

# Methods of information systems synthesis

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## PARAMETRIC SYNTHESIS OF A NON-STATIONARY AUTOMATIC CONTROL SYSTEM OF THE COURSE STABILITY OF THE CAR

**Abstract.** Widely used in the practice of analysis and synthesis of automatic control systems of non-stationary dynamic systems, the method of "frozen coefficients" does not have a rigorous theoretical justification and does not always lead to the desired results. In this regard, to solve the problem of parametric synthesis of a non-stationary automatic control system of the course stability of the car, an algorithmic method is considered for choosing the variable parameters of the regulators of non-stationary objects, based on the direct calculation of the additive integral quadratic quality functional that reflects the set of requirements for the automatic regulator of a non-stationary object, followed by finding the values of the variable parameters a regulator delivering a minimum of quality functional, and the required values of the weight coefficients of the additive functional.

**Keywords:** non-stationary dynamic system; additive quality functional; variable controller parameters; weight coefficients of additive functional; dynamic accuracy of the course stability system of the car.

### Introduction

**Problem statement.** In [1], the problem of choosing the values of the variable parameters of the electronic unit of the automatic control system (ACS) of the course stability of the car was considered, the structural diagram of which is shown in Fig. 1, where the following notation: CIC – command instrument complex; OC – onboard computer; EHA – electrohydraulic amplifier; BP – brake pedal; A – automobile. In the process of braking the car, external disturbances acting from the side of the motion surface, as well as skid and slip of the wheels of the car, lead to the deviation of the car body from a given direction of travel and possibly to an emergency situation. The objective of the automatic control system of the course stability of the car (international designation VSC) is to minimize the possible deviation of the skid of the car body when braking in difficult road conditions.

The CIC contains three gyroscopic sensors of angular velocity AVS and three linear acceleration sensors LAS, the sensitivity axes of which coincide in the direction with the three main central axes of inertia of the car body:  $Ox$  – longitudinal,  $Oy$  – transverse and  $Oz$  – vertical. The output signals of the AVS  $\omega_x(t)$ ,  $\omega_y(t)$ ,  $\omega_z(t)$  and LAS  $w_x(t)$ ,  $w_y(t)$ ,  $w_z(t)$  are fed to the inputs of the analog-code converter (ACC) of the OC, where they are converted into lattice functions  $\omega_x[nT]$ ,  $\omega_y[nT]$ ,  $\omega_z[nT]$ ,  $w_x[nT]$ ,  $w_y[nT]$  and  $w_z[nT]$ . High-frequency interference of lattice functions at the output of the ACC is filtered by Butterworth recursive digital filters [2]. From the output of the filters FB1 – FB6, the filtered lattice functions  $\tilde{\omega}_x[nT]$ ,  $\tilde{\omega}_y[nT]$ ,  $\tilde{\omega}_z[nT]$ ,  $\tilde{w}_x[nT]$ ,  $\tilde{w}_y[nT]$  and  $\tilde{w}_z[nT]$  go to the algorithm block A1, which implements the algorithms of strapdown inertial systems SINS [3], which calculates the angular deviation of the

car body  $\psi[nT]$  and the linear displacement of the center of mass of the body  $y[nT]$  from a given vehicle trajectory in braking process. The output signal of block A1  $\psi[nT]$  is fed to the input of the Lanczos digital filter [4], at the output of which there is a lattice function  $\omega[nT]$  corresponding to the angular velocity of rotation of the car body in its perturbed movement during braking.

Block A2 implements the stabilization algorithm of the car body in the channel of angular stabilization

$$u_\psi[nT] = k_\psi \psi[nT] + k_{\dot{\psi}} \omega[nT], \quad (1)$$

and block A3 implements the stabilization algorithm in the channel of lateral displacement

$$u_y[nT] = k_y y[nT], \quad (2)$$

as well as the stabilization algorithm of the two-channel system

$$u[nT] = k_\psi \psi[nT] + k_{\dot{\psi}} \omega[nT] + k_y y[nT]. \quad (3)$$

The control signal in the form of a lattice function (3) is fed to the input of a code-analog converter (CAC), from the output of which a continuous signal  $u(t)$  is fed to the input of the control winding of the EHA, which forms the brake fluid pressure  $p_r(t)$  and  $p_l(t)$  which is supplied to the brake cylinders of the wheels of the right and left sides of the car.

In relations (1)–(3), through  $k_\psi$ ,  $k_{\dot{\psi}}$  and  $k_y$  are designated the variable constants of the control algorithm. The task of the parametric synthesis of the ACS of the course stability of the car is to find the values of the variable constants  $k_\psi$ ,  $k_{\dot{\psi}}$  and  $k_y$  of the algorithms (1)–(3) that deliver the required properties to a closed ACS. So, in [1], these constants are selected from the condition of maximum margin of stability and maximum speed of closed ACS.

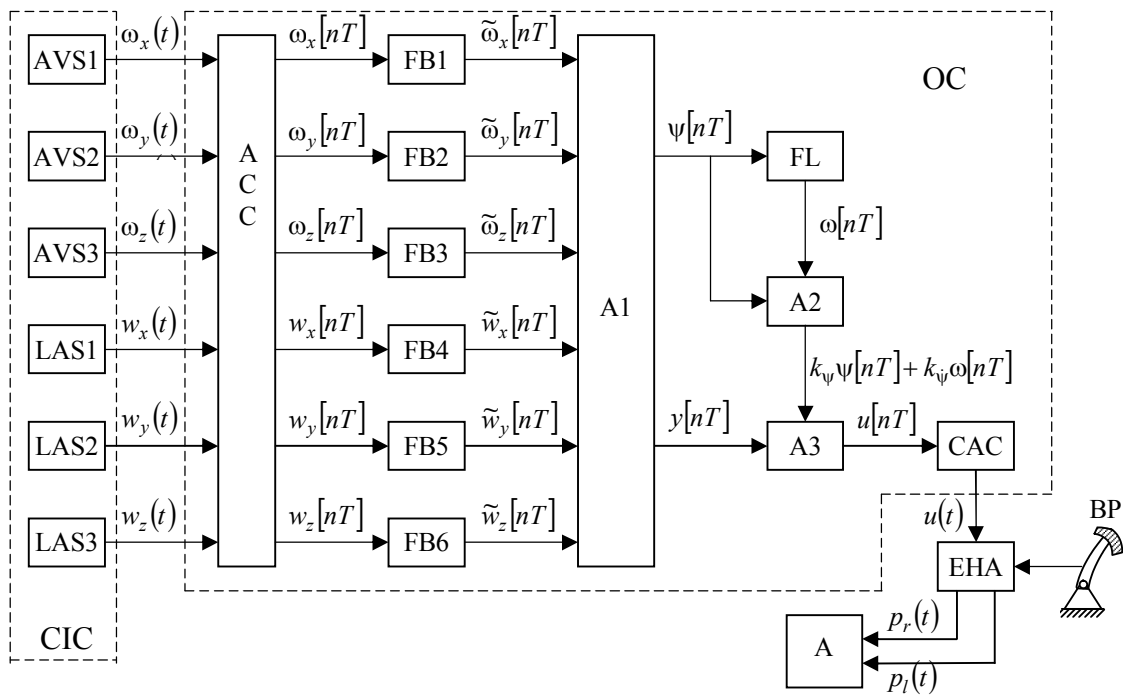


Fig. 1. Block diagram of the automatic control system of the course stability of the car

The control object under consideration – a car with an electro-hydraulic brake amplifier is non-stationary. Indeed, the mathematical model of the perturbed motion of the control object developed in [1] has the following form:

$$I_a \frac{d^2\psi(t)}{dt^2} = k_a \Delta p(t) + M_f(t); \quad (4)$$

$$\frac{dy(t)}{dt} = -v(t)\psi(t); \quad (5)$$

$$L_0 \frac{di(t)}{dt} + r_0 i(t) = u(t); \quad (6)$$

$$I_k \frac{d^2\gamma(t)}{dt^2} + f_k \frac{d\gamma(t)}{dt} + c_k \gamma(t) = k_e i(t); \quad (7)$$

$$\Delta p(t) = k_p \gamma(t), \quad (8)$$

where  $\psi(t)$  is the current angle of deviation of the longitudinal axis of the car body from a given direction of movement during braking;  $y(t)$  is the lateral displacement of the center of mass of the body relative to a given trajectory of the car during braking;  $i(t)$  is the current value of electric current in the control winding of the EHA;  $\gamma(t)$  is the angular deviation of the rocker arm of the EHA electromagnet;  $\Delta p(t)$  is the differential pressure of the working fluid in the brake lines of the right and left sides of the car;  $u(t)$  is the control signal at the output of the CAC converter of the OC;  $M_f(t)$  is the disturbing moment acting on the car body relative to its own vertical axis of inertia;  $v(t)$  is the current speed of the center of mass of the car during braking;  $I_a$

is the moment of inertia of the car body relative to its own vertical axis;  $L_0$  is the inductance of the control winding of the EHA;  $r_0$  is the resistance of the control winding;  $I_k$  is the moment of inertia of the rocker arm of the EHA;  $f_k$  is the coefficient of viscous friction in the axis of rotation of the rocker arm;  $c_k$  is the stiffness coefficient of the fixing spring of the EHA rocker arm;  $k_a, k_e, k_p$  are the proportionality coefficients.

Equations (4)–(8) describe the perturbed motion of the continuous part of closed digital ACS of the course stability of the car. Equation (4) describes the change in the mismatch angle  $\psi(t)$  of the own longitudinal axis of the car body from a given direction of movement; equation (5) is the deviation of the center of mass of the body  $y(t)$  from a given trajectory; equation (6) is the change in the electric current in the control winding of the EHA  $i(t)$ , to which a control signal  $u(t)$  is supplied from the output of the CAC; equation (7) is the change in the angle of rotation of the rocker arm of the EHA electromagnet  $\gamma(t)$  under the action of electric current  $i(t)$ , and equation (8) is the mismatch of the working fluid pressures in the brake lines of the right and left sides of the car when the rocker arm is rotated through an angle  $\gamma(t)$ .

The system of differential equations (4)–(8) is non-stationary, since in equation (5) the coefficient at  $\psi(t)$  is the current velocity of the center of mass of the car  $v(t)$ , which changes in time during braking. In [1], to solve the problem of parametric synthesis of ACS of the course stability of the car, the “frozen coefficients” method is used, according to which the trajectory of a non-stationary object is divided into a number of intervals, on each of which the time-variable coefficients of the mathematical model of the non-stationary object are “frozen”, that is, are assumed to be constant over time. At each of the intervals, the problem of choosing the variable parameters of the ACS for a stationary object is solved,

and then the change in each of the variable parameters in time is extrapolated by the corresponding function of time, which is realized by an analog or digital ACS. Despite the simplicity of the method of "frozen coefficients", this method is not theoretically justified and often leads to unsatisfactory results.

In [5, 6], an algorithmic method for the parametric synthesis of dynamical systems is described, based on the direct calculation of the additive integral quadratic functional, which reflects the set of requirements for ACS, on the solutions of the mathematical model of a closed ACS and on the simulation model of external perturbations acting on the control object with subsequent finding the minimum mathematical expectation of the functional in the space of variable parameters using the Optimization Toolbox software package MATLAB or Minimize software MATHCAD. Moreover, the solution to the problems of analysis and synthesis of ACS of non-stationary technical objects can be obtained without the use of the "frozen coefficients" method and, as a rule, leads to higher quality of controlled processes in a closed ACS.

The purpose of this article is to solve the problem of parametric synthesis of the ACS of the course stability of the car using the algorithmic method of parametric synthesis of dynamic systems, followed by a comparative analysis of the obtained results with the results of use of the "frozen coefficients" method for solving the same problem.

**Main material**

The first step of the algorithmic method of parametric synthesis of dynamical systems, described in [5,6], is the formulation of requirements to system and their formalization. With regard to a closed ACS of the course stability of the car these requirements are reduced to the fact that, firstly, the closed ACS should be stable and, secondly, have high dynamic accuracy. In the theory of analytical construction of optimal regulators (ACOR), developed back in the mid-60s of the previous century, it was concluded that the formalization of the requirements of stability and high dynamic accuracy of the system reduces to the requirement of a minimum of the additive integral quadratic functional calculated on the solutions of dynamic system. In the work [7] of the authors of this article, it was shown that this functional can contain only the so-called "main" coordinates of the dynamic system, which mainly characterize the dynamic process under consideration.

From the consideration of the mathematical model of the perturbed movement of the control object (4)–(8) it follows that the "main" coordinates, which mainly characterize the movement of a car with an electro-hydraulic brake amplifier, are the angle  $\psi(t)$  of deviation of the longitudinal axis of the body from a given direction of movement, the angular velocity  $\dot{\psi}(t)$  of rotation of the body and the current deviation of the center of mass of the body  $y(t)$  from a given trajectory of the car. Then the additive integral quadratic functional, the value of which characterizes the accuracy of the closed ACS of the course stability of the car, is written as

$$I = M_{(j)} \left\{ \int_0^\tau \left[ \beta_1^2 \psi_j^2(t) + \beta_2^2 \dot{\psi}_j^2(t) + \beta_3^2 y_j^2(t) \right] dt \right\}, \quad (9)$$

where  $j = \overline{1, N}$  is the implementation of random process  $M_{(j)}^j(t)$ ;  $M_{(j)}$  – symbol of mathematical expectation of random process implementations  $M_{(j)}^j(t)$ ;  $\tau$  – random process analysis time  $\psi_j(t)$ ,  $\dot{\psi}_j(t)$  and  $y_j(t)$ ;  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  – weighting coefficients of functional (9) to be selected.

The problem of the parametric synthesis of the ACS of the course stability of the car is to find the values of the variable parameters of the ACS  $k_\psi$ ,  $k_{\dot{\psi}}$  and  $k_y$ , which deliver on solutions of the closed ACS (3)–(8) a minimum of the integral quadratic functional (9). It is assumed that the digital ACS uses a zero-order CAC, which converts the lattice function  $u[nT]$  into a piecewise constant function  $u(t)$  [8]:

$$u(t) = \begin{cases} u[nT], & nT \leq t < (n+1)T; \\ u[(n+1)T], & (n+1)T \leq t < (n+2)T. \end{cases} \quad (10)$$

We represent functional (9) in the following form

$$I(k_\psi, k_{\dot{\psi}}, k_y) = \beta_1^2 M_{(j)} \left\{ \int_0^\tau \psi_j^2(t) dt \right\} + \beta_2^2 M_{(j)} \left\{ \int_0^\tau \dot{\psi}_j^2(t) dt \right\} + \beta_3^2 M_{(j)} \left\{ \int_0^\tau y_j^2(t) dt \right\} \quad (11)$$

and introduce the following notation:

$$\begin{aligned} I_1(k_\psi, k_{\dot{\psi}}, k_y) &= M_{(j)} \left\{ \int_0^\tau \psi_j^2(t) dt \right\}; \\ I_2(k_\psi, k_{\dot{\psi}}, k_y) &= M_{(j)} \left\{ \int_0^\tau \dot{\psi}_j^2(t) dt \right\}; \\ I_3(k_\psi, k_{\dot{\psi}}, k_y) &= M_{(j)} \left\{ \int_0^\tau y_j^2(t) dt \right\}. \end{aligned} \quad (12)$$

In contrast to the additive functional (11), relations (12) are called partial functionals. In view of relations (12), the additive functional (11) is written

$$I(k_\psi, k_{\dot{\psi}}, k_y) = \beta_1^2 I_1(k_\psi, k_{\dot{\psi}}, k_y) + \beta_2^2 I_2(k_\psi, k_{\dot{\psi}}, k_y) + \beta_3^2 I_3(k_\psi, k_{\dot{\psi}}, k_y). \quad (13)$$

In [9], a methodology was presented for choosing the values of the weight coefficients of the additive functional, in accordance with which, for the problem under consideration, these values are

$$\begin{aligned} \beta_1 &= \frac{\psi_{\max}}{I_1^* \left( \psi_{\max}^2 / I_1^* + \dot{\psi}_{\max}^2 / I_2^* + y_{\max}^2 / I_3^* \right)}; \\ \beta_2 &= \frac{\dot{\psi}_{\max}}{I_2^* \left( \psi_{\max}^2 / I_1^* + \dot{\psi}_{\max}^2 / I_2^* + y_{\max}^2 / I_3^* \right)}; \\ \beta_3 &= \frac{y_{\max}}{I_3^* \left( \psi_{\max}^2 / I_1^* + \dot{\psi}_{\max}^2 / I_2^* + y_{\max}^2 / I_3^* \right)}, \end{aligned} \quad (14)$$

where  $\psi_{\max}$ ,  $\dot{\psi}_{\max}$ ,  $y_{\max}$  are the maximum possible

values of the "main" coordinates  $\psi_j(t)$ ,  $\dot{\psi}_j(t)$  and  $y_j(t)$ , achieved by realizing a random process  $M_f^j(t)$ ,  $j = \overline{1, N}$ :

$$\begin{aligned} \Psi_{\max} &= \max_{j=1, N; t \in [0, \tau]} \psi_j(t); \dot{\Psi}_{\max} = \max_{j=1, N; t \in [0, \tau]} \dot{\psi}_j(t); \\ y_{\max} &= \max_{j=1, N; t \in [0, \tau]} y_j(t); \end{aligned} \quad (15)$$

$I_1^*$ ,  $I_2^*$ ,  $I_3^*$  – minimum values of partial functionals (12) obtained by minimizing each of them individually

$$\begin{aligned} I_1^* &= \min_{K \in G_k} I_1(K); I_2^* = \min_{K \in G_k} I_2(K); \\ I_3^* &= \min_{K \in G_k} I_3(K), \end{aligned} \quad (16)$$

где  $K$  – vector of variable parameters of the ACS

$$K = [k_\psi \quad k_{\dot{\psi}} \quad k_y]^T; \quad (17)$$

$G_k$  – the set of permissible values of variable parameters.

Finding the minimum of each of the partial functionals (12) under constraint

$$K \in G_k \quad (18)$$

is a solution to the nonlinear programming problem.

The Nelder-Mead method [10], implemented in the Optimization Toolbox software package MATLAB and Minimize software MATHCAD, allows you to find the local minimum of each functional (12) closest to the starting point  $K^0 \in G_k$ . In [11], it was shown that the integral quadratic functional calculated on the solutions of a linear system of differential equations has a single minimum on the set of possible values of the variable parameters  $G_k$ , for which it is recommended to choose the stability region of the closed ACS in the space of variable parameters.

After evaluating the maximum values of the "main" coordinates (15) and calculating the minimum values of the partial functionals (16), the values of the weighting coefficients of the additive functional (9) are calculated, the functional (9) is formed, its mathematical expectation is calculated for  $N$  realizations of the random process  $M_f^j(t)$ ,  $j = \overline{1, N}$ , and then using Optimization Toolbox software package MATLAB or Minimize software MATHCAD it is searched for a vector of variable parameters  $K \in G_k$  that delivers a minimum of functional (9).

$$A(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_a k_p / I_a & 0 \\ -v(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{r_0}{L_0} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & k_e / I_k & -c_k / I_k & -f_k / I_k \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/L_0 \\ 0 \end{bmatrix}; C = \begin{bmatrix} 0 \\ 1/I_a \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Consider the method of constructing the set  $G_k$ , for which we present the system of differential equations (4)–(8) in normal form, introducing the six-dimensional state vector of the control object

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} = \begin{bmatrix} \psi(t) \\ \dot{\psi}(t) \\ y(t) \\ i(t) \\ \gamma(t) \\ \dot{\gamma}(t) \end{bmatrix} \quad (19)$$

and resolving system (4)–(8) relatively to highest derivatives

$$\begin{aligned} \frac{d^2 \psi(t)}{dt^2} &= \frac{k_a k_p}{I_a} \gamma(t) + \frac{1}{I_a} M_f(t); \\ \frac{dy(t)}{dt} &= -v(t) \psi(t); \end{aligned} \quad (20)$$

$$\frac{di(t)}{dt} = -\frac{r_0}{L_0} i(t) + \frac{1}{L_0} u(t);$$

$$\frac{d^2 \gamma(t)}{dt^2} = -\frac{c_k}{I_k} \gamma(t) - \frac{f_k}{I_k} \frac{d\gamma(t)}{dt} + \frac{k_e}{I_k} i(t).$$

Given the notation (19), we represent the system of four differential equations (20) of the first and second orders in the form of a system of six differential equations of the first order:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t); \\ \dot{x}_2(t) &= \frac{k_a k_p}{I_a} x_5(t) + \frac{1}{I_a} M_f(t); \\ \dot{x}_3(t) &= -v(t) x_1(t); \\ \dot{x}_4(t) &= -\frac{r_0}{L_0} x_4(t) + \frac{1}{L_0} u(t); \\ \dot{x}_5(t) &= x_6(t); \\ \dot{x}_6(t) &= -\frac{c_k}{I_k} x_5(t) - \frac{f_k}{I_k} x_6(t) + \frac{k_e}{I_k} x_4(t). \end{aligned} \quad (21)$$

We write system (21) in the vector-matrix form

$$\begin{aligned} \dot{X}(t) &= \\ &= A(t) \cdot X(t) + B \cdot u(t) + C \cdot M_f(t), \end{aligned} \quad (22)$$

where the matrices  $A(t)$ ,  $B$  and  $C$  are equal:

The stability of the dynamic system does not depend on external influences on the system, therefore, in equation (22) we put  $M_f(t) = 0$ , and we take the value of the speed of the center of mass of the car to be maximum and equal  $v_0$ , assuming the change in speed during braking in accordance with the formula

$$v(t) = v_0 - a \cdot t, \quad (23)$$

where  $a$  is the deceleration of the center of mass during braking. At the time  $t = 0$ , the movement of the car is most stable. The stability of the car begins to decrease with the beginning of braking, with a decrease in the current speed of the center of mass  $v(t)$ , when  $v(t) = v_0$ , the stability region of a closed system in the space of variable parameters is most voluminous and begins to decrease with the beginning of the braking process. When  $t = 0$  the equation of the perturbed motion of the control object takes the form

$$\dot{X}(t) = A \cdot X(t) + B \cdot u(t)$$

where the matrix  $A$  is equal

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_a k_p / I_a & 0 \\ -v_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r_0 / L_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & k_e / I_k & -c_k / I_k & -f_k / I_k \end{bmatrix}. \quad (24)$$

The difference vector-matrix equation connecting the initial state of the continuous part of system (21)  $X[kT]$  with its final state  $X[(k+1)T]$  at each discreteness period is written in the form [12]

$$X[(k+1)T] = \Phi \cdot X[kT] + H \cdot u[kT], \quad (25)$$

where the matrices  $\Phi$  and  $H$  are respectively determined by the formulas

$$\Phi = \sum_{i=0}^{\infty} \frac{1}{i!} A^i T^i; \quad H = \sum_{i=0}^{\infty} \left[ \frac{1}{(i+1)!} A^i T^{i+1} \right] B. \quad (26)$$

The number of considered members of the matrix series (25) and (26) depends on the value of the quantization period  $T$ .

Usually, when using modern onboard computers with a short quantization period, with a sufficient degree of accuracy it is assumed

$$\Phi = E + A \cdot T; \quad H = B \cdot T. \quad (27)$$

We write the control algorithm (3) in the vector-matrix form

$$\begin{aligned} & (64 - 32a_1T + 16a_2T^2 - 8a_3T^3 - 4a_4T^4k_{\psi} + 2a_4T^5k_{\psi} + a_4v_0T^6k_y)w^6 + (32a_1T - 32a_2T^2 + 24a_3T^3 + \\ & + 16a_4T^4k_{\psi} - 10a_4T^5k_{\psi} - 6a_4v_0T^6k_y)w^5 + (16a_2T^2 - 24a_3T^3 - 24a_4T^4k_{\psi} + 20a_4T^5k_{\psi} + \\ & + 15a_4v_0T^6k_y)w^4 + (8a_3T^3 + 16a_4T^4k_{\psi} - 20a_4T^5k_{\psi} - 20a_4v_0T^6k_y)w^3 + \\ & + (-4a_4T^4k_{\psi} + 10a_4T^5k_{\psi} + 15a_4v_0T^6k_y)w^2 + (-2a_4T^5k_{\psi} - 6a_4v_0T^6k_y)w + a_4v_0T^6k_y = 0. \end{aligned} \quad (34)$$

$$U[nT] = K \cdot X[nT], \quad (28)$$

where  $K$  is the matrix of variable parameters of the ACS, and

$$K = [k_{\psi} \quad k_{\dot{\psi}} \quad k_y \quad 0 \quad 0 \quad 0]. \quad (29)$$

In the right side of difference equation (24), we substitute relations (27) and (28). As a result, we obtain a difference equation that describes in finite differences the own motion of the closed ACS of the course stability of the car

$$X[(k+1)T] = (E + A \cdot T + B \cdot K \cdot T)X[kT]. \quad (30)$$

Then the characteristic equation of the closed ACS is written in the form

$$\det[A \cdot T + B \cdot K \cdot T + E(1-z)] = 0, \quad (31)$$

where  $z$  is the complex variable of the  $Z$ -transformation of lattice functions.

Substituting the matrices  $A$ ,  $B$ ,  $K$  in equation (31) and revealing the determinant obtained, we write the characteristic equation of the closed discrete ACS of the course stability of the car

$$\begin{aligned} & (1-z)^6 - (1-z)^5 T \times \\ & \times \left( \frac{r_0}{L_0} + \frac{f_k}{I_k} \right) + \left( \frac{r_0}{L_0} \cdot \frac{f_k}{I_k} + \frac{c_k}{I_k} \right) (1-z)^4 T^2 - \\ & - \frac{r_0}{L_0} \cdot \frac{c_k}{I_k} (1-z)^3 T^3 - \frac{k_e k_a k_p}{I_a I_k L_0} (1-z)^2 T^4 k_{\psi} + \\ & + \frac{k_e k_a k_p}{I_a I_k L_0} (1-z) T^5 k_{\psi} + \frac{k_e k_a k_p}{I_a I_k L_0} v_0 T^6 k_y = 0. \end{aligned} \quad (32)$$

According to the notation

$$\begin{aligned} a_1 &= \frac{r_0}{L_0} + \frac{f_k}{I_k}; \quad a_2 = \frac{r_0}{L_0} \cdot \frac{f_k}{I_k} + \frac{c_k}{I_k}; \\ a_3 &= \frac{r_0}{L_0} \cdot \frac{c_k}{I_k}; \quad a_4 = \frac{k_e k_a k_p}{I_a I_k L_0} \end{aligned}$$

characteristic equation (32) takes the form

$$\begin{aligned} & (1-z)^6 - a_1(1-z)^5 T + a_2(1-z)^4 T^2 - \\ & - a_3(1-z)^3 T^3 - a_4(1-z)^2 T^4 k_{\psi} + \\ & + a_4(1-z) T^5 k_{\psi} + a_4 v_0 T^6 k_y = 0. \end{aligned} \quad (33)$$

Using the  $W$ -transformation method [13], in equation (33) we set  $z = \frac{1+w}{1-w}$ .

As a result, we obtain a new characteristic equation for a complex variable  $w$ :

In characteristic (34), we make a replacement  $w = j\omega$ , select the real and imaginary part, and equate them to zero. As a result, we obtain the relations:

$$\begin{aligned}
 & 2a_4T^5\omega^2(-\omega^4+10\omega^2-5)k_{\psi} + \\
 & +4a_4T^4\omega^2(\omega^4-6\omega^2+1)k_{\dot{\psi}} + a_4v_0T^6 \cdot k_y \times \\
 & \times(-\omega^6+15\omega^4-15\omega^2+1) = \omega^4(A_6\omega^2 - A_4); \\
 & 2a_4T^5\omega(-5\omega^4+10\omega^2-1)k_{\psi} + \\
 & +16a_4T^4\omega^3(\omega^2-1)k_{\dot{\psi}} + 2a_4v_0T^6\omega \times \\
 & \times(-3\omega^4+10\omega^2-3)k_y = \omega^3(A_3 - A_5\omega^2),
 \end{aligned} \tag{35}$$

where  $A_3, A_4, A_5$  and  $A_6$  are equal

$$\begin{aligned}
 A_3 &= 8a_3T^3; \quad A_4 = 16a_2T^2 - 24a_3T^3; \\
 A_5 &= 32a_1T - 32a_2T^2 + 24a_3T^3; \\
 A_6 &= 64 - 32a_1T + 16a_2T^2 - 8a_3T^3.
 \end{aligned}$$

In relations (35) we set  $k_y = 0$ . Then relations (35) are written as

$$\begin{aligned}
 & 2a_4T^5(-\omega^4+10\omega^2-5)k_{\psi} + \\
 & +4a_4T^4(\omega^4-6\omega^2+1)k_{\dot{\psi}} = \omega^2(A_6\omega^2 - A_4); \\
 & 2a_4T^5(-5\omega^4+10\omega^2-1)k_{\psi} + \\
 & +16a_4T^4\omega^2(\omega^2-1)k_{\dot{\psi}} = \omega^2(A_3 - A_5\omega^2).
 \end{aligned} \tag{36}$$

Using relations (36), in the plane of the variable parameters  $(k_{\psi}, k_{\dot{\psi}})$  of algorithm (3) we construct the boundary of the stability region of the closed ACS. In accordance with the hatching rule [8], we consider the determinant of the system (36)

$$\begin{aligned}
 \Delta &= \begin{vmatrix} 2a_4T^5 \times & 4a_4T^4 \times \\ \times(-\omega^4+10\omega^2-5) & \times(\omega^4-6\omega^2+1) \\ 2a_4T^5 \times & 16a_4T^4\omega^2 \times \\ \times(-5\omega^4+10\omega^2-1) & \times(\omega^2-1) \end{vmatrix} = \\
 &= 8a_4^2T^9 \begin{vmatrix} -\omega^4+10\omega^2-5 & \omega^4-6\omega^2+1 \\ -5\omega^4+10\omega^2-1 & \omega^2(\omega^2-1) \end{vmatrix}.
 \end{aligned} \tag{37}$$

For small values  $\omega$ , the determinant (37) is positive; therefore, moving along the boundary of the stability region in the direction of increase  $\omega$ , the boundary should be hatched on the left. In this case, the hatching is directed inside the stability region (Fig. 2).

Inside the stability region  $G_{\psi}$ , we choose a point  $(k_{\psi}^*, k_{\dot{\psi}}^*)$ , substitute the values  $k_{\psi}^*$  and  $k_{\dot{\psi}}^*$  in the characteristic equation (34), and solve the equation (34) with respect to the variable parameter  $k_y$ :

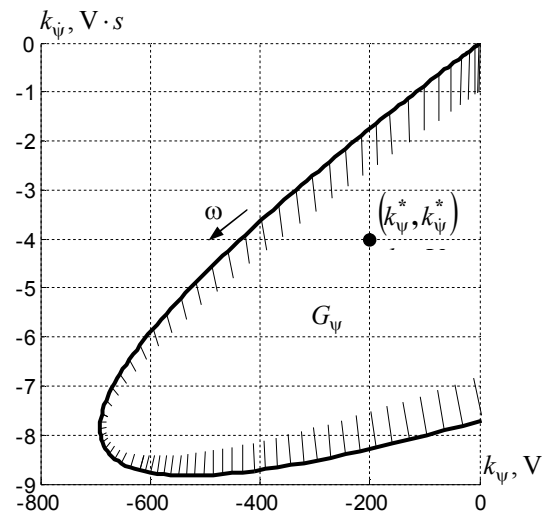


Fig. 2. Construction of stability region  $G_{\psi}$  in plane  $(k_{\psi}, k_{\dot{\psi}})$

$$k_y = \frac{\left( B_6w^6 + B_5w^5 + B_4w^4 + B_3w^3 + B_2w^2 + B_1w \right)}{a_4v_0T^6 \cdot \left( -w^6 + 6w^5 - 15w^4 + 20w^3 - 15w^2 + 6w - 1 \right)}. \tag{38}$$

In relation (38) we substitute  $w = j\omega$  and select the real and imaginary parts. As a result, we have:

$$\text{Re}k_y = \frac{C_y(w)E_y(w) + D_y(w)F_y(w)}{a_4v_0T^6 [E_y^2(w) + F_y^2(w)]}; \tag{39}$$

$$\text{Im}k_y = \frac{D_y(w)E_y(w) - C_y(w)F_y(w)}{a_4v_0T^6 [E_y^2(w) + F_y^2(w)]}, \tag{40}$$

where

$$\begin{aligned}
 B_6 &= 64 - 32a_1T + 16a_2T^2 - \\
 &- 8a_3T^3 - 4a_4T^4k_{\dot{\psi}}^* + 2a_4T^5k_{\psi}^*;
 \end{aligned}$$

$$\begin{aligned}
 B_5 &= 32a_1T - 32a_2T^2 + 24a_3T^3 + \\
 &+ 16a_4T^4k_{\dot{\psi}}^* - 10a_4T^5k_{\psi}^*;
 \end{aligned}$$

$$B_4 = 16a_2T^2 - 24a_3T^3 - 24a_4T^4k_{\dot{\psi}}^* + 20a_4T^5k_{\psi}^*;$$

$$B_3 = 8a_3T^3 + 16a_4T^4k_{\dot{\psi}}^* - 20a_4T^5k_{\psi}^*;$$

$$B_2 = -4a_4T^4k_{\dot{\psi}}^* + 10a_4T^5k_{\psi}^*; \quad B_1 = -2a_4T^5k_{\psi}^*;$$

$$C_y(w) = -B_6w^6 + B_4w^4 - B_2w^2;$$

$$D_y(w) = B_5w^5 - B_3w^3 + B_1w;$$

$$E_y(w) = w^6 - 15w^4 + 15w^2 - 1;$$

$$F_y(w) = 6w^5 - 20w^3 + 6w.$$

Using relations (39) and (40), we construct the boundary of the stability region  $G_y$  of the closed ACS of the course stability of the car in the plane of the complex parameter  $k_y$  (Fig. 3). This region represents a segment of the real axis, concluded between the origin and the point of intersection of the boundary of the stability region with the real axis of the complex plane  $(\text{Re}k_y, \text{Im}k_y)$ .

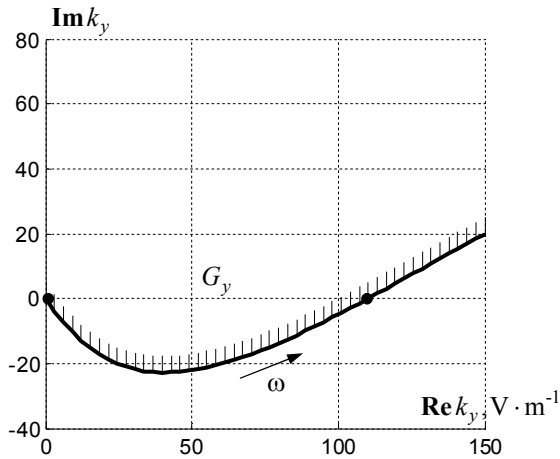


Fig. 3. Construction of stability region  $G_y$  in plane  $(\text{Re}k_y, \text{Im}k_y)$

In the future, to clarify the areas of stability  $G_\psi$  and  $G_y$ , we use the iterative process developed by the authors described in [1]. As a result, we obtain the set of possible values of the varied parameters of the ACS  $G_k$ , which is the intersection of the sets  $G_\psi$  and  $G_y$ :

$$G_k = G_\psi \cap G_y.$$

In this example, the values of the parameters of the mathematical model (4)–(8) were taken equal:

$$L_0 = 10^{-3} \text{ H}; r_0 = 30 \text{ } \Omega;$$

$$I_k = 0,98 \cdot 10^{-2} \text{ N} \cdot \text{m} \cdot \text{s}^2; f_k = 0,55 \text{ N} \cdot \text{m} \cdot \text{s};$$

$$c_k = 1,01 \cdot 10^2 \text{ N} \cdot \text{m}; k_e = 10^3 \text{ N} \cdot \text{m} \cdot \text{A}^{-1};$$

$$k_a = -0,5 \cdot 10^{-4} \text{ N} \cdot \text{m} \cdot \text{Pa}^{-1}; k_p = 3,5 \cdot 10^8 \text{ Pa};$$

$$I_a = 1750 \text{ N} \cdot \text{m} \cdot \text{s}^2; T = 0,003 \text{ s}.$$

Using the algorithmic method of parametric synthesis described above leads to the following optimal values of the variable constants of the stabilization algorithm (3):

$$k_\psi^0 = -13,65 \text{ V}; k_\psi^0 = -4,07 \text{ V} \cdot \text{s}; k_y^0 = 41,02 \text{ V} \cdot \text{m}^{-1}.$$

It was shown in [14] that a random process  $M_f(t)$  is determined by the relation

$$M_f(t) = M_{fl}(t) - M_{fr}(t); \quad (41)$$

where  $M_{fl}(t)$ ,  $M_{fr}(t)$  are the moments of resistance to the rolling of the wheels of the left and right side of the car, which are random functions that depend on the properties of the soil and the microprofile of the surface of movement under the corresponding side. Method for generating implementations of a random function  $M_f^j(t)$ ,  $j = \overline{1, N}$ , where  $j$  is the number of the implementation of a random function  $M_f^j(t)$ ,  $N$  – the number of implementations described in [15] for various types of the motion surfaces and various speeds of the center of mass of the car.

The accuracy of solving the problem of parametric synthesis of ACS depends on the accuracy of estimating the additive functional (9), which is determined by the number of implementations  $N$  of the random process  $M_f^j(t)$ ,  $j = \overline{1, N}$ . Functional (9) is calculated by adding one more differential equation

$$\dot{x}_7^j(t) = \beta_1^2 \psi_j^2(t) + \beta_2^2 \dot{\psi}_j^2(t) + \beta_3^2 y_j^2(t); \quad j = \overline{1, N}. \quad (42)$$

to the mathematical model of the closed (21), (3).

For  $N$  implementations of a random process  $M_f^j(t)$  on solutions of the closed ACS, we find  $N$  implementations of a random function (42). From relations (9) and (42) it follows

$$I_j(K) = x_7^j(\tau, K), \quad (43)$$

therefore,

$$I(K) = M_{(j)} \left\{ x_7^j(\tau, K) \right\} = \frac{1}{N} \sum_{j=1}^N x_7^j(\tau, K). \quad (44)$$

We estimate the variance of a random variable (43) [16]:

$$D(K) = \frac{1}{N-1} \sum_{j=1}^N \left[ I_j(K) - I(K) \right]^2. \quad (45)$$

We set the necessary accuracy of the estimation of functional (44) or the quantities  $\varepsilon$  and  $\beta$ , for which

$$P \left\{ \left| I_j(K) - I(K) \right| \leq \varepsilon \right\} = \beta.$$

In accordance with [16], for a given value  $\beta$ , we find the coefficient  $t_\beta$  and the required number of implementations of the random function  $M_f^j(t)$ ,  $j = \overline{1, N}$

$$\bar{N} = \frac{D(K) t_\beta^2}{\varepsilon^2}; \quad N = \lceil \bar{N} \rceil,$$

where  $\lceil (\cdot) \rceil$  is the Iverson symbol, meaning rounding the number  $(\cdot)$  to the nearest larger integer.

### Conclusions

The “frozen coefficients” method, which is widely used in the practice of developing systems for the

automatic control of non-stationary objects, is not theoretically substantiated and often leads to high dynamic errors of closed ACS.

The main requirements for the automatic control system of the course stability of the car are stability and high accuracy of the stabilized process.

Quantitatively, these requirements can be estimated by the value of the mathematical expectation of the additive functional, the integrand of which is a quadratic form of the “main coordinates” of the mathematical model. The model describes the random stabilized process of the course deviation of the body of the car from a given direction of movement and lateral displacement of the center of mass of the body from a given trajectory during braking.

An alternative to the “frozen coefficients” method is the algorithmic method for the parametric synthesis of automatic control systems for non-stationary objects, based on the direct calculation of the mathematical expectation of the additive integral quadratic functional, calculated on the basis of the mathematical model of the closed ACS with the subsequent selection of the weight coefficients of the additive functional and its optimization using the Optimization Toolbox software package MATLAB and Minimize software MATHCAD. The optimal solution to the parametric synthesis problem is found on the set of permissible vectors of variable constants of the control algorithm, which is the intersection of the stability regions of a closed system in each of the control channels.

#### REFERENCES

1. Aleksandrov, Ye., Aleksandrova, T. and Morhun, Ya. (2019), Parametric synthesis of the electronic control unit of the course stability system of the car, *Eastern-European Journal of Enterprise Technologies*, No. 6(102), pp. 39–45.
2. Aleksandrova, I.Ye., Aleksandrova, T.Ye. and Lazarenko, A.A. (2014), “Tsifrovyye filtry v sistemah avtomobilnoy avtomatiki” [Digital filters in automotive automation systems], *Vestnik Moskovskogo avtomobilno-dorozhnogo gosudarstvennogo tekhnicheskogo universiteta*, No. 1(37), pp. 25–28 (in Russian)
3. Aleksandrova, I.Ye., Aleksandrova, T.Ye. and Kleis, D.M. (2013), “Ispolzovanie parametrov Rodriga-Gamiltona v sistemah avtomaticheskogo upravleniya tormozheniem avtomobilya” [Using Rodrigue-Hamilton parameters in systems of automatic control of car braking], *Vestnik Moskovskogo avtomobilno-dorozhnogo gosudarstvennogo tekhnicheskogo universiteta*, No. 2(34), pp. 22–27 (in Russian).
4. Aleksandrova, T.Ye., Kononenko, V.A. and Lazarenko, A.A. (2011), “Sravnitelnyy analiz PD-stabilizatorov podvizhnykh ob'ektov s nizkochastotnymi filtrami Batterworta i Lantsosha” [Comparative analysis of PD-stabilizers for moving objects with low-frequency Butterworth and Lanczos filters], *Radioelektronika. Informatyka. Upravlinnia*, No. 2, pp. 148–152 (in Russian).
5. Aleksandrov, Ye.Ye., Aleksandrova, T.Ye. and Ovcharenko, Yu.Ye. (2019), *Pidvyshchennia tekhnichnykh ta erhonomichnykh kharakterystyk ob'ektiv viiskovoho pryznachennia* [Improvement of technical and ergonomic characteristics of military objects], KhNADU, Kharkiv, 175 p. (in Ukrainian).
6. Aleksandrov, Ye.Ye. and Aleksandrova, T.Ye. (2015), “Parametric Synthesis of Digital Stabilization System of Tank Gun”, *Journal of Automation and Information Sciences*, No. 41 (11), pp. 1–17.
7. Aleksandrov, Ye.Ye. and Aleksandrova, T.Ye. (2017), “The Method of Main Coordinate in the Theory of Parametric Synthesis of the Linear Stabilized Systems”, *Journal of Automation and Information Sciences*, No. 49 (3), pp. 34–45.
8. Aleksandrov, Ye.Ye., Kozlov, Ye.P. and Kuznyeczov, B.I. (2002), *Avtomatychne keruvannya ruhomykh obyektamy i tehnologichnykh procesamy. Tom 1. Teoriia avtomatichnoho keruvannya* [Automatic control of moving objects and technological processes. Volume 1. Automatic control theory], NTU «KhPI», Kharkiv, 490 p. (in Ukrainian).
9. Aleksandrov, Ye.Ye., Aleksandrova, T.Ye. and Severin, V.P. (2019), *Osnovyi sovremennoy teorii upravleniya* [Fundamentals of modern control theory], KhNADU, Kharkiv, 324 p. (in Russian).
10. Himmelblau, D. (1975), *Prikladnoe nelineynoe programmirovaniye* [Applied Nonlinear Programming], Mir, Moscow, 534 p. (in Russian).
11. Aleksandrov, Ye.Ye. and Aleksandrova, T.Ye. (2014), *Matematicheskoe modelirovaniye, sistemnyy analiz i sintez dinamicheskikh sistem* [Mathematical modeling, systems analysis and synthesis of dynamic systems], NTU «KhPI», Kharkiv, 200 p. (in Russian).
12. Vasilev, S.K., Zaharov, V.N. and Prohorov, Yu.F. (1979), *Kibernetika v sistemah voennogo naznacheniya* [Cybernetics in military systems], Voenizdat, Moscow, 263 p. (in Russian).
13. Tu, Yu. (1964), *Tsifrovyye i impulsnyye sistemy avtomaticheskogo upravleniya* [Digital and pulse automatic control systems], Mashinostroeniye, Moscow, 703 p. (in Russian).
14. Alesandrov, Ye. Ye. (2010). *Osnovy avtomobilnoy avtomatiki* [Automotive Fundamentals], KhNADU, Kharkiv, 172 p. (in Russian).
15. Turenko, A.N., Lebedev, A.T. and Volontsevych, D.O. (2001), *Dinamika transportno-tyagovykh kolesnykh i gusenichnykh mashin* [The dynamics of transport and traction wheeled and tracked vehicles], KhNADU, Kharkiv, 642 p. (in Russian).
16. Venttsel, E.S. (1964), *Teoriya veroyatnostey* [Probability theory], Nauka, Moscow, 576 p. (in Russian).

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### Параметричний синтез нестационарної системи автоматичного керування курсовою стійкістю автомобіля

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**Анотація.** Метод «заморожених коефіцієнтів», що широко використовується в практиці аналізу і синтезу систем автоматичного керування нестационарних динамічних систем, не має строгого теоретичного обґрунтування і не завжди приводить до бажаних результатів. Основними вимогами, що пред'являються до системи автоматичного керування курсовою стійкістю автомобіля, є стійкість і висока точність процесу стабілізації. Кількісно ці вимоги оцінюються значенням математичного очікування адитивного функціоналу якості, підінтегральна функція якого – квадратична форма «головних координат» математичної моделі, яка описує випадковий процес стабілізації курсового відхилення корпусу автомобіля від заданого напрямку руху та бічного зсуву центру мас корпусу від заданої траєкторії в процесі гальмування. У зв'язку з цим для вирішення задачі параметричного синтезу нестационарної системи автоматичного керування курсовою стійкістю автомобіля розглядається алгоритмічний метод вибору варійованих параметрів регуляторів нестационарних об'єктів, заснований на прямому обчисленні математичного очікування адитивного інтегрального квадратичного функціоналу, що обчислюється на рішеннях математичної моделі замкненої системи автоматичного керування з подальшим вибором вагових коефіцієнтів адитивного функціоналу та його оптимізацією з використанням програмних продуктів Optimization Toolbox і Minimize програмних пакетів MATLAB та MATHCAD відповідно. Оптиміальне рішення задачі параметричного синтезу відшукується на множині допустимих векторів варійованих констант алгоритму керування, що представляє собою перетин областей стійкості замкненої системи в кожному з каналів керування. Алгоритмічний метод параметричного синтезу, що використовується в статті, підвищує точність рішення задачі, так як вільний від недоліків, які притаманні методу «заморожених коефіцієнтів».

**Ключові слова:** нестационарна динамічна система; адитивний функціонал якості; варійовані параметри регулятора; вагові коефіцієнти адитивного функціоналу; динамічна точність системи курсової стійкості автомобіля.

### Параметрический синтез нестационарной системы автоматического управления курсовой устойчивостью автомобиля

Е. Е. Александров, Т. Е. Александрова, И. В. Костяник, Я. Ю. Моргун

**Аннотация.** Широко используемый в практике анализа и синтеза систем автоматического управления нестационарных динамических систем метод «замороженных коэффициентов» не имеет строгого теоретического обоснования и не всегда приводит к желаемым результатам. Основными требованиями, предъявляемыми к системе автоматического управления курсовой устойчивостью автомобиля, являются устойчивость и высокая точность стабилизируемого процесса. Количественно эти требования оцениваются значением математического ожидания аддитивного функционала качества, подынтегральная функция которого представляет собой квадратичную форму «главных координат» математической модели, описывающей случайный стабилизируемый процесс курсового отклонения корпуса автомобиля от заданного направления движения и бокового смещения центра масс корпуса от заданной траектории в процессе торможения. В этой связи для решения задачи параметрического синтеза нестационарной системы автоматического управления курсовой устойчивостью автомобиля рассматривается алгоритмический метод выбора варьируемых параметров регуляторов нестационарных объектов, основанный на прямом вычислении математического ожидания аддитивного интегрального квадратичного функционала, вычисляемого на решениях математической модели замкнутой системы автоматического управления с последующим выбором весовых коэффициентов аддитивного функционала и его оптимизацией с использованием программных продуктов Optimization Toolbox и Minimize программных пакетов MATLAB и MATHCAD соответственно. Оптимальное решение задачи параметрического синтеза отыскивается на множестве допустимых векторов варьируемых констант алгоритма управления, представляющего собой пересечение областей устойчивости замкнутой системы в каждом из каналов управления. Используемый в статье алгоритмический метод параметрического синтеза повышает точность решения задачи, так как свободен от недостатков, присущих широко используемому методу «замороженных коэффициентов».

**Ключевые слова:** нестационарная динамическая система; адитивный функционал качества; варьируемые параметры регулятора; весовые коэффициенты аддитивного функционала; динамическая точность системы курсовой устойчивости автомобиля.