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THE INTELLIGENCE THEORY MATHEMATICAL APPARATUS FORMAL BASE

Purpose. The main task of the theory of intelligence is to describe mathematically the laws governing the intellectual activity of a human. This requires to obtain using physical and objective methods to obtain formal description of the subjective states of a human sufficiently complete for practical purposes. Human thoughts, sensations, perceptions and awareness are all subjective states. This paper is tasked to develop a multidimensional predicate model of comparator identification - the basic experimental method of the intelligence theory and to substantiate the axiomatics of this model.

Methods. The comparator identification method developed in this paper provides the possibility of obtaining objective knowledge of subjective states of human intelligence. According to the comparator identification method with his behavior the subject realizes some finite predicate, the properties of which are experimentally studied and mathematically described. The comparator identification method is based on the algebra of finite predicates, Boolean algebra and the axiomatic method. **Results.** As a result of the comparator identification method application, we obtain a mathematical description of the studied subjective states of a subject, as well as the form of the function underlying the transformation of physical objects into subjective images generated by them. **Conclusions.** The results of this paper provide a mathematical substantiation of the possibility of using the comparator identification method in human intelligence modeling.

Keywords: theory of intelligence, algebra of finite predicates, comparator identification.

Introduction

In this article some results aimed at developing the mathematical apparatus of the theory of intelligence are obtained.

As a model in the development of the theory of intelligence, we adopt modern Physics, which, like the theory of the intelligence, has two sides - formal and conceptual.

As a formal teaching, Physics is an experienced science that studies the laws of nature and expresses them in the form of equations.

The need to consider the theory of intelligence as a formal teaching is due to the fact that it needs a special mathematical language that is not sufficiently developed in the available sections of mathematics. Therefore, the theory of intelligence, along with a meaningful study of the mind of a human, is also compelled to develop the necessary formal apparatus. Here this theory of intelligence is not unique.

Thus, the needs of celestial mechanics gave rise to mathematical analysis, the doctrine of the logical abilities of a human stimulated the development of the predicate calculus.

The possibility to expound the theory of intellect in a deductive way, proceeding solely from the physically observed facts, is based on the method of the axiomatic description of the mind of a human. This is a comparison method, or a comparator identification method.

The algebra of finite predicates was developed in this article [1]. In [2-4] some aspects of the theory and practice of comparator identification are considered.

In this article, the development of the theory of comparator identification is continued.

A multidimensional predicate model of comparator identification is proposed, and the axiomatics of this model is justified.

1. Comparison method (comparator identification method)

The essence of the method consists in the fact that the subject (the person whose intellect is being investigated) in specially designed experiments by his physical reactions forms the meanings of some predicates P_1, P_2, \dots, P_r . In these experiments, the properties of predicates are revealed P_1, P_2, \dots, P_r , which are formally written in the form of logical equations connecting predicate variables X_1, X_2, \dots, X_r . Some of these equations are used in the role of axioms or the initial postulates of the theory of intelligence. From axioms, as from equations, there are values of predicate variables X_1, X_2, \dots, X_r , which are respectively predicates P_1, P_2, \dots, P_r .

The internal structure of the found predicates characterizes certain details of the mechanism of the human intellect.

The method of comparison was first used by Newton in the physical study of human color vision. Acting as a test subject, he observed on the comparison fields an arbitrary light radiation x_1, x_2 and recorded the equality or inequality of their color. The predicate formed this way $P(x_1, x_2)$ for the first time connected the Grassmann axioms with logical axioms. Based on Grassmann's postulates (laws), Schroedinger first constructed the deductive theory of human color vision.

In the study of human intelligence by comparison, the researcher influences the senses of the subject experienced by physical signals (stimuli) x_1, x_2, \dots, x_n , generating in his mind certain subjective experiences (states) y_1, y_2, \dots, y_n . It is assumed that the states y_1, y_2, \dots, y_n uniquely depend on the corresponding

expressing the properties of these predicates. In addition, it is necessary to have formal means for describing the internal structure of the stimuli presented to the subject and the states experienced by him, as well as the internal structure of the predicates that the subject realizes. Finally, it is necessary to have mathematical means of extraction from the properties of predicates of their internal structure. The foundations for developing the desired formal language are the concepts of set and relation.

Let's assume that a_1, a_2, \dots, a_k – are various subjects. Their totality is called a set. We will commonly denote sets by the bold uppercase Latin letters. The subjects a_1, a_2, \dots, a_k , which are part of the set, are called its elements. As a rule, elements will be denoted by lowercase Latin letters. Sets may differ from each other by a number k and the composition of the elements in them a_1, a_2, \dots, a_k . To write the set we will use the list of all its elements, enclosed in curly brackets: $\{a_1, a_2, \dots, a_k\}$. The sets can be built not only from the elements, but also from the sets, for example $\{\{a_1\}, \{a_1, a_2\}\}$. Such sets are called sets systems.

The elements in the set are unordered, so the order of enumeration of elements in the set record does not matter. In the record of a set, the same elements can be repeated, but the set itself does not change because it does not have the same elements. If the characters a and b denote the same element, it is said that the elements a and b are equal and is written $a = b$. Otherwise, it is written $a \neq b$. If the sets A and B consist of the same elements, then it is said that they are equal and written $A = B$. If it is false that $A = B$, then it is written $A \neq B$.

The sets just considered are named the finite. The number of elements in them can take any natural value $k = 1, 2, \dots$. Where $k = 0$ we get an empty set \emptyset , which does not contain any elements. Where $k = 1$ we get the singleton sets. Also can be considered the infinite sets for which the value k is not limited to the maximum value. The examples of infinite sets can be a countable set consisting of all natural numbers and the continual set of all real numbers. The power of a continual set is greater than the cardinality of a countable set. There are the sets cardinality of which exceeds the power of the continuum, for example, the set of all real functions.

For an infinite set the role of the number of its elements plays the cardinality of the set. Two sets A and B are named the equipotent, if for each element of the set A can be associated its element of the set B and vice versa. The power of a finite set is the number of its elements. The totality of all objects that are elements of all possible sets that are considered in a particular problem (reasoning, research, theory) is called a universal set or a universe of this problem and is denoted by the character U . It is possible to combine in the same universe, together with elements, also the sets formed from these elements. It is believed that in such a universe the sets differ from the elements, in particular $a \neq \{a\}$.

If the element a is a part of the set A , it is said, that a belongs to A and it is written $a \in A$. The record $a \bar{\in} A$ or $a \notin A$ means that the element a does not belong to the set A . The record $a_1, a_2, \dots, a_n \in A$ means that $a_1 \in A, a_2 \in A, \dots, a_n \in A$. In the role of elements of a set can be used any elements of the universe U . Each element of any set considered in any problem must be an element of the universe of this problem. The relation \in is named an element belonging to a set.

The relation of belonging of the element to the set and the equality of the elements are related by the law of Leibniz: for all a and b $a = b$ only if $a \in A$ is equally matched $b \in A$ at any A . The relation of an element to a set and the equality of sets are connected by the law of capacity or extensionality: for all A and B $A = B$ only if $a \in A$ is equally matched $a \in B$ at any a .

The set A is called a subset or part of the set B , and the set B – the superset of the set A , if every element of the set A belongs also to the set B . In this case it is said that the set A is included in the set B and is written $A \subseteq B$. In the role of sets of elements, can be used any subset of the universe U . Each set, considered in any problem, must be a subset of the universe of this problem:

$$A \subseteq U \quad (2)$$

for any A . Each element that appears in the problem must belong to the universe of this problem:

$$a \subseteq U \quad (3)$$

for any a . The empty set is a subset of any set:

$$\emptyset \subseteq A \quad (4)$$

for any A .

The relation \subseteq is called the inclusion of sets. It is reflexive:

$$A \subseteq A \quad (5)$$

for any A ; anti-symmetrically: $A \subseteq B$ and $B \subseteq A$ is equally matched $A = B$ for any A and B ; transitively: $A \subseteq B$ and $B \subseteq C$ entails $A \subseteq C$ for any A, B, C . If $A \subseteq B$ and $A \neq B$, then A are called proper subsets or regular parts of the set B and is written $A \subset B$. The relation \subset is called a strict inclusion of sets. The sets \emptyset and A are called improper subsets of the set A , all other subsets of the set A – its own subsets.

The totality or the sum $A \cup B$ of sets A and B is set consisting of all elements of the set A and all elements of the set B . The predicating $A \cup B$ is equally matched to the predicating $a \in A$ or $a \in B$ at any a, A, B . Intersection or common part $A \cap B$ of the sets A and B is a set consisting of all such elements, each of which is contained both in the set A , and in the set B . The predicating $a \in A \cap B$ is equally matched to the predicating $a \in A$ and $a \in B$ at any a, A, B .

The operations of union and intersection of sets are idempotent:

$$A \cup A = A, \quad (6) \quad \bar{U} = \emptyset. \quad (26)$$

$$A \cap A = A \quad (7)$$

for any A ; commutative:

$$A \cup B = B \cup A, \quad (8)$$

$$A \cap B = B \cap A \quad (9)$$

for any A and B ; associative:

$$(A \cup B) \cup C = A \cup (B \cup C), \quad (10)$$

$$(A \cap B) \cap C = A \cap (B \cap C), \quad (11)$$

and distributive:

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C), \quad (12)$$

$$(A \cap B) \cup C = (A \cup B) \cap (B \cup C) \quad (13)$$

for any A, B, C .

The unification and the intersections of sets obey the laws of absorption or elimination:

$$(A \cup (A \cap B)) = A, \quad (14)$$

$$(A \cap (A \cup B)) = A \quad (15)$$

for any A and B .

In combination with the universal and empty sets, the operations of union and intersection of sets have the following properties:

$$A \cup \emptyset = A, \quad (16)$$

$$A \cap U = A, \quad (17)$$

$$A \cup U = U, \quad (18)$$

$$A \cap \emptyset = \emptyset \quad (19)$$

at any A .

The sets A and B are called disjoint if $A \cap B = \emptyset$; otherwise these sets are called intersect. A set of B is called the complement of the set A , if $A \cap B = \emptyset$ and $A \cup B = U$. For every set A there is a single complement \bar{A} . at any a and A $a \in \bar{A}$ is equally matched $a \notin A$.

The operation of addition \bar{A} of the set A obeys the double complement law:

$$\bar{\bar{A}} = A \quad (20)$$

for any A ; the Morgan de:

$$\overline{A \cup B} = \bar{A} \cap \bar{B}, \quad (21)$$

$$\overline{A \cap B} = \bar{A} \cup \bar{B} \quad (22)$$

for any A and B . In combination with the universal and empty sets, the operations of union, intersection, and complement of sets have the following properties:

$$A \cup \bar{A} = U, \quad (23)$$

$$A \cap \bar{A} = \emptyset \quad (24)$$

for any A ;

$$\bar{\emptyset} = U, \quad (25)$$

At any A and B the equality $A \cup B = B$ is equally matched to the inclusion $A \subseteq B$, the following inclusions are valid:

$$A \subseteq A \cup B, \quad (27)$$

$$A \subseteq A \cap B. \quad (28)$$

The difference of sets A and B is called the set

$$A \setminus B = A \cap \bar{B}. \quad (29)$$

The system of all subsets of the universe U together with the operations of addition, union, and intersection of sets is called the algebra of sets. The relations (6)–(29) are called the basic identities of the algebra of sets.

Any set M , containing elements 0 and 1, on which two double operations $+$ and \cdot and one single $'$, satisfying at any $a, b, c \in M$ equalities:

$$a + a = a, \quad (30)$$

$$a \cdot a = a, \quad (31)$$

$$a + b = b + a, \quad (32)$$

$$a \cdot b = b \cdot a, \quad (33)$$

$$(a + b) + c = a + (b + c), \quad (34)$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c), \quad (35)$$

$$(a + b) \cdot c = (a \cdot c) + (b \cdot c), \quad (36)$$

$$(a \cdot b) + c = (a + c) \cdot (b + c), \quad (37)$$

$$a + (a \cdot b) = a, \quad (38)$$

$$a \cdot (a + b) = a, \quad (39)$$

$$a + 0 = a, \quad (40)$$

$$a \cdot 1 = a, \quad (41)$$

$$a + 1 = 1, \quad (42)$$

$$a \cdot 0 = 0, \quad (43)$$

$$(a')' = a, \quad (44)$$

$$(a + b)' = a' \cdot b', \quad (45)$$

$$(a \cdot b)' = a' + b', \quad (46)$$

$$a + a' = 1, \quad (47)$$

$$a \cdot a' = 0, \quad (48)$$

$$\bar{0} = 1, \quad (49)$$

$$\bar{1} = 0 \quad (50)$$

is called a Boolean algebra. Relations (30)–(50) are called basic identities of a Boolean algebra.

Not all basic identities of Boolean algebra are independent of each other. Some of them can be derived from the totality of the others.

Thus, from the identities:

$$a + a = a,$$

$$a + b = b + a,$$

$$(a + b) + c = a + (b + c),$$

$$\begin{aligned}
 (a + b) \cdot c &= (a \cdot c) + (b \cdot c), \\
 (a')' &= a, \\
 (a + b)' &= a' \cdot b', \\
 a + (b \cdot b') &= a
 \end{aligned}
 \tag{51}$$

all the other basic identities of Boolean algebra are derived. The identity (51), which is absent in the list of basic identities of a Boolean algebra, follows from the identities $a + 0 = a$ and $a \cdot a' = 0$.

The recently given seven identities (51) are logically independent from each other, they are called axioms of Boolean algebra.

Any non-empty set M , on which the operations $+$ and \cdot are given, subordinate to these axioms, is a Boolean algebra. From the axioms of Boolean algebra follows the existence and uniqueness of zero $0 = a \cdot a'$ and a figure $1 = a + a'$.

If 0 is taken as a set \emptyset , 1 is taken as a set U , $+$ is taken as an operation, \cdot , $'$ – correspondingly the operations \cup , \cap , $\bar{}$ over the sets of a set U , then the Boolean algebra turns into one of its varieties – the algebra of sets. Operations \cup , \cap , $\bar{}$ are called Boolean operations over sets. The axioms of Boolean algebra now play the role of axioms of algebra of sets, which can be written in the form of identities:

$$\begin{aligned}
 A \cup B &= B \cup A, \\
 (A \cup B) \cup C &= A \cup (B \cup C), \\
 (A \cup B) \cap C &= (A \cap C) \cup (B \cap C), \\
 \overline{\overline{A}} &= A, \\
 \overline{A \cup B} &= \overline{A} \cap \overline{B}, \\
 A \cup (B \cap \overline{B}) &= A.
 \end{aligned}
 \tag{52}$$

Conclusions

From an applied point of view, the language of finite mathematics seems quite acceptable for the theory of intelligence, since any artificial intelligence systems have a finite complexity. With their help, you can practically reproduce only those intellectual processes that allow a mathematical description in the language of finite mathematics.

So, let's focus on the final mathematics in the role of the universal language of the theory of intelligence.

But in which specific form of an algebraic system should it be used in the theory of intelligence. For this purpose, can be used the algebra of finite predicates.

This recommendation is based on the completeness of the algebra of finite predicates.

In the language of the algebra of finite predicates, can be written any finite relation and any finite function.

This means that in the language of the algebra of finite predicates, any law of intelligence and any intellectual activity realized on a computer can be expressed.

All that can be expressed in the language of the algebra of finite predicates can also be practically reproduced on a computer. And on the contrary, everything that can be implemented on a computer can also be written in the language of the algebra of finite predicates.

Thus, there is an exact correspondence between the descriptive possibilities of the algebra of finite predicates and the capabilities of computers to actually implement the descriptions of this algebra. The conclusion about the admissibility of the algebra of finite predicates for the theory of intelligence is also reinforced by the fact that literally all paths lead to the algebra of finite predicates.

So, if the language of graph theory is supplemented with a formal apparatus, then as a result it is obtained the algebra of finite predicates.

If the algebra of logic is generalized and go from binary to alphabetic ones, it is also obtained the algebra of finite predicates.

If a multivalued logic is supplemented with a language for writing relations, we again come to the algebra of finite predicates. Finally, if we take a finite fragment of the logic of predicates and algebraize it, then in this case we are led to the same algebra of finite predicates.

It is very important that the algebra of finite predicates serves for the theory of intellect not only as a formal language for describing the laws of the intellect and intellectual activity of man. Its role is much more significant. Without exaggeration, we can say that the algebra of finite predicates in action is actually the intellect.

The structures of the algebra of finite predicates express the very essence of intellectual processes and phenomena, allowing the direct interpretation in psychological terms.

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Формальна база математичного апарату теорії інтелекту

Кудхаїр Абед Тамер

Мета Головне завдання теорії інтелекту - математично описати закони, що регулюють інтелектуальну діяльність людини. Для цього необхідно отримати фізичні та об'єктивні методи отримання формального опису суб'єктивних станів людини, достатньо повних для практичних цілей. Людські думки, відчуття, сприйняття та усвідомлення - це всі суб'єктивні стани. У цій статті поставлено завдання розробити багатовимірну предикатну модель компараторної ідентифікації - основного експериментального методу теорії інтелекту, і обґрунтувати аксіоматику цієї моделі. **Методи.** Метод компараторної ідентифікації, розроблений в даній статті, дає можливість отримати об'єктивне знання суб'єктивних станів людського інтелекту. За методом компараторної ідентифікації з його поведінкою суб'єкт реалізує деякий кінцевий предикат, властивості якого експериментально вивчені та математично описані. Метод компараторної ідентифікації заснований на методах алгебри та скінченних предикатів, булевої алгебри і аксіоматичному методі. **Результати.** Застосування методу компараторної ідентифікації дає математичний опис досліджуваних суб'єктивних станів людини, а також вид функції, що лежить в основі перетворення фізичних предметів в породжувані нею суб'єктивні образи. **Висновки.** Результати роботи математично обґрунтовують можливості застосування методу компараторної ідентифікації при моделюванні інтелекту людини.

Ключові слова: теорія інтелекту, алгебра скінченних предикатів, компараторна ідентифікація.

Формальная база математического аппарата теории интеллекта

Кудхаир Абед Тамер

Цель Главная задача теории интеллекта – математически описать законы, регулирующие интеллектуальную деятельность человека. Для этого необходимо получить физические и объективные методы получения формального описания субъективных состояний человека, достаточно полных для практических целей. Человеческие мысли, чувства, восприятие и осознание - это все субъективные состояния. В этой статье поставлена задача разработать многомерную предикатную модель компараторной идентификации – основного экспериментального метода теории интеллекта, и обосновать аксиоматику этой модели. **Методы.** Метод компараторной идентификации, разработанный в данной статье, дает возможность получить объективное знание субъективных состояний человеческого интеллекта. По методу компараторной идентификации с его поведением субъект реализует некоторое конечное предикат, свойства которого экспериментально изучены и математически описаны. Метод компараторной идентификации основан на методах алгебры конечных предикатов, булевой алгебры и аксиоматическом методе. **Результаты.** Применение метода компараторной идентификации дает математическое описание исследуемых субъективных состояний человека, а также вид функции, лежащий в основе преобразования физических предметов в порождаемые ей субъективные образы. **Выводы.** Результаты работы математически обосновывают возможности применения метода компараторной идентификации при моделировании интеллекта человека.

Ключевые слова: теория интеллекта, алгебра конечных предикатов, компараторная идентификация.