\frac{T_2}{2T_1^2}T\right)^2 \end{array}\right\}; \\ A_{32}^{\mathrm{II}} = a_{\psi\delta}k\left\{\begin{array}{c} \frac{T^4}{2T_1^4}\left[T_2+\frac{T}{2}\left(1-\frac{T_2^2}{T_1^2}\right)\right] + \\ +\frac{T^3}{T_1^2}\left(1-\frac{T_2}{2T_1^2}T\right)^2 \end{array}\right\}. \end{gathered} \tag{23}$$

The characteristic equations (18) and (20) of both considered variants will be reduced to a single form

$$(1-z)^4 + A_1^{\mathrm{I}}(1-z)^3 + \left[A_{20}^{\mathrm{I}} + A_{21}^{\mathrm{I}}k_1 + A_{22}^{\mathrm{I}}k_2\right]\times \times(1-z)^2 + \left[A_{31}^{\mathrm{I}}k_1 + A_{32}^{\mathrm{I}}k_2\right](1-z) + A_4^{\mathrm{I}} = 0. \quad (24)$$

for variant I and

$$(1-z)^4 + A_1^{\mathrm{II}}(1-z)^3 + \left[A_{20}^{\mathrm{II}} + A_{21}^{\mathrm{II}}k_1 + A_{22}^{\mathrm{II}}k_2\right]\times \times(1-z)^2 + \left[A_{31}^{\mathrm{II}}k_1 + A_{32}^{\mathrm{II}}k_2\right](1-z) + A_4^{\mathrm{II}} = 0. \quad (25)$$

для варианта II. При этом коэффициенты уравнения (24) составляют: for variant II. In this case, the coefficients of equation (24) are:

$$A_1^{\mathrm{I}} = -\frac{T_2}{T_1^2}T;\ \ A_{20}^{\mathrm{I}} = \frac{T^2}{T_1^2};\ A_{21}^{\mathrm{I}} = 0;\ A_{22}^{\mathrm{I}} = 0;$$
$$A_{31}^{\mathrm{I}} = 0;\ A_{32}^{\mathrm{I}} = a_{\psi\delta}\frac{k}{T_1^2}T^3;\ \ A_4^{\mathrm{I}} = -a_{\psi\delta}\frac{k}{T_1^2}T^4k_1. \quad (26)$$

We omit the superscripts in the coefficients of the characteristic equations (24) and (25), using the universal form of representation of each of the equations

$$(1-z)^4 + A_1(1-z)^3 + \left[A_{20} + A_{21}k_1 + A_{22}k_2\right]\times \times(1-z)^2 + \left[A_{31}k_1 + A_{32}k_2\right](1-z) + A_4 = 0. \quad (27)$$

In the characteristic equation (27), we replace (22). As a result, we obtain a new characteristic equation of the closed-loop digital stabilization system with respect to the complex variable w:

$$\begin{aligned}&16w^4 + 8A_1w^4 + 4\left(A_{20} + A_{21}k_1 + A_{22}k_2\right)w^4 + \\ &+2\left(A_{31}k_1 + A_{32}k_2\right)w^4 + A_4k_1w^4 - 8A_1w^3 - \\ &-8\left(A_{20} + A_{21}k_1 + A_{22}k_2\right)w^3 - 4A_4k_1w^3 - \\ &-6\left(A_{31}k_1 + A_{32}k_2\right)w^3 + + + 6\left(A_{31}k_1 + A_{32}k_2\right)w^2 + \\ &4\left(A_{20} + A_{21}k_1 + A_{22}k_2\right)w^2 + 6A_4k_1w^2 - \\ &-2\left(A_{31}k_1 + A_{32}k_2\right)w - 4A_4k_1w + A_4k_1 = 0.\end{aligned} \quad (28)$$

In the new characteristic equation (28), we make a replacement $w = j\omega$, select the real and imaginary parts, equate them to zero. As a result, we obtain a system of two algebraic equations with two unknowns k_1 and k_2:

$$\begin{aligned}&K(T,\omega)k_1 + L(T,\omega)k_2 = M(T,\omega);\\ &P(T,\omega)k_1 + Q(T,\omega)k_2 = N(T,\omega),\end{aligned} \quad (29)$$

where the corresponding coefficients of system (29) are determined by the following relations:

$$\begin{aligned}&K(T,\omega) = \left(4A_{21} + 2A_{31} + A_4\right)\omega^4 - \\ &\quad -\left(4A_{21} + 6A_{31} + 6A_4\right)\omega^2 + A_4;\\ &L(T,\omega) = \left(4A_{22} + 2A_{32}\right)\omega^4 - \left(4A_{22} + 6A_{32}\right)\omega^2;\\ &M(T,\omega) = -\left(16 + 8A_1 + 4A_{20}\right)\omega^4 + 4A_{20}\omega^2;\\ &P(T,\omega) = \left(8A_{21} + 6A_{31} + 4A_4\right)\omega^2 - \left(2A_{31} + 4A_4\right);\\ &Q(T,\omega) = \left(8A_{22} + 6A_{32}\right)\omega^2 - 2A_{32};\\ &N(T,\omega) = -8\left(A_1 + A_{20}\right)\omega^2.\end{aligned} \quad (30)$$

In accordance with Cramer's rule [10], solutions of system (29) are written in the form

$$k_1 = \frac{\Delta_1}{\Delta};\ \ k_2 = \frac{\Delta_2}{\Delta}, \quad (31)$$

where the corresponding determinants are equal:

$$\begin{aligned}&\Delta = \begin{vmatrix} K(T,\omega) & L(T,\omega) \\ P(T,\omega) & Q(T,\omega)\end{vmatrix} = K(T,\omega)Q(T,\omega) - \\ &\quad -P(T,\omega)L(T,\omega);\\ &\Delta_1 = \begin{vmatrix} M(T,\omega) & L(T,\omega) \\ N(T,\omega) & Q(T,\omega)\end{vmatrix} = M(T,\omega)Q(T,\omega) - \\ &\quad -N(T,\omega)L(T,\omega);\\ &\Delta_2 = \begin{vmatrix} K(T,\omega) & M(T,\omega) \\ P(T,\omega) & N(T,\omega)\end{vmatrix} = K(T,\omega)N(T,\omega) - \\ &\quad -P(T,\omega)M(T,\omega).\end{aligned} \quad (32)$$

Calculation results and conclusions

We choose the numerical values of the parameters of the stabilization object equal to [11]:

$$a_{\psi\delta} = -0,25\ s^{-2};\ T_1 = 0,02\ s;$$
$$T_2 = 0,04\ s;\ k = 0,01\ V.$$

Changing ω from zero to infinity, using relations (15), we construct the boundary of the stability region of a closed analog stabilization system (curve 1), and also using relations (31) and (32) - the boundaries of the stability regions of a closed digital stabilization system for the two considered accounting variants the terms of the matrix series (4) and (5) (curves 2 and 3) for different periods of quantization of the on-board computer corresponding to Fig. 1-3.

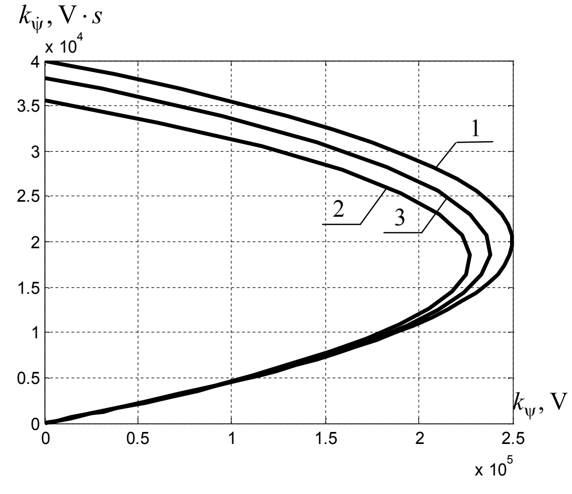

Fig. 1. Construction of boundary of stability region of closed stabilization system for $T = 0{,}001$ c :
1 – analog system; 2 – first option; 3 – second option

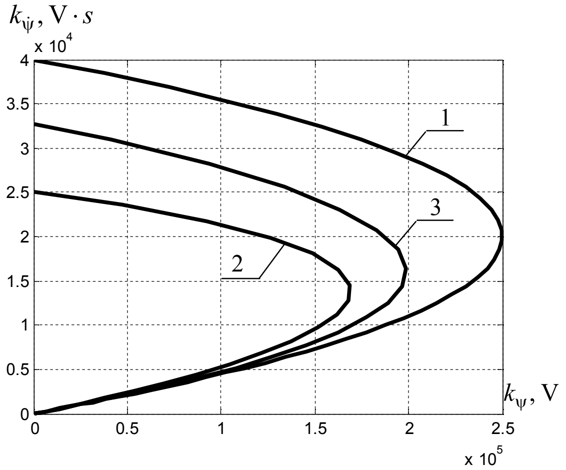

Fig. 2. Construction of boundary of stability region of closed stabilization system for $T = 0{,}005$ c :
1 – analog system; 2 – first option; 3 – second option

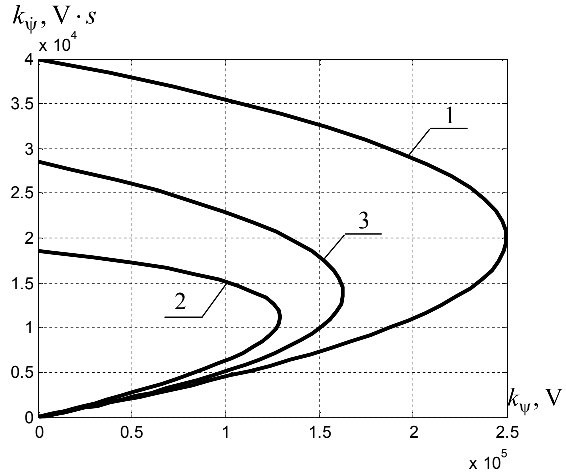

Fig. 3. Construction of boundary of stability region of closed stabilization system for $T = 0{,}01$ c :
1 – analog system; 2 – first option; 3 – second option

Simultaneously with the construction of the boundaries of the stability region, the sign of the determinant Δ is calculated. If the determinant Δ is positive, then the boundary of the stability region is hatched from the left; if the determinant Δ is negative, then the boundary is hatched from the right. In this case, the hatching is directed towards the inside of the stability region. Analysis of the above figures allows us to draw the following conclusions:

- an increase in the quantization period of the on-board computer leads to a decrease in the stability region of the closed digital stabilization system of the cosmic stage of solid-propellant carrier rocket;
- taking into account the terms of the matrix series containing quadratic terms leads to an increase in the stability region of the closed digital stabilization system of the cosmic stage of solid-propellant carrier rocket.

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До питання про побудову області дозволених значень варійованих параметрів цифрового стабілізатора рухомого об’єкта

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Анотація. Рішення задач аналізу і синтезу замкнутих цифрових систем стабілізації рухомих об'єктів пов'язане із значними труднощами, зумовленими тим, що збурений рух безперервної частини замкнутої системи описується системою звичайних диференціальних рівнянь, а функціонування дискретної частини - алгоритмами в кінцевих різницях і різницевими рівняннями. Отримання характеристичного рівняння замкнутої дискретної системи шляхом z-перетворення решітчастої функції, яка відповідає перехідній функції безперервної частини системи, для складних об'єктів, що описуються диференціальними рівняннями високого порядку, часто не представляється можливим. Одним з можливих шляхів вирішення проблеми є перехід від математичної моделі континуально-дискретної замкнутої системи стабілізації до наближеної математичної моделі дискретної замкнутої системи з використанням нескінченних матричних рядів, що містять власну матрицю і матрицю керування безперервної частини системи, а також період квантування дискретної частини. При цьому точність задач аналізу і синтезу замкнутої системи стабілізації визначається кількістю врахованих членів матричних рядів. На прикладі замкнутої цифрової системи стабілізації космічного ступеня твердопаливної ракети-носія, що здійснює політ в безповітряному просторі з включеним маршовим двигуном, вирішена задача побудови областей стійкості замкнутої цифрової системи стабілізації в площині варійованих параметрів цифрового стабілізатора і проведено порівняльний аналіз цих областей при різній кількості врахованих членів матричних рядів і різних значеннях періоду квантування цифрового стабілізатора.

Ключові слова: континуально-дискретна система стабілізації; цифровий стабілізатор; область стійкості замкнутої дискретної системи; період квантування стабілізатора.

К вопросу о построении области допустимых значений варьируемых параметров цифрового стабилизатора подвижного объекта

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Аннотация. Решение задач анализа и синтеза замкнутых цифровых систем стабилизации подвижных объектов связано со значительными трудностями, обусловленными тем, что возмущенное движение непрерывной части замкнутой системы описывается системой обыкновенных дифференциальных уравнений, а функционирование дискретной части – алгоритмами в конечных разностях и разностными уравнениями. Получение характеристического уравнения замкнутой дискретной системы путем z-преобразования решетчатой функции, которая соответствует переходной функции непрерывной части системы, для сложных объектов, описываемых дифференциальными уравнениями высокого порядка, часто не представляется возможным. Одним из возможных путей решения проблемы является переход от математической модели континуально-дискретной замкнутой системы стабилизации к приближенной математической модели дискретной замкнутой системы с использованием бесконечных матричных рядов, содержащих собственную матрицу и матрицу управления непрерывной части системы, а также период квантования дискретной части. При этом точность задач анализа и синтеза замкнутой системы стабилизации определяется количеством учитываемых членов матричных рядов. На примере замкнутой цифровой системы стабилизации космической ступени твердотопливной ракеты-носителя, осуществляющей полет в безвоздушном пространстве с включенным маршевым двигателем, решена задача построения областей устойчивости замкнутой цифровой системы стабилизации в плоскости варьируемых параметров цифрового стабилизатора и проведен сравнительный анализ этих областей при различном числе учитываемых членов матричных рядов и различных значениях периода квантования цифрового стабилизатора.

Ключевые слова: континуально-дискретная система стабилизации; цифровой стабилизатор; область устойчивости замкнутой дискретной системы; период квантования стабилизатора.