Information systems modeling

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MATHEMATICAL MODEL OF OPTIMAL DISTRIBUTION OF APPLIED PROBLEMS OF SAFETY-CRITICAL SYSTEMSOVER THE NODES OF THE INFORMATION AND TELECOMMUNICATION NETWORK

The **subject matter** of the article is the processes of synthesis of the information and telecommunication network (ITN) for solving applied problems of safety-critical systems (SCS). The **goal** is to develop a mathematical model for the optimal distribution of applied tasks of safety-critical systems over the ITN nodes. The **tasks** to be solved are: to formalize the procedure of distribution of applied tasks and SCS software over the ITN nodes; to develop a mathematical model of optimal distribution in order to minimize the cost of network resources; to select an effective algorithm for solving it. The **methods** used are: alternative-graph approach, mathematical optimization models, methods for solving nonlinear integer programming problems with Boolean variables. The following **results** were obtained: the task of selecting the ITN optimal structure was formulated according to the alternative-graph model of information processing; in addition to structural characteristics, the requirements for the parameters necessary for performing applied tasks were taken into account while constructing a mathematical model; when minimizing the cost of a computing resource, constraints related to the capabilities for financing the development and operation of the network are taken into account; the costs for organizing additional connections among the network nodes are considered as well. As a result, a mathematical model of distributing SCS applied problems over the ITN environment was obtained in order to minimize the total costs of computing resource, data transmission, network setting and maintenance. This model is a non-linear integer programming problem with Boolean variables. Taking into account the specific nature of the objective function and model constraints with the use of pseudo-Boolean functions, the original task is reduced to a linear form. The obtained model represents the canonical form of the linear optimizing problem of large-dimensional Boolean programming, for which the method of the recession vector is effective. **Conclusions**. The scientific novelty of the results obtained is as follows: 1) the optimization model of distributing applied tasks over the nodes of the computer network was improved by defining the objective function in order to minimize the costs of both computational and data transmission and the constraints caused by the requirements for the technical and information structure of the network; 2) methods for solving the problems of optimizing the ITN structure on the basis of models of nonlinear Boolean programming by transforming the initial task into a linear form and applying the recession vector method was further developed, which makes obtaining a quasi-optimal solution of the problem in the context of large dimension possible.

Keywords: information and telecommunication network; applied problems; optimal distribution; cost minimization; nonlinear model; Boolean variables.

Introduction

High-quality information and telecommunications processes should be provided so that safety-critical systems (SCS), which belong to complex distributed systems, can operate. In the context of continuously improving concepts of developing information and telecommunication networks (ITN) and new network technologies there is a tendency of their "convergence", i.e. integration into more complex structures and technologies. Due to the heterogeneity and inequality of hardware and software, there is an interpenetration of information and software environments that are different in origin and principles of operation. While developing distributed and local information and telecommunication networks there is a number of unresolved tasks that is a complex scientific and technical problem.

Problem analysis and task setting

The development of a reliable and efficient ITN on the basis of available hardware and software is very important. At the same time, the problem of distribution of applied tasks and SCS special software over the nodes of ITN should be solved [1]. Requirements for efficiency, reliability, continuity and completeness of information should be ensured.

The literature review [2 – 5] showed that the heterogeneity and convergence of the network are not taken into consideration in existing approaches to the distribution of applied tasks of complex systems over ITN nodes. Besides, the main index of ITN performance is data timeliness. Little attention is paid to the problem of minimizing the total costs of network resources [6].

Mathematical models based on the use of results of graph theory and queuing theory [2] do not take into account the dependence of the characteristics of the network structure on the parameters of applied tasks that are solved in a networked environment, which leads to the loss of accuracy in the results of modelling.

Therefore, the goal of this article is to develop a mathematical model for the optimal distribution of applied tasks of safety-critical systems (SCS) over ITN nodes, based on minimizing the cost of network resources. Taking into account the fact that the task is characterized by a large dimension (due to a large number of applied tasks, program modules and nodes of a distributed network), an effective algorithm of its solution should be selected.

Problem solution

When developing the functional structure of the ITN of a distributed system, it is necessary to solve the following problem [5, 9, 10]: to distribute the applied tasks and SCS software modules over the network nodes taking into consideration the network technical structure (as a set of nodes and the links among them). The synthesis of the ITN structure is closely related to the task of optimizing its efficiency in terms of minimizing the costs of network resources [11].

Thus, it is necessary to develop a mathematical model that enables solving the task of selecting:

– variants of performing various types of applied tasks of the distributed system;

- variants of the ITN development in view of mapping a set of applied SCS tasks and software for their implementation at a number of interconnected ITN nodes, optimal in the context of the cost minimization criterion.

Alternative-graph model of information processing is the basis for describing the process of synthesizing the ITN structure and software [12].

The tasks of the SCS information and communication support are solved with the help of a set of system and application software. Let us define graph $G_P = (P, \Gamma_P)$ for a number of applied tasks $P = \{p_k\}, k = \overline{1,l}$, and Γ_P that is a set of arcs representing the interrelations among them.

To implement the information and telecommunication tasks, the ITN technical structure was developed. Let us define graph $G_U = (U, \Gamma_U)$ that describes the structure of the ITN nodes, where $U = \{u_c\}, c = \overline{1,d}$ is a set of graph vertices corresponding to the nodes of the network, Γ_U is a set of arcs representing the system of nodes commutation.

It is necessary to assign the applied SCS tasks to the nodes of the network taking into account special software and channels of information interaction. The result can be represented as a mutual mapping of the following sets: $G_P \leftrightarrow G_U$ are the options for assigning application tasks to the network nodes.

The considered sets of elements of graph structures P and U have large dimensions and include applied tasks, software, hardware, means of communication, etc. Subsets of homogeneous objects are distinguished at each set. Each stage can be considered as a separate element of the structure for performing tasks that consist of several stages and are performed at the different nodes of the ITN in a homogeneous software environment, each stage can be considered to be a separate element of the set. To establish the correspondence, it is necessary to know the set of required properties of the structure and to select such an option among the set of admissible ones that will ensure the achievement of these properties in the context of the given criterion of optimality.

Let us define matrix $X = \|x_{kc}\|$ with the dimensions of $l \times d$, where

$$x_{kc} = \begin{cases} \text{is 1 } if \text{ task k is completed at node } c; \\ \text{otherwise is 0.} \end{cases}$$

Since each applied task is solved only at one node of the network,

$$\sum_{c=1}^{d} x_{kc} = 1, k = \overline{1,l}. \quad (1)$$

For the subset of tasks $P' \subset P$ that are solved only at certain nodes of the system, let us define the matrix $A = \|a_{kc}\|$ with the dimensions of $l \times d$:

$$a_{kc} = \begin{cases} \text{is 1 if the node admits} \\ \quad \text{the solution of task } k; \\ \text{otherwise is 0.} \end{cases} \quad (2)$$

The equation (2) results in

$$\sum_{k=1}^{l} \sum_{c=1}^{d} a_{kc} x_{kc} = l. \quad (3)$$

In addition to the structural characteristics, let us take into account the requirements to the parameters for performing the applied tasks, in particular, the constraints on the permissible tasks performance time at specific nodes of the network. Let us introduce matrix $T = \|t_{kc}\|$ with the dimensions of $l \times d$, whose element t_{kc} represents the duration that is not greater than the time necessary for performing the kth task at the cth node. If in vector

$$T^{dop} = \left\{ t_k^{dop} \right\}$$

element t_k^{dop} denotes the time limit for performing the kth task, then

$$\sum_{c=1}^{d} t_{kc} x_{kc} \le t_k^{dop}, k = \overline{1,l}. \quad (4)$$

Let us define the cost matrix of the network processor time $Z = \|z_{kc}\|$ with the dimensions of $l \times d$, where z_{kc} represents the processor time spent for performing the kth task at the cth node. In this case, the total network processor time spent is

$$\sum_{k=1}^{l} \sum_{c=1}^{d} z_{kc} x_{kc} = z^{sum}.$$

Let us select the subset of tasks $P_v \subset P$ whose input data should be entered only from a certain network node, and the task can be performed at another node. While data transmitting, there happens the network computational burden. To describe these costs, let us introduce matrix $V = \|v_{cj}\|$ with the dimensions of $l \times d$ whose element v_{cj} represents the cost of the computing resource for transmitting the information unit of the applied level from node c to node j. Note that if $c=j$, then $v_{jj}=0$.

Matrix $B=\|b_{kc}\|$ with the dimensions of $l\times d$ for tasks $p_k\in P_v$ describes the amount of input information from nodes c. In the case, if $p_{k*}\notin P_v$ or $p_{k*}\in P_v$, but the information from node $j*$ ($j*\in U$) is not entered, $b_{k*j*}=0$. Then the total cost of the computing resource for transmitting the input information (denoted as z'_{kc}) from task p_k to node u_c is:

$$\sum_{j*=1}^{d} b_{kj*}v_{cj*}=z'_{kc}, k=\overline{1,l}, c=\overline{1,d}.$$

Note that:

a) $p_k\notin P_v$, $z_{kc}=0, c=\overline{1,d}$;

b) $p_k\in P_v$, $z_{kc}=0$ when $b_{kc}=0, c\neq j*$.

When the kth task is completed, the transmitted data is distributed over the network nodes that make up subset $U_{vih}\subset U$. Let us introduce matrix $B'=\|b'_{kc}\|$ with the dimensions of $l\times d$, whose element b'_{kc} denotes the amounts of information of the applied level obtained after its solution that is transmitted by the kth task to the cth node of the network.

Since set U consists of elementary units, the computing facilities at the nodes of the network make only one-program mode possible. A real network node will be subset of $U_1\in U$, whose power is at least 1. Then the total costs for the distribution of information of the ith task over the nodes will be:

$$\sum_{c=1}^{d} v_{kc}b'_{kc}=z_k, k=\overline{1,l}.$$

Let us introduce matrix $B''=\|b''_{km}\|$ with the dimensions of $l\times l$, whose element b''_{km} is equal to the number of information units of the applied level transmitted from task p_k to task p_m. If $b''_{km}=0$, while performing the task the information for the task is not transmitted. In this case, the amount of information transmitted for the kth task at the cth node will be:

$$\sum_{k*=1}^{l}\sum_{c*=1}^{d} b''_{cc*}b''_{kk*}x_{k*c*}=z''_{kc}, k=\overline{1,l}, c=\overline{1,d}.$$

Now let us take into account the cost resources for the ITN development and operation. When minimizing the cost of a computing resource, constraints related to the financing capabilities for the development and operation of the network are considered. If the amount of expenses for hardware at the nodes should not exceed amount F^p, the constraint will be:

$$\sum_{k=1}^{l}\sum_{c=1}^{d} f_c^p x_{kc}\le F^p, \tag{5}$$

where f_c^p is the cost for re-equipment or installation of the cth node.

Let the amount sum that does not exceed F^u be given for the ITN operation for a certain period of time. Consequently,

$$\sum_{k=1}^{l}\sum_{c=1}^{d} f_c^u x_{kc}\le F^u, \tag{6}$$

where f_c^u is the cost of technical operation of unit c.

Thus, the total costs will be $F=F^p+F^u$,

$$f_c=f_c^p+f_c^u, c=\overline{1,d}.$$

After summing (5) and (6)

$$\sum_{k=1}^{l}\sum_{c=1}^{d} f_c\, x_{kc}\le F \text{ is obtained.} \tag{7}$$

When developing a mathematical model, the costs for organizing additional connections among different nodes of the network should be taken into consideration. When the nodes are significantly removed from one another the costs for technical operation of the communication line should be considered as well. Let Z^t be maximum allowable one-off costs for the communication among nodes, $Z^t(\tau)$ is the upper limit of the cost for maintaining communication for a certain period of time τ, then the total costs are:

$$Z=Z^t+Z^t(\tau).$$

Let's denote the costs for the organization and operation of communication between nodes c and j as z^t_{cj}. Then the costs for the node c are:

$$\sum_{j=1}^{d}\sum_{k=1}^{l} z^t_{cj}x_{kc}=z^t_c,$$

In this case, the total costs must satisfy the constraint

$$\sum_{c=1}^{d}\sum_{j=1}^{d}\sum_{k=1}^{l} z^t_{cj}x_{kc}\le Z. \tag{8}$$

The costs at the cth node of the network for solving problem $\tilde Z_{kc}$ consist of direct costs of computing resources for its solution summed up with the costs for receiving and transmitting data while preparing for performing, while performing the task, and after performing. They are defined by the expression:

$$\tilde Z_{kc}=z_{kc}+z_k+z'_{kc}+z''_{kc}=z_{kc}+$$
$$+\sum_{\substack{j=1\\ j\ne c}}^{d} v_{kj}b'_{kj}+\sum_{\substack{j*=1\\ j*\ne c}}^{d} b_{kj*}v_{cj*}+\sum_{k*=1}^{l}\sum_{c*=1}^{d} b''_{cc*}b''_{kk*}x_{k*c*}.$$

Let us develop a mathematical model, according to which the total costs for solving the kth problem at node $\tilde Z_{kc}$ over all the elements of sets P and U are minimized:

$$\sum_{k=1}^{l}\sum_{c=1}^{d}\left(\begin{array}{l} z_{kc}+\sum\limits_{j=1}^{d}(v_{kj}b'_{kj}+v_{cj}b_{kj})+ \\ +\sum\limits_{k^*=1}^{l}\sum\limits_{c^*=1}^{d} b''_{cc^*}b''_{kk^*}x_{k^*c^*} \end{array}\right)x_{kc}\to\min, \quad (9)$$

taking into account the system of constraints (1), (3), (4), (7) and (8).

As a result, a mathematical model for distributing SCS applied tasks in the ITN environment is obtained; while performing it, the task of minimizing the total costs of the computing resource, data transmission, network settings and maintenance should be solved. This mathematical model is a non-linear integer programming problem with Boolean variables of dimension

$$R=ld+2d+1.$$

The constraints in the model are linear, and in the objective function the fourth augends is nonlinear. Such problems are solved basically by the search method[13]. However, the complexity of the method increases exponentially when the problem dimension increases. To reduce the computational complexity of the method of its solution, an algorithm that has a polynomial complexity should be selected, i.e. the model should be considered as P-task whose performance time depends on its dimension in the following way:

$$t(n)=f\left(n^r\right),$$

where r is a polynomial factor, and a is the problem dimension.

Taking into account the specific nature of the objective function and the constraints of the model (9), pseudo-Boolean functions should be used and the initial task should be reduced to a linear form. However, that will lead to a significant increase in the number of variables and constraints, the problem dimension will become equal to

$$R=(ld+1)^2+l(d+2)+2.$$

However, under real-life conditions, a number of non-zero elements of a simplex table of a transformed mathematical model is not great for applied tasks that are solved at the ITN nodes.

So, for the considered problem, the dimension does not increase significantly, and therefore, the objective function (9) should be reduced to the linear P-task if the computational complexity of the algorithm for its solution decreases. Let us replace the nonlinear factor that is the last augends in expression (9):

$$x_{k^*c^*}x_{kc}=y_{kk^*cc^*}.$$

Then the objective function is:

$$\sum_{k=1}^{l}\sum_{c=1}^{d}\left(z_{kc}+\sum_{j=1}^{d}(v_{kj}b'_{kj}+v_{cj}b_{kj})\right)x_{kc}+$$

$$+\sum_{k,k^*=1}^{l}\sum_{c,c^*=1}^{d} b''_{cc^*}b''_{kk^*}y_{kk^*cc^*}\to\min$$

and two more types of inequalities will be added to the initial constraint system:

$$x_{kc}+x_{k^*c^*}-2y_{kk^*cc^*}\ge 0,$$

$$x_{kc}+x_{k^*c^*}-y_{kk^*cc^*}\le 1,$$

$$x_{kc}\in\{0,1\},\, y_{kk^*cc^*}\in\{0,1\},\, k=\overline{1,l}, c=\overline{1,d}.$$

Let us reduce this problem to the canonical form. To do this, the following variables are determined $\left(e=\overline{1,e'},\ c'=1,\overline{d+1}\right)$:

$$\xi_e=\begin{cases} x_{kc}, \text{ when e}=l(k-1)+c, \\ y_{kk^*cc^*}, \text{ when e}=ld+kk^*cc^*; \end{cases}$$

$$\gamma_e=\begin{cases} z_{kc}+\sum\limits_{j=1}^{d}(v_{kj}b'_{kj}+v_{cj}b_{kj}), \text{ when e}=l(k-1)+c; \\ \sum\limits_{k,k^*=1}^{l}\sum\limits_{c,c^*=1}^{d} b''_{cc^*}b''_{kk^*}, \text{ when e}=ld+kk^*cc^*, \end{cases}$$

$$\alpha_{c'e}=\begin{cases} 0, \text{ when } \left[c'\le d,\ c'k<e\le(c'-1)\right] \\ \qquad \text{or } \left[c'=d+1,\ e>kd\right]; \\ 1, \text{ when } \left[\mathrm{c}'\le d,\ (c'-1)l+1\le l\le c'l\right]; \\ a_{kc}, \text{ when } \left[c'=d+1,\ e=d(k-1)+c\right], \end{cases}$$

$$\alpha'_{c'e}=\begin{cases} 0, \text{ when } \left[c'\le d+2,\ c'd<e\le d(c'-1)\right]; \\ t_{kc}, \text{ when } \left[c'\le d,\ d(c'-1)+1<e\le c'd\right]; \\ f_{kc}, \text{ when } \left[c'=d+1,\ e\le ld\right]; \\ \sum_{j=1}^{n} z_{cj}^{t}, \text{ when } \left[c'=d+2,\ e\le ld\right]; \\ \alpha''_{kk^*cc^*}, \text{ when e>d}, \end{cases}$$

where $\alpha''_{kk^*cc^*}$ are coefficients of additional constraints.

Let us also define the variables

$$e_{c'}=\begin{cases} 1, \ if \text{ c}'\le d; \\ d, \ if \text{ c}'\text{=d+1}; \end{cases}$$

$$e_j=\begin{cases} \tau_j, \text{ если j}\le d; \\ \beta_j, \text{ если j>d}, \end{cases}$$

where τ_j and β_j are free members of additional constraints.

Then the linear model of assigning the applied SCS tasks to the ITN nodes in order to minimize total costs is reduced to finding the values of vector $(\xi_1,\ldots\xi_e,\ldots\xi_{e'})$, that provide a minimum of the objective function

$$C_{\min} = \sum_{e=1}^{e'} \gamma_e \xi_e , \qquad (10)$$

and satisfying the constraint system:

$$\begin{gathered} \sum_{e=1}^{e'} \alpha'_{c'e} \xi_e = e_{c'}, c' = \overline{1, d+1}; \\ \sum\nolimits_{e=1}^{e'} \alpha_{je} \xi_e \le e_j, j = \overline{1, d}, \\ \xi_e \in \{0, 1\}, e = \overline{1, e'}. \end{gathered} \qquad (11)$$

The mathematical model (10), (11) represents the canonical form of the linear optimization problem of large-dimensional Boolean programming; there are a large number of methods and algorithms for solving it [14]:

- cutting-plane methods;
- branches and bounds method;
- methods for sequential analysis of options;
- additive methods;
- approximate methods of local optimization;
- lexicographical methods.

The method of the recession vector is relative to approximate methods of local optimization of discrete programming problems. It enables finding the optimal solution by narrowing the local open neighborhoods of the initial approximation in the direction of decreasing the values of the specially formed vector.

Conclusions

According to the suggested pattern of formalization, the problem of selecting the ITN optimal structure is formulated, which enables considering the problem of developing a mathematical model for the distribution of application software over the ITN environment.

The scientific novelty of the results obtained includes:

1) the improvement of optimization model of distributing the applied tasks over the nodes of the computer network by determining the objective function for minimizing the costs for both computing and transmitting the data, and the constraints caused by the requirements for the technical and information structure of the network. The result is a non-linear optimization problem of Boolean programming, whose solution is the basis for the synthesis of the network structure, taking into account the applied tasks of the distributed safety-critical system;

2) further development of methods for solving the problems of optimizing the ITN structure on the basis of models of nonlinear Boolean programming by converting the initial problem into a linear one and applying the method of the recession vector, which makes it possible to obtain a quasi-optimal solution of the problem in the context of large dimensions.

REFERENCES

1. Ageev, D.V., Ignatenko, A.A. and Kopylev, A.N. (2011), "Metodika opredeleniya parametrov potokov na raznykh uchastkakh mul'tiservisnoy telekommunikatsionnoy seti s uchetom effekta samopodobiya" ["Method for determining the parameters of flows in different sections of a multiservice telecommunications network, taking into account the self-similarity effect"], *Problemy telekommunikatsiy,* [*Telecommunications problems*], No. 3 (5), pp. 18-37, available at: http://pt.journal.kh.ua/2011/3/1/113_ageyev_method.pdf (last accessed May 24, 2017).
2. Losev, Yu. I. and Rukkas, K. M. (2007), "Sravnitel'nyy analiz matematicheskogo apparata modelirovaniya telekommunikatsionnykh setey ["Comparative analysis of mathematical apparatus of modeling of telecommunication networks"], *Systemy obrobky informatsiyi [Information Processing Systems]*, No. 8 (66), pp. 55-60.
3. Gelenbe, E. and Pujolle, G. (2010), *Analysis and synthesis of computer systems (2nd Edition)*, Advances in Computer Science and Engineering, 309 p.
4. RFC 1122 – Requirements for Internet Hosts - Communication Layers, available at: http://rfc2.ru/1122.rfc (last accessed May 24, 2017).
5. Marder, N.S. (2006), *Sovremennye telekommunikatsii* [*Modern telecommunications*], Moscow, IRIAS, 255 p.
6. Kuchuk, G.A., Gakhov, R.P. and Pashnev, A.A. (2006), *Upravlenie resursami infokommunikatsiy* [*Information and telecommunication resources management*], Moscow, Fizmatlit, 220 p.
7. Pepelnjak, I. (2000), *EIGRP Network Design Solutions: The Definitive Resource for EIGRP Design, Deployment, and Operation*, CiscoPress, 384 p.
8. Paulsen, S. and Boens, J. (2012), *Summary of the Workshop on information and communication technologies supply chain risk management,* National Institute of Standarts and Technology, 21 p.
9. Kuchuk, G.A. and Pashnev, A.A. (2007), "Klassifikatsiya zadach upravleniya mul'tiservisnymi setyami raspredelennykh informatsionno-upravlyayushchikh system kriticheskogo primeneniya" ["Classification of control tasks for multi-service networks of distributed information-control systems for critical applications"], *Problemy upravlinnyay edynoyu derzhavnoyu systemoyu tsyvil'noho zakhystu: zb. mat. NPK, 4.04.2007*, Kharkiv, MNSU, UTsZU, pp. 104-106.
10. Olifer, V.G. and Olifer, N.A. (2012), *Komp'yuternye seti. Printsipy, tekhnologii, protokoly* [*Computer networks. Principles, technologies, protocols*], SPb. : Piter, 943 p.
11. Kosenko, V.V. and Kuchuk, N.H. (2016), "Modelyuvannya tekhnichnoy i struktury informatsiyno-telekomunikatsiynoyi merezhi na osnovi konkretnoyi realizatsiy iinformatsiynoyi struktury" ["Modeling the structure of technical information and telecommunications network based on the specific implementation of information structure"], *Systemy obrobky informatsiyi* [*Information Processing Systems*], Kharkiv, No. 9(146), pp. 167-171.
12. Kosenko, V.V. and Kuchuk, N.H. (2016), "Vzayemodiya tekhnichnykh I prohramnykh zasobiv pry upravlinni rozpodilom trafika" ["The interaction of hardware and software in the management of traffic distribution"], *Systemy ozbroyennya I viys'kova tekhnika ["Systems of Arms and Military Equipment"]*, No. 3 (47), pp. 72-75.

13. Sobol', I.M. (2006), *Vybor optimal'nykh parametrov v zadachakh so mnogimi kriteriyami* [*The choice of optimal parameters in problems with many criteria*], Moscow, Drofa, 175 p.
14. Ovezgel'dyev, A.O., Petrov, E.G. and Petrov, K.E. (2002), *Sintez i identifikatsiya modeley mnogofaktornogo otsenivaniya i optimizatsii* [*Synthesis and identification of models of multifactorial estimation and optimization*], Kyiv, Naukova dumka, 163 p.

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МАТЕМАТИЧНА МОДЕЛЬ ОПТИМАЛЬНОГО РОЗПОДІЛУ ПРИКЛАДНИХ ЗАДАЧ СИСТЕМ КРИТИЧНОГО ПРИЗНАЧЕННЯ ПО ВУЗЛАХ ІНФОКОМУНІКАЦІЙНОЇ МЕРЕЖІ

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Предметом вивчення в статті є процеси синтезу інформаційно-телекомунікаційної мережі (ІТМ) для вирішення прикладних завдань систем критичного призначення (СКН). **Метою** є розробка математичної моделі оптимального розподілу прикладних задач систем критичного призначення по вузлах ІТМ. **Завдання:** формалізувати процедуру розподілу прикладних задач і програмного забезпечення СКН по вузлах ІТМ; розробити математичну модель оптимального розподілу для мінімізації вартості мережевих ресурсів; вибрати ефективний алгоритм її вирішення. Використовуваними **методами** є: альтернативно-графовий підхід, математичні моделі оптимізації, методи розв'язання нелінійних задач цілочисельного програмування з булевими змінними. Отримані такі **результати.** Згідно альтернативно-графової моделі процесу обробки інформації сформульована задача вибору оптимальної структури ІТМ. При побудові математичної моделі крім структурних характеристик враховані вимоги до параметрів виконання прикладних завдань. При мінімізації витрат обчислювального ресурсу враховуються обмеження, пов'язані з можливостями фінансування розвитку та експлуатації мережі. Враховуються також витрати на організацію додаткових зв'язків між вузлами мережі. В результаті отримана математична модель розподілу прикладних задач СКН в середовищі ІТМ для мінімізації сумарних витрат обчислювального ресурсу, передачі даних, налаштування та обслуговування мережі. Дана модель являє собою нелінійну задачу цілочисельного програмування з булевими змінними. З урахуванням специфіки цільової функції і обмежень моделі з використанням псевдобулевих функцій вихідна задача приводиться до лінійної формі. Отримана модель являє канонічний вид лінійної оптимізаційної задачі булевого програмування великої розмірності, для вирішення якої ефективним є метод вектора спаду. **Висновки.** Наукова новизна отриманих результатів полягає в наступному: ми вдосконалили оптимізаційну модель розподілу прикладних задач по вузлах обчислювальної мережі шляхом визначення цільової функції для мінімізації витрат як обчислювальних, так і передачі даних, і обмежень, зумовлених вимогами до технічної та інформаційної структурі мережі; отримали подальший розвиток методи розв'язання задач оптимізації структури ІТМ на основі моделей нелінійного булева програмування шляхом перетворення вихідної задачі в лінійний вид і застосування методу вектора спаду, що дозволяє отримати квазіоптимальні рішення задачі в умовах великої розмірності.

Ключові слова: інформаційні та телекомунікаційні мережі; прикладні завдання; оптимальний розподіл; мінімізація витрат; нелінійна модель; булеві змінні.

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОПТИМАЛЬНОГО РАСПРЕДЕЛЕНИЯ ПРИКЛАДНЫХ ЗАДАЧ СИСТЕМ КРИТИЧЕСКОГО НАЗНАЧЕНИЯ ПО УЗЛАМ ИНФОКОММУНИКАЦИОННОЙ СЕТИ

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Предметом изучения в статье являются процессы синтеза информационно-телекоммуникационной сети (ИТС) для решения прикладных задач систем критического назначения (СКН). **Целью** является разработка математической модели оптимального распределения прикладных задач систем критического назначения по узлам ИТС. **Задачи:** формализовать процедуру распределения прикладных задач и программного обеспечения СКН по узлам ИТС; разработать математическую модель оптимального распределения для минимизации стоимости сетевых ресурсов; выбрать эффективный алгоритм ее решения. Используемыми **методами** являются**:** альтернативно-графовый подход, математические модели оптимизации, методы решения нелинейных задач целочисленного программирования с булевыми переменными. Получены следующие **результаты.** Согласно альтернативно-графовой модели процесса обработки информации сформулирована задача выбора оптимальной структуры ИТС. При построении математической модели кроме структурных характеристик учтены требования к параметрам выполнения прикладных задач. При минимизации затрат вычислительного ресурса учитываются ограничения, связанные с возможностями финансирования развития и эксплуатации сети. Учитываются также расходы на организацию дополнительных связей между узлами сети. В результате получена математическая модель распределения прикладных задач СКН в среде ИТС для минимизации суммарных затрат вычислительного ресурса, передачи данных, настройки и обслуживания сети. Данная модель представляет собой нелинейную задачу целочисленного программирования с булевыми переменными. С учетом специфики целевой функции и ограничений модели с использованием псевдобулевых функций исходная задача приводится к линейной форме. Полученная модель представляет канонический вид линейной оптимизационной задачи булевого программирования большой размерности, для решения которой эффективным является метод вектора спада. **Выводы.** Научная новизна полученных результатов состоит в следующем: усовершенствована оптимизационная модель распределения прикладных задач по узлам вычислительной сети путем определения целевой функции для минимизации затрат как вычислительных, так и передачи данных, и ограничений, обусловленных требованиями к технической и информационной структуре сети; получили дальнейшее развитие методы решения задач оптимизации структуры ИТС на основе моделей нелинейного булева программирования путем преобразования исходной задачи в линейный вид и применения метода вектора спада, что позволяет получить квазиоптимальное решение задачи в условиях большой размерности.

Ключевые слова: информационные и телекоммуникационные сети; прикладные задачи; оптимальное распределение; минимизация затрат; нелинейная модель; булевы переменные.