UDC 004.9 DOI: https://doi.org/10.20998/2522-9052.2021.2.11

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STRUCTURAL OPTIMIZATION IN A MULTI-CHANNEL DISTRIBUTED MASS SERVICE SYSTEM

Abstract. Problem of structural optimization in a distributed service system is solved by the example of system "Production - delivery - consumption" for mass market product. In this regard, the purpose of work is to develop a method for structural optimization of "Production - delivery - mass consumption" system, by introducing and rational placement of intermediate production points based on solving clustering problems with taking into account the peculiarities of calculating distances between city objects. To achieve the goal of the work, it is necessary to solve the following tasks: clustering of city objects, using the metric of city blocks, for a given number of groups for selected location of production and grouping centers; finding the best location for a given number of clustering centers; determination of a rational number of clustering centers. Task was solved in three stages. First stage - clustering a set of consumption objects for given intermediate delivery centers locations. The second stage - finding the best locations for a given number of intermediate delivery centers. The third stage - determination of the rational number of intermediate centers. Formulated problem is solved according to two criteria: combined length of delivery routes product consumers and the probability that a random delivery time exceeds the critical value. The numerical value of the second criterion is calculated on the assumption that for each path may be estimated value of the mean and variance delivery time. The appropriate number of production centers is determined by a simple comparison of system efficiency for several realistically possible options. An example of clustering problem solving in the metric of "city blocks" on a directed graph by both criteria is given.

Keywords: distributed system "production - delivery - consumption"; clustering; directed graph; shortest path.

Introduction

Consider one of the many, important in practical sense, management tasks, come up in system "production - delivery - consumption". Let the city has l production centers of some consumer product. This product must be delivered to n its consumers. The natural desire to make this system as efficient as possible leads to the expediency of dividing the entire set of objects into groups, each of them can be serviced by the "nearest" production center.

This problem is solved by the methods of cluster analysis [1-4].

Cluster analysis is one of the central positions among data analysis techniques and is a set of methods and algorithms, designed to find some partition of the studied set of objects into subsets similar objects. In this case, the following requirements are usually imposed on the clustering results:

– each cluster must contain objects with similar values of properties or attributes;

– the set of all clusters must be exhaustive, that is, contain all objects of the studied population;

– none of the objects should belong to different clusters at the same time.

The position of each object is specified, in the simplest particular case, a point in a two-dimensional Cartesian coordinate system by vector $X^T = (x_1, x_2)$. Distance between a pair of points x_1 and x_2 measured, for example, in the Euclidean metric

$$R(x_1, x_2) = \sqrt{(x_{11} - x_{21})^2 + (x_{12} - x_{22})^2} . \qquad (1)$$

In practice, other measures of points proximity are also used in [5, 6]:

– metric of "city blocks"

$$R(x_1, x_2) = |x_{11} - x_{21}| + |x_{12} - x_{22}| , \qquad (2)$$

– metric P.L. Chebyshev

$$R(x_1, x_2) = \max\{|x_{11} - x_{21}| + |x_{12} - x_{22}|\} , \qquad (3)$$

– Minkowski metric

$$R(x_1, x_2) = \left((x_{11} - x_{21})^P + (x_{12} - x_{22})^P\right)^{1/q} . \qquad (4)$$

Metric (4) in special case when $p = q$, is converted to metric (2). But if $p = 2$, then the Euclidean metric will be obtained (1). Solving a specific clustering problem for each of points to be grouped using one of the formulas (1) – (4), found distance to the grouping centers is calculated and selected closest of them.

Returning to formulated task of structural organization in system "production - delivery - consumption", it should be noted that this task has a fundamental feature that distinguishes it from standard, traditional grouping tasks.

Difference lies in the method of distance calculating between grouping objects, and the distance for any two points of the city can not be correctly calculated in any of metrics shown above. It is determined by total length of the route, consisting from relevant sections of urban highways.

In this regard, **the purpose of work** is to develop a method for structural optimization of "Production - delivery - mass consumption" system, by introducing and rational placement of intermediate production points based on solving clustering problems with taking into account the peculiarities of calculating distances between city objects.

Formulation of the problem.

To achieve the goal of the work, it is necessary to solve the following tasks:

– clustering of city objects, using the metric of city blocks, for a given number of groups for selected location of production and grouping centers;

– finding the best location for a given number of clustering centers;

– determination of a rational number of clustering centers.

Main results

Let's start by solving the first task from the list. An adequate mathematical model of this problem can be obtained using the graph, containing a set of vertices and arcs connecting these vertices. Moreover, for an arc connecting directly a pair of vertices i and j, its length is indicated by r_{ij}, the totality of which all the arcs of the graph shows the matrix $R^{(1)} = \left(r_{ij}\right)$. In solving the clustering problem of vertices there is a need to calculate the distance between any pair of vertices. To calculate these distances, it is natural to use algorithm Floyd - Warshall [7], the essence of which is as follows. If the transition from the top i to the top j possible through one of the many intermediate vertices $k \in \{1, 2, ..., l\}$, then shortest route from i to j determined by relation

$$r_{ij} = \min_{k} \left\{ \begin{matrix} r_{i1} + r_{1j},\ r_{i2} + r_{2j}, ..., \\ r_{ik} + r_{kj}, ..., r_{ie} + r_{ej} \end{matrix} \right\}. \qquad (5)$$

Let introduce a matrix generalization of relation (5):

$$R^{(2)} = R^{(1)} \otimes R^{(1)}, \qquad (6)$$

where the operation $\otimes$ implements calculation by formula (5), i.e

$$r_{ij}^{(2)} = \min_{k} \left\{ \begin{matrix} r_{i1}^{(1)} + r_{1j}^{(1)},\ r_{i2}^{(1)} + r_{2j}^{(1)}, ..., \\ r_{ik}^{(1)} + r_{kj}^{(1)}, ...,\ r_{ie}^{(1)} + r_{ej}^{(1)} \end{matrix} \right\}. \qquad (7)$$

The matrix $R^{(2)} = \left(r_{ij}^{(2)}\right)$ defines the set of shortest two-step paths between the graph vertices. Further, to find the matrix of shortest three-step paths, we use a matrix relation similar to (6)

$$R^{(3)} = R^{(1)} \otimes R^{(2)}, \qquad (8)$$

where

$$r_{ij}^{(3)} = \min_{k} \left\{ \begin{matrix} r_{i1}^{(1)} + r_{1j}^{(2)},\ r_{i2}^{(1)} + r_{2j}^{(2)}, ..., \\ r_{ik}^{(1)} + r_{kj}^{(2)}, ...,\ r_{ie}^{(1)} + r_{ej}^{(2)} \end{matrix} \right\}. \qquad (9)$$

Note that relation (9) essentially embodies the well-known principle of dynamic programming [8], according to which the optimal multistep control is determined by the best choice of options set: { initial step + optimal continuation of state, occurring after the initial step }.

Consider an example of introduced relations using to solve the following simple clustering problem. Let the locations of two certain production centers, of a product, and nine centers of its consumption be determined in the city, as well as the highways of the city along with their intersections (Fig. 1).

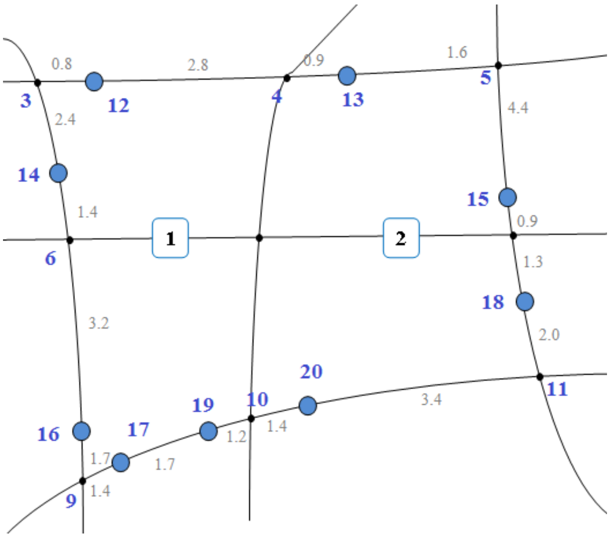

Fig. 1. Transport network configuration
(2 suppliers, 9 consumers)

The collection of all these points defines a graph with 20 vertices along with a set of arcs, connecting some of them. The lengths of these arcs are defined by the matrix (Tabl. 1). In this matrix the symbol *M*, located in i-th line and j-th column, corresponds to a situation where points i and j are not interconnected directly.

Table 1 – **Matrix of distances between transport network points**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	M	M	M	M	1,5	1,5	M	M	M	M	M	M	M	M	M	M	M	M	M
2	M	0	M	M	M	M	1,4	1,6	M	M	M	M	M	M	M	M	M	M	M	M
3	M	M	0	3,6	M	3,8	M	M	M	M	M	0,8	M	2,4	M	M	M	M	M	M
4	M	M	4,2	0	2,3	M	4,1	M	M	M	M	2,8	0,9	M	M	M	M	M	M	M
5	M	M	M	2,3	0	M	M	5,1	M	M	M	M	1,6	M	4,4	M	M	M	M	M
6	M	M	3,7	M	M	0	3,1	M	4,9	M	M	M	M	1,4	M	3,2	M	M	M	M
7	1,5	1,4	M	4,1	M	3,1	0	3,0	M	4,2	M	M	M	M	M	M	M	M	M	M
8	M	1,6	M	M	5,1	M	3,0	0	M	M	3,3	M	M	M	0,9	M	M	1,3	M	M

End of Table 1

9	M	M	M	M	M	4,9	M	M	0	4,3	M	M	M	M	M	1,7	1,4	M	2,9	M
10	M	M	M	M	M	M	4,2	M	4,3	0	5,1	M	M	M	M	M	3,0	M	1,2	1,4
11	M	M	M	M	M	M	M	3,3	M	5,1	0	M	M	M	M	M	M	2,0	M	3,4
12	M	M	0,8	2,8	M	M	M	M	M	M	M	0	M	M	M	M	M	M	M	M
13	M	M	M	0,9	0,9	1,6	M	M	M	M	M	M	0	M	3,3	M	M	M	M	M
14	M	M	2,4	M	1,4	M	M	M	M	M	M	M	M	0	M	M	M	M	M	M
15	M	M	M	M	4,4	M	M	0,9	M	M	M	M	3,3	M	0	M	M	M	M	M
16	M	M	M	M	M	3,2	M	M	1,7	M	M	M	M	M	M	0	M	M	M	M
17	M	M	M	M	M	M	M	M	1,4	2,7	M	M	M	M	M	M	0	M	1,7	3,8
18	M	M	M	M	M	M	M	1,3	M	M	2,0	M	M	M	M	M	M	0	M	M
19	M	M	M	M	M	M	1,4	M	M	1,2	M	M	M	M	M	M	1,7	M	0	M
20	M	M	M	M	M	M	M	M	M	1,4	3,4	M	M	M	M	M	M	M	M	0

We calculate the matrix of two-step shortest paths between points using (7). We have:

$$r_{12}^{(2)} = \min\left\{r_{16}^{(1)} + r_{62}^{(1)},\ r_{17}^{(1)} + r_{72}^{(1)}\right\} =$$
$$= \min\{1.5 + M,\ 1.5 + 1.4\} = 2.9;$$

$$r_{13}^{(2)} = \min\left\{r_{16}^{(1)} + r_{63}^{(1)},\ r_{17}^{(1)} + r_{73}^{(1)}\right\} =$$
$$= \min\{1.5 + 3.8,\ 1.5 + M\} = 5.3;$$

$$r_{14}^{(2)} = \min\left\{r_{16}^{(1)} + r_{64}^{(1)},\ r_{17}^{(1)} + r_{74}^{(1)}\right\} =$$
$$= \min\{1.5 + M,\ 1.5 + 4.1\} = 5.6;$$

$r_{15}^{(2)} = M$, since there is no two-step path between points 1 and 5,

$$r_{16}^{(2)} = r_{16}^{(1)} = 1.5,\ r_{17}^{(2)} = r_{17}^{(1)} = 1.5;$$

$$r_{18}^{(2)} = \min\left\{r_{16}^{(1)} + r_{68}^{(1)},\ r_{17}^{(1)} + r_{78}^{(1)}\right\} =$$
$$= \min\{1.5 + M,\ 1.5 + 3.0\} = 4.5;$$

$$r_{19}^{(2)} = \min\left\{r_{16}^{(1)} + r_{69}^{(1)},\ r_{17}^{(1)} + r_{79}^{(1)}\right\} =$$
$$= \min\{1.5 + 4.9,\ 1.5 + M\} = 6.4;$$

$$r_{1,10}^{(2)} = \min\left\{r_{16}^{(1)} + r_{6,10}^{(1)},\ r_{17}^{(1)} + r_{7,10}^{(1)}\right\} =$$
$$= \min\{1.5 + M,\ 1.5 + 4.2\} = 5.7;$$

$r_{1,11}^{(2)} = r_{1,12}^{(2)} = r_{1,13}^{(2)} = M$, since there are no corresponding two-step paths;

$$r_{1,14}^{(2)} = \min\left\{r_{16}^{(1)} + r_{6,14}^{(1)},\ r_{17}^{(1)} + r_{7,14}^{(1)}\right\} =$$
$$= \min\{1.5 + 1.4,\ 1.5 + M\} = 2.9;$$

$$r_{15}^{(2)} = M;$$

$$r_{1,16}^{(2)} = \min\left\{r_{16}^{(1)} + r_{6,16}^{(1)},\ r_{17}^{(1)} + r_{7,16}^{(1)}\right\} =$$
$$= \min\{1.5 + 3.2,\ 1.5 + M\} = 4.7;$$

$r_{1,17}^{(2)} = r_{1,18}^{(2)} = r_{1,18}^{(2)} = M$, since there are no corresponding two-step paths;

$$r_{21}^{(2)} = \min\left\{r_{27}^{(1)} + r_{71}^{(1)},\ r_{28}^{(1)} + r_{81}^{(1)}\right\} =$$
$$= \min\{1.4 + 1.5,\ 1.6 + M\} = 2.9;$$

$$r_{23}^{(2)} = M;$$

$$r_{24}^{(2)} = \min\left\{r_{27}^{(1)} + r_{74}^{(1)},\ r_{28}^{(1)} + r_{84}^{(1)}\right\} =$$
$$= \min\{1.4 + 4.1,\ 1.6 + M\} = 5.5;$$

$$r_{25}^{(2)} = \min\left\{r_{27}^{(1)} + r_{75}^{(1)},\ r_{28}^{(1)} + r_{85}^{(1)}\right\} =$$
$$= \min\{1.4 + M,\ 1.6 + 5.3\} = 6.9;$$

$$r_{26}^{(2)} = \min\left\{r_{27}^{(1)} + r_{76}^{(1)},\ r_{28}^{(1)} + r_{86}^{(1)}\right\} =$$
$$= \min\{1.4 + 3.0,\ 1.6 + M\} = 4.4;$$

$$r_{27}^{(2)} = r_{27}^{(1)} = 1.4,\quad r_{28}^{(2)} = r_{28}^{(1)} = 1.6,\quad r_{29}^{(2)} = M;$$

$$r_{2,10}^{(2)} = \min\left\{r_{27}^{(1)} + r_{7,10}^{(1)},\ r_{28}^{(1)} + r_{8,10}^{(1)}\right\} =$$
$$= \min\{1.4 + 4.2,\ 1.6 + M\} = 5.6;$$

$$r_{2,11}^{(2)} = \min\left\{r_{27}^{(1)} + r_{7,11}^{(1)},\ r_{28}^{(1)} + r_{8,11}^{(1)}\right\} =$$
$$= \min\{1.4 + M,\ 1.6 + 3.3\} = 4.9;$$

$r_{2,12}^{(2)} = r_{2,13}^{(2)} = r_{2,14}^{(2)} = M$, since there are no corresponding two-step paths;

$$r_{2,15}^{(2)} = \min\left\{r_{27}^{(1)} + r_{7,15}^{(1)},\ r_{28}^{(1)} + r_{8,15}^{(1)}\right\} =$$
$$= \min\{1.4 + M,\ 1.6 + 0.9\} = 2.5;$$

$$r_{2,16}^{(2)} = r_{2,17}^{(2)} = M;$$

$$r_{2,18}^{(2)} = \min\left\{r_{27}^{(1)} + r_{7,18}^{(1)},\ r_{28}^{(1)} + r_{8,18}^{(1)}\right\} =$$
$$= \min\{1.4 + M,\ 1.6 + 1.3\} = 2.9;$$

$r_{2,19}^{(2)} = r_{2,20}^{(2)} = M$, since there are no corresponding two-step paths.

Continuing similarly for the remaining possible paths, we fill in the matrix $R^{(2)}$ (Table 2).

Table 2 – **Matrix of the lengths for shortest two-step paths**

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0	2,9	5,3	5,6	6,1	1,5	1,5	4,5	6,4	5,7	M	M	M	2,9	M	4,7	M	M	M	M
2	2,9	0	M	5,5	6,9	4,4	1,4	1,6	M	5,6	4,9	M	M	M	2,5	M	M	2,9	M	M
3	5,3	M	0	3,6	6,7	3,8	6,8	M	8,7	M	M	0,8	4,5	2,4	M	7,0	M	M	M	M
4	5,6	5,5	3,6	0	2,5	8,0	4,1	7,8	M	8,3	M	2,8	0,9	6,0	6,9	M	M	M	M	M
5	6,1	6,9	6,7	2,5	0	M	6,6	5,3	M	M	8,6	5,3	1,6	M	4,4	M	M	6,6	M	M
6	1,5	4,4	3,8	8,0	M	0	3,0	6,0	4,9	7,2	M	4,6	M	1,4	M	3,2	6,3	M	8,0	M
7	1,5	1,4	6,8	4,1	6,6	3,0	0	3,0	7,9	4,2	6,3	6,9	5,0	4,4	3,9	6,2	7,1	4,3	5,4	5,6
8	4,5	1,6	M	7,9	5,3	6,0	3,0	0	M	7,2	3,3	M	6,9	M	0,9	M	M	1,3	M	6,7
9	6,4	M	8,7	M	M	4,9	7,9	M	0	4,3	9,1	M	M	6,3	M	1,7	1,4	M	3,1	5,7
10	5,7	5,6	M	8,3	M	7,2	4,2	7,2	4,3	0	4,8	M	M	M	M	6,0	2,9	6,8	1,2	1,4
11	M	4,9	M	M	8,6	M	6,3	3,3	9,1	4,8	0	M	M	M	4,2	M	7,7	2,0	6,0	3,4
12	M	M	0,8	2,8	5,3	4,6	6,9	M	M	M	M	0	3,7	3,2	M	M	M	M	M	M
13	M	M	4,5	0,9	1,6	M	5,0	6,9	M	M	M	3,7	0	M	6,0	M	M	M	M	M
14	2,9	M	2,4	6,0	M	1,4	4,4	M	6,3	M	M	3,2	M	0	M	4,6	M	M	M	M
15	M	2,5	M	6,9	4,4	M	3,9	0,9	M	M	4,2	M	6,0	M	0	M	M	2,2	M	M
16	4,7	M	7,0	M	M	3,2	6,0	M	1,7	6,0	M	M	M	4,6	M	0	3,1	M	4,8	M
17	M	M	M	M	M	6,3	7,1	M	1,4	2,9	7,7	M	M	M	M	3,1	0	M	1,7	4,3
18	M	2,9	M	M	6,6	M	4,3	1,3	M	6,8	2,0	M	M	M	2,2	M	M	0	M	5,4
19	M	M	M	M	M	8,0	5,4	M	3,1	1,2	6,0	M	M	M	M	4,8	1,7	M	0	2,6
20	M	M	M	M	M	M	5,6	6,7	5,7	1,4	3,4	M	M	M	M	M	4,3	5,4	2,6	0

From the resulting matrix analysis it follows, that using of shortest two-step paths from production points (№1 and №2) most consumption points (12-20) is unattainable.

Continuing of procedure, we calculate three-step matrix paths using the formula (9).

We get:

$$r_{12}^{(3)} = r_{12}^{(2)}, \quad r_{13}^{(3)} = r_{13}^{(2)},$$

$$r_{14}^{(3)} = r_{14}^{(2)}, \quad r_{15}^{(3)} = r_{15}^{(2)},$$

$$r_{16}^{(3)} = r_{16}^{(2)}, \quad r_{17}^{(3)} = r_{17}^{(2)},$$

$$r_{18}^{(3)} = r_{18}^{(2)}, \quad r_{19}^{(3)} = r_{19}^{(2)}, \quad r_{1,10}^{(3)} = r_{1,10}^{(2)},$$

$$r_{1,11}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,11}^{(2)},\ r_{17}^{(1)} + r_{7,11}^{(2)}\right\} = \\ = \min\{1.5 + M,\ 1.5 + 6.3\} = 7.8,$$

$$r_{1,11}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,11}^{(2)},\ r_{17}^{(1)} + r_{7,11}^{(2)}\right\} = \\ = \min\{1.5 + M,\ 1.5 + 6.3\} = 7.8,$$

$$r_{1,12}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,12}^{(2)},\ r_{17}^{(1)} + r_{7,12}^{(2)}\right\} = \\ = \min\{1.5 + 4.6,\ 1.5 + 6.9\} = 6.1,$$

$$r_{1,13}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,13}^{(2)},\ r_{17}^{(1)} + r_{7,13}^{(2)}\right\} = \\ = \min\{1.5 + M,\ 1.5 + 5.0\} = 6.5,$$

$$r_{1,15}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,15}^{(2)},\ r_{17,15}^{(1)} + r_{7,15}^{(2)}\right\} = \\ = \min\{1.5 + M,\ 1.5 + 3.9\} = 5.4,$$

$$r_{1,16}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,16}^{(2)},\ r_{17}^{(1)} + r_{7,16}^{(2)}\right\} = \\ = \min\{1.5 + 3.2,\ 1.5 + 6.2\} = 4.7,$$

$$r_{1,17}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,17}^{(2)},\ r_{17}^{(1)} + r_{7,17}^{(2)}\right\} = \\ = \min\{1.5 + 6.3,\ 1.5 + 7.1\} = 7.8,$$

$$r_{1,18}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,18}^{(2)},\ r_{17}^{(1)} + r_{7,18}^{(2)}\right\} = \\ = \min\{1.5 + M,\ 1.5 + 4.3\} = 5.8,$$

$$r_{1,19}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,19}^{(2)},\ r_{17}^{(1)} + r_{7,19}^{(2)}\right\} \\ = \min\{1.5 + 8.0,\ 1.5 + 5.4\} = 6.9,$$

$$r_{1,20}^{(3)} = \min\left\{r_{16}^{(1)} + r_{6,20}^{(2)},\ r_{17}^{(1)} + r_{7,20}^{(2)}\right\} \\ = \min\{1.5 + M,\ 1.5 + 5.6\} = 7.1.$$

Continuing in the same way, we get:

$$r_{2,12}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,12}^{(2)},\ r_{28}^{(1)} + r_{8,12}^{(2)}\right\}$$
$$= \min\{1.4 + 6.9,\ 1.6 + M\} = 8.3,$$

$$r_{2,13}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,13}^{(2)},\ r_{28}^{(1)} + r_{8,13}^{(2)}\right\}$$
$$= \min\{1.4 + 5.0,\ 1.6 + 6.9\} = 6.4,$$

$$r_{2,14}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,14}^{(2)},\ r_{28}^{(1)} + r_{8,14}^{(2)}\right\}$$
$$= \min\{1.4 + 4.4,\ 1.6 + M\} = 5.8,$$

$$r_{2,15}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,15}^{(2)},\ r_{28}^{(1)} + r_{8,15}^{(2)}\right\}$$
$$= \min\{1.4 + 3.9,\ 1.6 + 0.9\} = 2.5,$$

$$r_{2,16}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,16}^{(2)},\ r_{28}^{(1)} + r_{8,16}^{(2)}\right\}$$
$$= \min\{1.4 + 6.2,\ 1.6 + M\} = 7.6,$$

$$r_{2,17}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,17}^{(2)},\ r_{28}^{(1)} + r_{8,17}^{(2)}\right\}$$
$$= \min\{1.4 + 7.1,\ 1.6 + M\} = 8.5,$$

$$r_{2,18}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,18}^{(2)},\ r_{28}^{(1)} + r_{8,18}^{(2)}\right\}$$
$$= \min\{1.4 + 4.3,\ 1.6 + 1.3\} = 2.9,$$

$$r_{2,19}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,19}^{(2)},\ r_{28}^{(1)} + r_{8,19}^{(2)}\right\}$$
$$= \min\{1.4 + 5.4,\ 1.6 + M\} = 6.8,$$

$$r_{2,20}^{(3)} = \min\left\{r_{27}^{(1)} + r_{7,20}^{(2)},\ r_{28}^{(1)} + r_{8,20}^{(2)}\right\}$$
$$= \min\{1.4 + 5.6,\ 1.6 + 6.7\} = 7.0.$$

Performed calculations show us that for each of consumption points there is exists and is found the shortest three-step route from each of production centers.

The respective paths lengths are calculated, summarized in Table 3 and can be used for comparison for clustering purposes.

Table 3 – **Lengths of the shortest routes from production centers to consumption points**

Production centers	Consumption points								
	12	13	14	15	16	17	18	19	20
1	6,1	6,5	2,9	5,4	4,7	7,8	5,8	6,9	7,1
2	8,3	6,4	5,8	2,5	7,6	8,5	2,9	6,8	7,0

Distances comparison from production centers (1 and 2) to delivery points (12 - 20) makes it possible to form clusters as follows:

Cluster 1: points 12, 14, 16, 17,

Cluster 2: points 13, 15, 18, 19, 20.

Solution obtained.

In delivery system of perishable products (or products of immediate use), another criterion is more important than distance to the delivery points - delivery time. The values calculation of this parameter must be carried out taking uncertainty into account, arising due to differences in some section's throughput of highways, dynamics of the flow density for transport units, road surface quality, depending on weather conditions, time of day, etc. In suppose that according to results of preliminary processing of the corresponding data for all transport network sections, shown in Fig. 1, average values of the duration in overcoming these sections and their variance have been determined. Then, using this data for any route, the probability that random duration of the delivery time will exceed a critical value can be calculated. Moreover, if for some specific route, the average time to overcome it is m, and the variance is $-\sigma^2$, then, assuming a normal distribution of this time, probability of exceeding the critical value is calculated by formula

$$P\left(T \geq T_{кр}\right) = \int_{T_{кр}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} - \exp\left\{-\frac{(T-m)^2}{\sum \sigma^2}\right\} dT .$$

Calculation results are summarized in Table 4.

Table 4 – **Probabilities of critical value exceeding by the random delivery time**

Production centers	Consumption points								
	12	13	14	15	16	17	18	19	20
1	0,45	0,41	0,16	0,47	0,43	0,61	0,44	0,47	0,51
2	0,72	0,54	0,51	0,13	0,68	0,79	0,19	0,59	0,68

Data comparison presented in table 4 leads to the following clustering of consumption points:

Cluster 1: points 12, 13, 14, 16, 17, 19, 20,

Cluster 2: points 15, 18.

Thus, the result of clustering by the time criterion is radically different from the previous one, obtained taking into account only the distances between the centers of production and points of consumption.

The considered clustering problem becomes more complicated if the proximity measures of objects are determined indistinctly [9-12]. Application methods of fuzzy mathematics in this case, is absolutely justified, since practical tasks, similar to the above, are solved in a situation where actual available initial data is insufficient to obtain a correct theoretical and probabilistic description of them. Methods of the fuzzy mathematics theory are less demanding and are better suited for constructing adequate mathematical models in a small sample of these data, since they do not need to correctly reconstruct the unknown distribution density of the corresponding random variables. Consider a possible approach to clustering problem solving in terms of fuzzy mathematics. We will assume that the duration measures of overcoming network sections are determined by fuzzy numbers (L-R) type with their own membership functions. Consider the technology for fuzzy value calculating of the integral metric for a fragment of a route composed from two consecutive sections. We introduce two corresponding fuzzy numbers ղ1 ղ2 @(L-R) type, that determine performance of these two sections in the network:

$$\eta_1 = \langle m_1, \alpha_1, \beta_1 \rangle,\ \eta_2 = \langle m_2, \alpha_2, \beta_2 \rangle. \qquad (7)$$

Here: m_1, m_2 – modal values of the numbers F1 and F2, α_1, α_2 –left fuzziness coefficients values of the membership functions in numbers F1 and F2 ; β_1, β_2 - right fuzziness coefficients values of the membership functions in numbers F1 and F2.

To calculate integral metric that determines the level of preference for a network fragment, consisting of given two sections, it is necessary to define the rules for performing the following operations on fuzzy numbers (L-R) type: addition, multiplication, selection of the minimum value, division. We define these rules by formulas justified in [13]:

– addition:

$$\begin{gathered}\langle m, \alpha, \beta \rangle = \langle m_1, \alpha_1, \beta_1 \rangle + \langle m_2, \alpha_2, \beta_2 \rangle, \\ m = m_1 + m_2;\ \alpha = \alpha_1 + \alpha_2;\ \beta = \beta_1 + \beta_2;\end{gathered} \qquad (8)$$

– multiplication

$$\begin{gathered}\langle m, \alpha, \beta \rangle = \langle m_1, \alpha_1, \beta_1 \rangle \cdot \langle m_2, \alpha_2, \beta_2 \rangle, \\ m = m_1 \cdot m_2;\ \alpha = m_1\alpha_2 + m_2\alpha_1 - \alpha_1\alpha_2; \\ \beta = m_1\beta_2 + m_2\beta_1 + \beta_1\beta_2;\end{gathered} \qquad (9)$$

– division

$$\begin{gathered}\langle m, \alpha, \beta \rangle = \langle m_1, \alpha_1, \beta_1 \rangle : \langle m_2, \alpha_2, \beta_2 \rangle, \\ m = \frac{m_1}{m_2};\ \alpha = \frac{m_2\alpha_1 + m_1\beta_2}{m_2(m_2 + \beta_2)};\ \beta = \frac{m_1\alpha_2 + m_2\beta_1}{m_2(m_2 - \alpha_2)};\end{gathered} \qquad (10)$$

– selection of the minimum value

$$\langle m, \alpha, \beta \rangle = \min\{\langle m_1, \alpha_1, \beta_1 \rangle, \langle m_2, \alpha_2, \beta_2 \rangle\}.$$

A natural and easily interpreted rule for choosing the lesser of two numbers is formulated as follows: if the difference between these two numbers is positive, then the subtracted is the smaller; if the difference between these numbers is negative, the minuend is smaller..

Define a rule perform the subtraction operation:

$$\begin{gathered}\langle m, \alpha, \beta \rangle = \langle m_1, \alpha_1, \beta_1 \rangle - \langle m_2, \alpha_2, \beta_2 \rangle, \\ m = m_1 - m_2;\ \alpha = \alpha_1 + \beta_2;\ \beta = \beta_1 + \alpha_2.\end{gathered} \qquad (11)$$

Thus, the problem of determining the smaller of two fuzzy numbers is reduced to analyzing subtraction result. In accordance with this, a simple and understandable rule for comparing two fuzzy numbers is formulated as follows.

For numbers being compared η_1 и η_2 calculate values of the left (b_1, b_2) and right (c_1, c_2) their carriers borders:

$$\begin{gathered}b_1 = m_1 - \alpha_1,\ c_1 = m_1 + \beta_1, \\ b_2 = m_2 - \alpha_2,\ c_2 = m_2 + \beta_2.\end{gathered}$$

Now comparing rule is formulated as follows:

a) if $\min\{(b_1 - b_2), (c_1 - c_2)\} > 0$, then $F_1 > F_2$,

b) if $\max\{(b_1 - b_2), (c_1 - c_2)\} < 0$, then $F_1 < F_2$,

c) if $\min\{(b_1 - b_2), (c_1 - c_2)\} < 0$ and $|\min\{(b_1 - b_2), (c_1 - c_2)\}| > \max\{(b_1 - b_2), (c_1 - c_2)\}$, then $F_1 < F_2$,

d) if $\min\{(b_1 - b_2), (c_1 - c_2)\} < 0$ and $|\min\{(b_1 - b_2), (c_1 - c_2)\}| < \max\{(b_1 - b_2), (c_1 - c_2)\}$, then $F_1 > F_2$.

The above ratios provide a calculation by the formula (7) values of fuzzy measures of the distance for each point to the clusters centers. These values are used when performing clustering procedure.

Problem of clustering is even more problematic if the initial data are specified inaccurately in the sense of Pavlak. [14-16]. Real way to solve problem in this case is to use the method of constructing fuzzy models for objects that are determined inaccurately [17].

Method for solving the second problem for a given number l clusters uses an iterative procedure consisting of preliminary and subsequent stages. At the preliminary stage, the initial location of grouping centers is found. To do this, a Cartesian coordinate system is applied to the city map so that the abscissa of the leftmost of the grouping objects and the ordinate of the lowest of the objects are equal to zero. Then a rectangle is constructed, the lower left vertex of which is placed at the origin. (0,0), and the upper right vertex is chosen so that all the objects of the city lie within this rectangle, and he had a minimum area. Now we find the center of the rectangle at the intersection of its diagonals, from which we draw l rays with an angle $\alpha = 360^\circ / l$ between them. Then we find point of rays intersection with the rectangular sides and segments midpoints formed in this case. The resulting points are used to determine the initial grouping centers by finding the nearest point on the nearest highway. Each of the subsequent iterations consists of two steps. At the first step, according to the method described above, clustering problem is solved. According to results of this task is calculated clustering quality criterion, equal to sum of the distances from grouping objects to corresponding grouping centers.

Let be:

(X_{KO}, Y_{KO}) – grouping center coordinates K -th cluster, $K = 1, 2, ..., l$,

(X_{KS}, Y_{KS}) – coordinates S -th object from K -th cluster.

Then

$$\eta_K = \sum_{S \in N_K} \sqrt{(X_{K0} - X_{KS})^2 + (Y_{K0} - Y_{KS})^2} \quad -(12)$$

sum of distances from grouping center K -th cluster to objects of this cluster, $S \in N_K$,

N_K - many numbers of objects that fall into the K-th cluster;

$$\eta = \sum_{K=1}^{l} \eta_K \quad - \tag{13}$$

sum of all routes lengths, which were obtained as a result of clustering.

It is clear that criterion (13) value is lower, the higher quality of clustering. In the second step, the grouping results are used to correct the positions of the grouping centers. To do this, within each cluster, it is necessary to find a point, the sum of distances from which to the objects of this cluster is minimal. This task can be solved as follows. From center of the cluster will hold K лучей rays (for example $K = 4$) with an angle between them $\beta = 360^\circ / K$. Then, for each of objects in each of resulting sectors, we find the shortest distance to grouping center (by formula (7), we sum them up and among the resulting sums we find the greatest. Correction of cluster grouping center position consists in its movement along bisector of the sector with greatest sum of distances to intersection with nearest highway. New positions of the grouping centers obtained in this case are used at the first step of next iteration. Effectiveness of this clustering is estimated by formulas (12), (13). The procedure continues if new value of criterion (13) is better than the previous one. As a result of solving this problem, the best position of clustering centers for a given number of clusters will be obtained.

Finally, let's move on to the third task. Simplest for implementation and a reliable way to solve it is to solve the complex of first and second problems sequentially, first for two clusters, then for three, etc. Comparison of criterion calculated values (13) for these options provides choice of the best.

Conclusions

1. Problem of structural optimization in the distributed system "production - delivery - consumption" is considered and solved. The problem is solved in three stages.

2. At the first stage, problem of distributing a set of consumers into clusters was solved. Optimization criterion - is total length of delivery routes in the city blocks metric. To solve the problem, proposed method that implements the technology of dynamic programming.

The method is generalized for case when the initial data is not clearly specified. An alternative approach to solving problem of clustering according to criterion is also considered - minimum probability that the delivery time will exceed a given threshold value.

3. At the second stage, problem of rational location finding a for a given number of production points was solved. To solve that problem, a procedure for sequential improvement of initial plan is implemented.

The appropriate number of production centers is determined by a simple comparison of system efficiency for several realistically possible options.

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Received (Надійшла) 26.02.2021
Accepted for publication (Прийнята до друку) 19.05.2021

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Структурна оптимізація у багатоканальній розподіленій системі масового обслуговування

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Анотація. Задача структурної оптимізації в розподіленій системі обслуговування вирішена на прикладі системи «виробництво - доставка - споживання» продукту масового попиту. У зв'язку з цим метою роботи є розробка методики структурної оптимізації системи «Виробництво - доставка - масове споживання» шляхом впровадження і раціонального розміщення проміжних точок виробництва на основі рішення задач кластеризації з урахуванням особливостей розрахунку відстаней. між міськими об'єктами. Постановка задачі. Для досягнення мети роботи вирішені наступні завдання: кластеризація міських об'єктів по метриці міських кварталів по заданій кількості груп для обраного розташування виробничих і групових центрів; пошук найкращого місця розташування для заданої кількості центрів кластеризації; визначення оптимальної кількості центрів кластеризації. Задача вирішена в три етапи. Перший етап - кластеризація множини об'єктів споживання для заданих місць розташування проміжних центрів доставки. Другий етап - відшукання найкращих місць розташування заданої кількості проміжних центрів доставки. Третій етап - визначення оптимальної кількості проміжних центрів. Сформульована задача вирішується за двома критеріями: сумарна довжина маршрутів доставки продукту споживачам; вірогідність того, що випадковий час доставки перевищить критичне значення. Чисельне значення другого критерію обчислюється за припущенням, що для кожного маршруту можуть бути оцінені значення середнього значення і дисперсії часу доставки. Відповідна кількість виробничих центрів визначається простим порівнянням ефективності системи для декількох реально можливих варіантів. Наведено приклад рішення задачі кластеризації в метриці «міських кварталів» на орієнтованому графі за обома критеріями.

Ключові слова: розподілена система «виробництво – доставка - споживання»; кластерізація; орієнтований граф; найкоротший шлях.

Структурная оптимизация в многоканальной распределенной системе массового обслуживания

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Аннотация. Задача структурной оптимизации в распределенной системе обслуживания решена на примере системы «производство – доставка - потребление» продукта массового спроса. В связи с этим целью работы является разработка методики структурной оптимизации системы «Производство - доставка - массовое потребление» путем внедрения и рационального размещения промежуточных точек производства на основе решения задач кластеризации с учетом особенностей расчета расстояний. между городскими объектами. Постановка задачи. Для достижения цели работы решены следующие задачи: кластеризация городских объектов по метрике городских кварталов по заданному количеству групп для выбранного расположения производственных и групповых центров; поиск наилучшего местоположения для заданного количества центров кластеризации; определение рационального количества центров кластеризации. Задача решена в три этапа. Первый этап – кластеризация множества объектов потребления для заданных мест расположения промежуточных центров доставки. Второй этап – отыскание наилучших мест расположения заданного числа промежуточных центров доставки. Третий этап – определение рационального числа промежуточных центров. Сформулированная задача решается по двум критериям: суммарная длина маршрутов доставки продукта потребителям; вероятность того, что случайное время доставки превысит критическое значение. Численное значение второго критерия вычисляется в предположении, что для каждого маршрута могут быть оценены значения среднего значения и дисперсии времени доставки. Подходящее количество производственных центров определяется простым сравнением эффективности системы для нескольких реально возможных вариантов. Приведен пример решения задачи кластеризации в метрике «городских кварталов» на ориентированном графе по обоим критериям.

Ключевые слова: распределенная система «производство – доставка - потребление»; кластеризация; ориентированный граф; кратчайший путь.