Information systems research

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DOA ESTIMATION BASED ON PROXIMITY OF THE ROOTS OF SEVERAL POLYNOMIALS OF SUPERRESOLUTION METHODS

Abstract. **Subject of study** is the performance of methods of the spectral analysis in the presence of outliers. The purpose of this paper is to increase the efficiency of spectral analysis (i.e. to reduce the root mean square error (RMSE) of direction-of-arrival (DOA) estimation based on root similarity approach initially proposed by A. Gershman. The used **methods** are: spectral analysis methods, pattern recognition methods, digital statistical modeling methods. The following **results** were obtained. The root classification approach is used in the case of joint application of two types of data covariance matrix (standard covariance matrix (CM) and estimate of CM with Toeplitz structure). This approach removes the outliers (outlying roots) from preliminary DOA estimates (roots corresponding to the preliminary DOAs). The modification of initial root classification approach is proposed. It consists of avoiding averaging of DOA estimates obtained by estimator for the different CM at high signal-to-noise ratios (SNRs). This step for considered case allows to improve the performance of DOA estimation using the root classification approach. Simulation results are presented confirming the performance of proposed approach. **Conclusions.** The performance improvement of the subspace-based methods of spectral analysis can be attained by removing the outliers from the initial DOA estimates. The simultaneous application of classical second-order CM and estimate of structured CM gives two sets of DOA estimates (roots of polynomials). Root classification approach processes these sets and improves the performance of DOA estimation. The modification proposed in the paper gives the additional advantage at high SNR. The considered approach is also can be used together with other polynomial rooting methods of DOA estimation.

Keywords: Direction-of–arrival estimation; Karhunen-Loève transformation; spectral decomposition of correlation matrix; spectral analysis methods; pattern recognition.

Introduction

Problem formulation in general form. The theory of superresolution has the fundamental results and many practical applications in the radiotechnical and information systems [1-3]. The methods and principles of the superresolution are widely used in the problems of direction-of-arrival (frequency) estimation, time of arrival estimation, delay estimation, multipath parameter estimation, signal separation and localization, signal (pattern) recognition in radar and communication, image processing, passive millimeter-wave imaging, frequency multiplexing, channel estimation, precoding in communication, collision resolution in packet radio networks, directional medium-access-control (MAC) protocol for mobile ad hoc networks (MANET), physical-layer secrecy in wireless systems with MIMO (multiple input-multiple output) [4]. This list can be extended by many applications arising in many areas. Furthermore, the resolution problem can be considered as consisting from the detection-resolution problem, measurement-detection-resolution. In the information systems the problem of resolution can be considered as information extraction from the set of nonorthogonal signals.

The traditional results are refined and extended based on results obtained in nonlinear dynamics, functional analysis, graph spectral theory, random matrix theory, properties of communication signals, sparse representation of signals and so on [4].

Modern methods of superresolution (so called subspace-based or eigenstructure methods) are related with Karhunen-Loève transforms and elements of the functional analysis, principal component analysis. Karhunen–Loève transformation is also widely used for

dimension reduction in signal processing, pattern recognition [1-6].

The performance degradation of the subspace-based methods can be explained by appearance of outliers in the parameter estimates in the practically important situations with small sample, signal-to-noise ratios and so on [1, 2, 4-10].

Performance of the parameter estimation by subspace-based methods can be improved by application of the group of the subspace-based methods with different characteristics (so called joint estimation strategy), subspace-based method with second-order and higher-order CM, resampling approaches (bootstrap, pseudo noise resampling, surrogate data technology), singular spectrum analysis (SSA) [4, 6-10]. The different levels of implementation of the concept of the outlier removing are known [6-10]. The process of elimination can be realized on the level of estimators (i.e. elimination of the estimators that gave parameter estimates with outliers) and on the level of roots of polynomial rooting methods [7].

The aim of the paper is improving the performance of DOA estimation by removing the outliers from the estimation process based on the elimination of the outlying roots from the underlying DOA estimators and using features characterizing the signal roots.

The analysis of the previous investigations and publications. The problem of the outlier reduction in DOA estimation was mentioned in the several works [1, 7-10]. In the general case the source angular sectors were used. This approach was used in the case of estimator bank formed using pseudorandom weighting of eigenvectors, pseudonoise resampling [7-10]. It can

be used in the case of traditional methods related with spatial spectrum (MUSIC, Min-Norm) and with parallel search methods (Root-MUSIC, ESPRIT). As an alternative way the substitution of the preliminary DOA estimates into the likelihood (ML) function is possible [1].

The approach with angular sector requires the information which is known a priori or these sectors must be estimated. In the first and second case (case with ML function) the performance is dependent on quality of CM estimate. Therefore, it is of interest to consider the using the root classification scheme [7] that can be applied to polynomial rooting methods (Root-MUSIC, Root-Min-Norm and so on).

The scheme was used in the context of joint application of two types of CM. The traditional second order and fourth order (cumulant) CMs with Root-MUSIC were considered in [7]. However, the possibility of application of the root classification approach for the case of combined application of the traditional matrix and estimate of Toeplitz matrix was not mentioned.

Approach from [7] consists of test of root similarity and proximity to the unit circle. The part of the approach was considered in [9] and named as distance detection strategy.

The motivation for using distance functions as a classification tool follows from the pattern recognition theory [5]. In turn, the most obvious way of determination of the similarity measure between pattern vectors is in estimation of their proximity.

It should be noted that problem of final root selection arises also for the case of DOA estimation in the case with interpolated arrays [10].

Joint application of the traditional CM and Toeplitz CM was considered in [4]. Source angular sectors were used in that work. However, the problem was the difference in the performance of proposed approach from one of the estimator with Toeplitz CM at low and medium SNRs. Therefore, it is of interest to find the ways of minimizing the mentioned drawback. This can be performed by using results of [7].

Signal model and proposed approach

Assume the uniform linear array of M elements receiving $V < M$ narrowband signals from far-field sources located at unknown DOAs $\theta_1, \cdots, \theta_V$. The number of sources is estimated by one of the known methods such as AIC, MDL [1]. The output of ULA can be modeled as

$$\mathbf{y}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{n}(t) . \qquad (1)$$

The problem is estimation of DOAs of the sources $\mathbf{и} = [\theta_1, \cdots, \theta_V]^T$ based on observations (1). Here $\mathbf{A} = [\mathbf{a}(\theta_1), ..., \mathbf{a}(\theta_V)]$ is the array direction matrix, $\mathbf{x}(t)$ is the vector of source waveforms, $\mathbf{n}(t)$ is the vector of white Gaussian noise, $(\cdot)^T$ denotes the transpose. Using the made assumptions the CM of array output can be written

$$\mathbf{R}_y = E[\mathbf{y}(t)\mathbf{y}^H(t)] = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2\mathbf{I} , \qquad (2)$$

where $\mathbf{R}_s = E[\mathbf{s}(t)\mathbf{s}^H(t)]$ is the $V \times V$ signal CM, $\mathbf{I}$ is the identity matrix, σ^2 is the noise variance, $E[\cdot]$ denotes the statistical expectation. Furthermore, $(\cdot)^H$ stands for the Hermitian transpose operator.

In practice instead of exact CM the sample CM is used [1]

$$\hat{\mathbf{R}} = (1/N)\sum_{t=1}^{N}\mathbf{y}(t)\mathbf{y}^H(t) . \qquad (3)$$

Here N is the number of snapshots.

Computation of the modern spectral analysis (SA) methods is related with spectral decomposition of $\hat{\mathbf{R}}$

$$\hat{\mathbf{R}} = \hat{\mathbf{E}}_s\hat{\mathbf{\Lambda}}_s\hat{\mathbf{E}}_s^H + \hat{\mathbf{E}}_n\hat{\mathbf{\Lambda}}_n\hat{\mathbf{E}}_n^H , \qquad (4)$$

Here $M \times \hat{\mathbf{V}}$ matrix $\hat{\mathbf{E}}$ and $M \times (M - \hat{\mathbf{V}})$ matrix $\hat{\mathbf{E}}_n$ are formed from the eigenvectors of $\hat{\mathbf{R}}$ corresponding to signal-subspace eigenvalues from $\hat{\mathbf{V}} \times \hat{\mathbf{V}}$ matrix $\hat{\mathbf{\Lambda}}_s$ and noise-subspace eigenvalues from $\hat{\mathbf{\Lambda}}_n$, respectively, $\hat{\mathbf{V}}$ is the estimate of the number of sources.

The Root-MUSIC method is based on ULA structure [1, 7, 8]. Root-MUSIC DOA estimates is obtained based on the $\hat{\mathbf{V}}$ roots of polynomial

$$P_{rm}(z) = \mathbf{a}^T(z^{-1})\mathbf{P}^\perp\mathbf{a}(z) , \qquad (5)$$

where $\mathbf{a}(z) = [1, z, ..., z^{M-1}]^T$, $z = \exp(j\omega)$, $\omega = 2\pi d \times \sin(\theta)/\lambda$, d is the distance between the elements, λ is the wave length. Moreover, $\mathbf{\Pi}^\perp = \left(\mathbf{I} - \hat{\mathbf{E}}_s\hat{\mathbf{E}}_s^H\right)$ is the projector on the noise subspace.

The $\hat{\mathbf{V}}$ roots of polynomial named as signal roots are selected base on their closeness to the unit circle. The order of polynomial is $2M - 2$ and roots are in conjugate reciprocal pairs. Only half of these roots (i.e. $M - 1$) is used usually for DOA estimation.

For the high SNR the roots corresponding to the signals lie exactly on the unit circle. However, for the medium and low SNRs these roots appear in conjugate reciprocal pairs. The roots lying inside the unit circle and closest to it are taken for the estimation process.

DOA estimates can be obtained from the corresponding signal roots $z_v, v = 1, ... V$

$$\hat{\theta}_v = \arcsin\left(\left(\frac{\lambda}{2\pi d}\right)\arg(z_v)\right) . \qquad (6)$$

The Toeplitz CM approximation [1, 4] is used to obtain the estimate of Toeplitz CM $\hat{\mathbf{R}}_{TA}$. The polynomial of the Root-MUSIC for such CM is

$$P_{rm}(z) = \mathbf{a}^T(z^{-1})(\mathbf{I} - \hat{\mathbf{U}}_s\hat{\mathbf{U}}_s^H)\mathbf{a}(z) , \qquad (7)$$

where $\hat{\mathbf{U}}_s$ is the matrix formed from the signal subspace eigenvectors of the $\hat{\mathbf{R}}_{TA}$.

The idea of [7] is to eliminate the outlying roots from the DOA estimation process based on root similarity and proximity to the unit circle. Root

similarity can be tested using the following criterion, checking whether two arbitrary roots of the estimators (5) and (7) correspond to the same DOA:

$$\left| \widetilde{\theta}_g^{(1)} - \widetilde{\theta}_f^{(2)} \right| \le \Delta_{sim} . \qquad (8)$$

Here $\widetilde{\theta}_v^{(i)} = \text{sort}\left\{ \widetilde{\theta}(\widetilde{z}_v^{(i)}) \right\}$ is the estimate of the v th DOA, $1 \le v \le V$, Δ_{sim} is the similarity parameter, $| \;|$ denotes the absolute value. The index i corresponds to DOA estimation method. Symbol $\text{sort}\{ \; \}$ is the sorting operator in the ascending order. Furthermore, $\widetilde{z}_v^{(i)}, v = 1,...,V$ are the signal roots which are the closest to unit circle in the complex plane.

Such approach is in some sense close to the known approach named as k-nearest neighbors (or nearest neighbor). The neighbors are used also in the attractor trajectory surrogates [6].

The example of signal roots is indicated in Fig. 1. The scenario with two equipower sources with $\theta_1 = 8°$, $\theta_2 = 12°$, ULA of $M = 9$, $N = 100$ and signal-to-noise ratio SNR=6 dB was used for this case. SNR was defined as $10\log(\sigma_s^2 / \sigma^2)$, where σ_s^2 is the power of sources.

The $M - 1$ roots in the unit circle (only the half of the $2M - 2$ roots) are indicated. Signal roots are also marked by arrows.
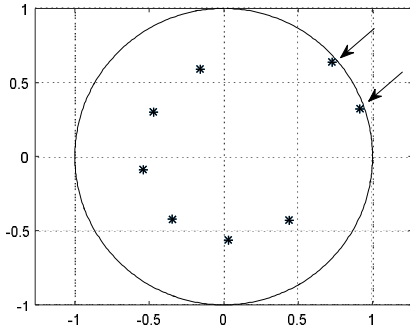

Fig. 1. Complex roots of Root-MUSIC polynomial inside of unit circle

In the mentioned approach the signal roots are sorted based on their proximity to the unit circle [7]

$$0 \le 1 - \left| z_1^{(i)} \right| \le 1 - \left| z_2^{(i)} \right| \le ... \le 1 - \left| z_V^{(i)} \right| . \qquad (9)$$

However, instead of the distance $1 - \left| z_v^{(i)} \right|$ the distance $\left| \left| z_v^{(i)} \right| - 1 \right|$ can be used giving the same result.

In order to explain the principle of similarity and obtain the second group of the roots the pseudonoise resampling was performed as in [8]. Two clusters corresponding to the sources are marked by dashed and dotted circles are shown in the Fig. 2.

It should be noted that traditional measures of similarity in pattern recognition and initial data processing in statistics are Euclidean distance between two patterns, Mahalanobis distance, Kolmogorov distance, Tanimoto coefficient distance measure, correlation distance, chi-square and so on [5].
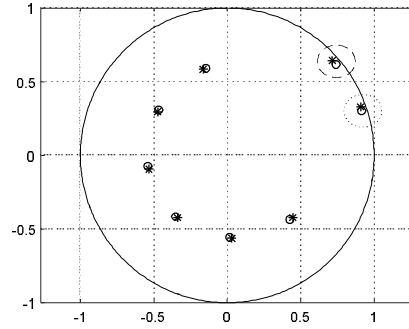

Fig. 2. Complex roots of traditional Root-MUSIC polynomial and one with pseudo-noise resampling

The approach from [7] (named in the paper as combined) adapted for the considered case of application of the traditional CM and estimate of Toeplitz CM can be described by the following sequence of steps:

Step 1. Define the initial value of the Δ_{sim}.

Step 2. Test V^2 possible pairs of signal roots $\left\{ z_g^{(1)}, z_f^{(2)} \right\}$, $g, f = 1,...,V$ using (8). If more than V root pairs satisfy the proximity measure (8) then reduce Δ_{sim}. Process of reduction is performed successively until $l \le V$ root pairs satisfy it.

Step 3. If V signal root pairs $\left\{ \widetilde{z}_1^{(1)}, \widetilde{z}_1^{(2)} \right\}$,..., $\left\{ \widetilde{z}_V^{(1)}, \widetilde{z}_V^{(2)} \right\}$ satisfy the root similarity condition (8) the signal DOAs can be estimated as

$$\hat{\theta}_v = (\widetilde{\theta}_v^{(1)} + \widetilde{\theta}_v^{(2)}) / 2, v = 1,...,V , \qquad (10)$$

where $\widetilde{\theta}_v^{(i)} = \widetilde{\theta}(\widetilde{z}_v^{(i)})$.

Step 4. If only $0 \le l < V$ signal root pairs $\left\{ \widetilde{z}_1^{(1)}, \widetilde{z}_1^{(2)} \right\}$,..., $\left\{ \widetilde{z}_l^{(1)}, \widetilde{z}_l^{(2)} \right\}$ satisfy the condition (8) the l signal DOAs are estimated as

$$\hat{\theta}_v = (\widetilde{\theta}_v^{(1)} + \widetilde{\theta}_v^{(2)}) / 2, v = 1,...,l . \qquad (11)$$

The residuary $V - l$ DOAs are obtained based on remaining $V - l$ signal roots of the estimator which has the smallest distance $1 - \left| z_V^{(i)} \right|$.

Step 5 End.

It should be noted that for the case when the distance between sources and Δ_{sim} are small we can have the situations when DOA estimates of estimators corresponding to different sources can satisfy the condition (8). However, the averaging of the DOA estimates of estimators should be performed for the same sources.

The simulation results of comparison of the traditional Root-MUSIC (with standard CM), Root-MUSIC with estimate of Toeplitz CM (Toeplitz CM approximation (TA)), combined approach based on root classification are presented in Fig. 3. The name combined is selected due to two methods (polynomials) are calculated simultaneously.

Two equipower sources with $\theta_1 = 10°$, $\theta_2 = 14°$ were taken, $N = 50$. The ULA of $M = 7$ antenna elements and

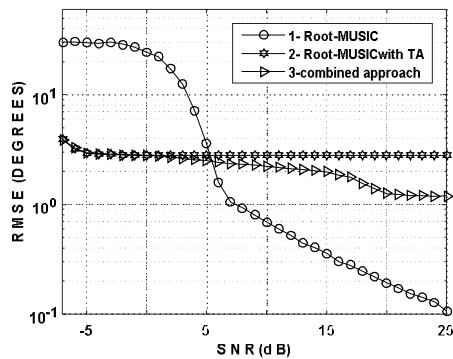**Fig. 3.** RMSE's of DOA
estimation vs. SNR

Fig. 4. Comparison of RMSE's of DOA estimation
vs. SNR for proposed variant of combined approach

1000 simulation runs were used. Parameter $\Delta_{sim} = 2.5°$ was used. RMSE was calculated as in [1].

Here we can see that using Toeplitz CM caused the saturation of RMSE of DOA estimation. Furthermore, performance of combined approach consisting in application of root comparison (similarity) approach is comparable with one of the Root-MUSIC using estimate of Toeplitz CM at medium SNR's and it is better at high SNR's. However, possibility of using the advantages of two estimates of CM is not used at full.

The proposed approach consists in changing the Step 3. The idea of proposed modification is avoiding the averaging of DOA estimates for the situations with high SNR. In such cases the performance of DOA estimation when using the standard CM is better than one with Toeplitz CM. Therefore, at high SNR we also define the distance of the roots of two polynomials to the unit circle. The roots of polynomial with smaller distance are used for DOA estimation.

Simulation results for the scenario which is the same as for the Fig. 3 are presented in the Fig. 4.

It can be seen that the performance of proposed approach improves as compared to initial approach after SNR=15dB. Here we can see the second threshold SNR=22 dB.

It is necessary to say that for such SNR the distances of roots of two polynomials (defined by equations 5 and 7) to the unit circle are approximately the same. Furthermore, the performance of proposed approach at medium and high SNRs is better as compared to Root-MUSIC with approximation of Toeplitz CM. However, such as the value of RMSE for combined approach is not the same as one of Root-
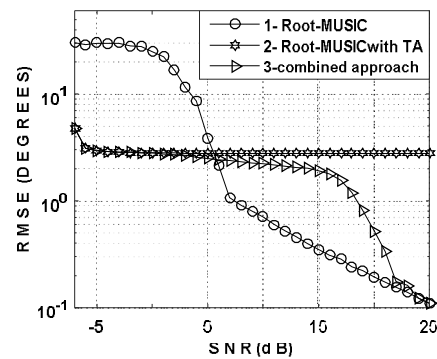
MUSIC it is possible to say that at high SNRs the additional features for defining of appropriate roots can be used.

Conclusions and directions of future investigation

In the paper the idea of outlier reduction based on roots classification and proximity of the roots of polynomial rooting methods is used for the case of joint application of two types of CM (traditional and Toeplitz CM). Furthermore, the modification of approach [7] is proposed. It is based on the fact that in the considered case at the high SNR instead of averaging of DOA estimates corresponding to the similar roots (roots with small distance between them) it is preferable to use the roots based on the their closeness to the unit circle.

The proposed approach can be extended to the case of Root-Min-Norm, Root-MUSIC (Root-Min-Norm) with pseudorandom weighting of eigenvectors, Root-MUSIC (Root-Min-Norm) with pseudonoise resampling and so on. Furthermore, it can be used for the case of interpolated array. The analysis of influence of different methods of distance calculation on the performance of the considered approach should be performed.

It is of interest to generalize the proposed approach also for the communication signals (BPSK, QAM and so on) using the properties of communication signals.

The ways of performance improvement of the Root-MUSIC when using several types of CM in the situations with low SNRs should be investigated.

The fast algorithms can be used for the computation of the roots of polynomial. Tricks allowing avoiding the spectral decomposition of CM or fast subspace decomposition should be considered.

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Відомості про авторів / About the Authors

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Оцінювання напрямків надходження радіохвиль, основане на близькості коренів декількох поліномів методів надрозділення

В. І. Василишин

Анотація. Предметом дослідження є ефективність методів спектрального аналізу при наявності аномальних оцінок (викидів). Метою даної статті є підвищення ефективності спектрального аналізу (зменшення середньоквадратичної похибки (СКП) оцінювання напрямків надходження (НН) радіохвиль) на основі підходу подібності коренів поліномів, запропонованого А. Гершманом. Методи, що використовуються: методи спектрального аналізу, методи розпізнавання образів, методи цифрового статистичного моделювання. Були отримані наступні **результати**. Підхід по класифікації коренів поліномів використовується у випадку спільного застосування двох типів коваріаційної матриці (КМ) даних (стандартної та оцінки КМ з тепліцевою структурою). Цей підхід усуває викиди (корені поліному, що відповідають викидам) з початкових оцінок НН радіохвиль (коренів поліному, що відповідають аномальним оцінкам). Запропонована модифікація початкового підходу класифікації коренів поліному. Вона полягає в усуненні усереднення оцінок НН, отриманих методом оцінювання для різних КМ при високих відношеннях сигнал/шум (ВСШ). Цей крок дозволяє для розглянутого випадку підвищити ефективність оцінювання НН при використанні методу класифікації коренів. Представлені результати моделювання, що підтверджують ефективність запропонованого підходу. **Висновки.** Покращення ефективності методів спектрального аналізу, основаних на використанні підпросторів власних векторів КМ даних може бути досягнуто шляхом видалення викидів з початкових оцінок НН радіохвиль. Одночасне застосування класичної КМ другого порядку та оцінки КМ з заданою структурою дає два набори оцінок НН радіохвиль (коренів полінома). Підхід по класифікації коренів здійснює обробку цих наборів і підвищує ефективність оцінювання НН. Запропонована в статті модифікація дає додаткову перевагу при високому ВСШ. Розглянутий підхід також може бути використаний разом з іншими методами оцінювання НН, основаними на пошуку коренів поліному.

Ключові слова: оцінювання напрямків надходження радіохвиль; теорема Карунена-Лоева; спектральне розкладення кореляційної матриці; методи спектрального аналізу; розпізнавання образів.

Оценивание направлений прихода радиоволн, основанное на близости корней нескольких полиномов методов сверхразрешения

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Аннотация. Предметом исследования является эффективность методов спектрального анализа при наличии аномальных оценок (выбросов). **Целью** данной статьи есть повышения эффективности спектрального анализа (уменьшение среднеквадратичной ошибки (СКО) оценивания направлений прихода радиоволн (НП)) на основе подхода сходства корней полиномов, предложенного А. Гершманом. **Используемые методы**: методы спектрального анализа, методы распознавания образов, методы цифрового статистического моделирования. Были получены следующие **результаты**. Подход по классификации корней полиномов используется в случае применения двух типов ковариационной матрицы (КМ) данных (стандартной и оценки КМ с теплицевой структурой). Этот подход удаляет выбросы (корни полинома, которые отвечают выбросам) из предварительных оценок НП радиоволн (корней полинома, соответствующих аномальным оценкам). Предложена модификация начального подхода классификации корней. Она заключается в устранении усреднения оценок НП, полученных методом оценивания для разных КМ при высоких отношениях сигнал/шум (ОСШ). Этот шаг для рассмотренного случая позволяет повысить эффективность оценивания НП при использовании метода классификации корней. Представленные результаты моделирования, которые подтверждают эффективность предложенного подхода. **Выводы.** Улучшение эффективности методов спектрального анализа, основанных на использовании подпространств собственных векторов КМ данных, может быть достигнуто путем удаления выбросов из начальных оценок НП радиоволн. Одновременное применение классической КМ второго порядка и оценки КМ с заданной структурой дает два набора оценок НП радиоволн (корней полинома). Подход по классификации корней осуществляет обработку этих наборов и повышает эффективность оценивания НП. Предложенная в статье модификация дает дополнительное преимущество при высоком ОСШ. Рассмотренный подход также может быть использован совместно с другими методами оценивания НП, основанными на поиске корней полинома.

Ключевые слова: оценивание направлений поступления радиоволн; теорема Карунена-Лоева; спектральное разложение корреляционной матрицы; методы спектрального анализа; распознавание образов.